Research Article

Properties of strong and complete Intuitionistic Fuzzy k-partite Hypergraphs

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Abstract: A k-partite hypergraph is a hypergraph whose vertices can be partitioned into k different independent sets. In this paper, operations on intuitionistic fuzzy k-partite hypergraph(IFk-PHG) are discussed and some properties were derived. The operations like union, intersection, join, structural subtraction, ringsum, product, cartesian product, composition and complement were discussed.

Keywords: Intuitionistic fuzzy k-partite hypergraph, Properties, Operations.

1. Introduction

Hypergraph is a generalization of graph theory which was originally developed by C. Berge in 1960. The notion of hypergraphs has been extended in fuzzy theory and the concept of fuzzy hypergraphs was proposed by Lee-Kwang and S. M. Chen. In **(Berge.C.1976)** the concepts of graph and hypergraph was introduced. **(Atanassov.K.T.1999)** introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. The notion of fuzzy graphs and fuzzy hypergraphs was developed in **(Mordeson.N.John, Nair.S.Premchand.2000)**.

Intuitionistic fuzzy graph, intuitionistic fuzzy hypergraph and its operations have been discussed in **(Parvathi.R et al.,2006, Parvathi.R et al.,2009, Parvathi.R et al.,2009, Parvathi.R et al.,2012 & Thilagavathi.S et al.,2008)**. **(Myithili.K.K., Parvathi.R., Akram.M.2014)** refined the ideas of intuitionistic fuzzy directed hypergraphs. Further in (**Myithili.K.K., Parvathi.R.2015)** operations for transversals of intuitionistic fuzzy directed hypergraphs was discussed.

Finally, in **(Myithili.K.K., Keerthika.R.2020)** the authors putforth the concepts of k-partite graphs in intuitionistic fuzzy hypergraphs. In this paper, the authors discussed about the operations on IFk-PHGs and worked on some of its results. Hence, operations like union, intersection, join, structural subtraction, ring sum, product, cartesian product, composition and complement were defined on IFk-PHG. Also it is proved that complement of a complete IFk-PHG is a complete IFk-PHG. Similarly, other properties has also been analysed and proved.

2. Preliminaries

In this section, basic definitions relating to intuitionistic fuzzy sets, intuitionistic fuzzy graphs, IFk-PHGs are dealt with.

Definition 2.1(Atanassov.K.T.1999) Let a set E be fixed. An *intuitionistic fuzzy set (IFS)* V in E is an object of the form $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i) \rangle / v_i \in E\}$, where the function $\mu_i : E \rightarrow [0,1]$ and $\nu_i : E \rightarrow [0,1]$ determine the degree of membership and the degree of non-membership of the element $v_i \in E$, respectively and for every $v_i \in E$, $0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$.

Definition 2.2(Parvathi.R et al.,2006) Let E be the fixed set and $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i) \rangle | v_i \in V\}$be an IFS. Six types of Cartesian products of n subsets (crisp sets) $V_1, V_2 ... V_n$ of V over E are defined as follows
$V_{i_1} \times_1 V_{i_2} \times_1 V_{i_3} ... \times_1 V_{i_n} = \{\langle (v_1, v_2 \cdots, v_n), \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \rangle | v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$,

$V_{i_1} \times_2 V_{i_2} \times_2 V_{i_3} \dots \times_2 V_{i_n} = \{\langle (v_1, v_2 \cdots, v_n), \sum_{i=1}^{n} \mu_i - \sum_{i \neq j} \mu_i \mu_j + \sum_{i \neq j \neq k} \mu_i \mu_j \mu_k - \cdots + (-1)^{n-2} \sum_{i \neq j \neq k \dots \neq n} \mu_i \mu_j \mu_k \dots \mu_n + (-1)^{n-1} \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \nu_i \rangle \mid v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$,

$V_{i_1} \times_3 V_{i_2} \times_3 V_{i_3} \dots \times_3 V_{i_n} = \{\langle (v_1, v_2 \cdots, v_n), \prod_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \nu_i - \sum_{i \neq j} \nu_i \nu_j + \sum_{i \neq j \neq k} \nu_i \nu_j \nu_k - \cdots + (-1)^{n-2} \sum_{i \neq j \neq k \dots \neq n} \nu_i \nu_j \nu_k \dots \nu_n + (-1)^{n-1} \prod_{i=1}^{n} \nu_i \rangle \mid v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$,

$V_{i_1} \times_4 V_{i_2} \times_4 V_{i_3} \dots \times_4 V_{i_n} = \{\langle (v_1, v_2 \cdots, v_n), \min(\mu_1, \mu_2, \dots, \mu_n), \max(\nu_1, \nu_2, \dots, \nu_n) \rangle \mid v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$,

$V_{i_1} \times_5 V_{i_2} \times_5 V_{i_3} \dots \times_5 V_{i_n} = \{\langle (v_1, v_2 \cdots, v_n), \max(\mu_1, \mu_2, \dots, \mu_n), \min(\nu_1, \nu_2, \dots, \nu_n) \rangle \mid v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$,

$V_{i_1} \times_6 V_{i_2} \times_6 V_{i_3} \dots \times_6 V_{i_n} = \left\{\langle (v_1, v_2 \cdots, v_n), \frac{\sum_{i=1}^{n} \mu_i}{n}, \frac{\sum_{i=1}^{n} \nu_i}{n} \rangle \mid v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\right\}$.

It must be noted that $v_i \times_s v_j$ is an IFS, where $s = 1, 2, 3, 4, 5, 6$.

Definition 2.3(Parvathi.R et al.,2006) An *intuitionistic fuzzy graph (IFG)* is of the form $G = \langle V, E \rangle$ where
(i) $V = \{v_1, v_2 \cdots, v_n\}$ such that $\mu_i : V \to [0,1]$ and $\nu_i : V \to [0,1]$ denote the degrees of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$ for every $v_i \in V, i = 1, 2, \cdots, n$
(ii) $E \subseteq V \times V$ where $\mu_{ij} : V \times V \to [0,1]$ and $\nu_{ij} : V \times V \to [0,1]$ are such that

$$\mu_{ij} \leq \mu_i \wedge \mu_j$$
$$\nu_{ij} \leq \nu_i \vee \nu_j \text{ and}$$
$$0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$$

where μ_{ij} and ν_{ij} are the membership and non-membership values of the edge (v_i, v_j); the values of $\mu_i \wedge \mu_j$ and $\nu_i \vee \nu_j$can be determined by one of its cartesian products $\times_s$, $s = 1, 2, \cdots, 6$ for all i and j given in above Definition.
Note: Throughout this paper, it is assumed that the fourth Cartesian product
$V_{i_1} \times_4 V_{i_2} \times_4 V_{i_3} \dots \times_4 V_{i_n} = \{\langle (v_1, v_2 \cdots, v_n), \min(\mu_1, \mu_2, \dots, \mu_n), \max(\nu_1, \nu_2, \dots, \nu_n) \rangle \mid v_1 \in V_1, v_2 \in V_2, \cdots, v_n \in V_n\}$,

is used to determine the edge membership μ_{ij} and the edge non-membership ν_{ij} .

Definition 2.4(Parvathi.R et al.,2009) An *intuitionistic fuzzy hypergraph (IFHG)* is an ordered pair $H = \langle V, E \rangle$ where
(i) $V = \{v_1, v_2 \cdots, v_n\}$, is a finite set of intuitionistic fuzzy vertices,
(ii) $E = \{E_1, E_2, \cdots, E_m\}$ is a family of crisp subsets of V,
(iii) $E_j = \left\{\left(v_i, \mu_j(v_i), \nu_j(v_i)\right): \mu_j(v_i), \nu_j(v_i) \geq 0 \; and \; \mu_j(v_i) + \nu_j(v_i) \leq 1\right\}, \; j = 1, 2, \cdots, m,$
(iv) $E_j \neq \emptyset, j = 1, 2, \cdots, m,$
(v) $\cup_j supp(E_j) = V, j = 1, 2, \cdots, m.$
Here, the hyperedges E_j are crisp sets of intuitionistic fuzzy vertices, $\mu_j(v_i)$ and $\nu_j(v_i)$ denote the degrees of membership and non-membership of vertex v_i to edge E_j. Thus, the elements of the incidence matrix of IFHG are of the form $(v_{ij}, \mu_j(v_i), \nu_j(v_j))$. The sets (V, E) are crisp sets.

Notations - list

- $\langle \mu(v_i), \nu(v_i) \rangle$ or simply $\langle \mu_i, \nu_i \rangle$ denote the degrees of membership and non-membership of the vertex $v_i \in V$, such that $0 \leq \mu_i + \nu_i \leq 1$.
- $\langle \mu(v_{ij}), \nu(v_{ij}) \rangle$ or simply $\langle \mu_{ij}, \nu_{ij} \rangle$ denote the degrees of membership and non-membership of the edge $(v_i, v_j) \in V \times V$, such that $0 \leq \mu_{ij} + \nu_{ij} \leq 1$.
- μ_{ij} is the membership value of i^{th} vertex in j^{th} edge and ν_{ij} is the non-membership value of i^{th} vertex in j^{th} edge.

Definition 2.5(Myithili.K.K., Keerthika.R.2020) The IFk-PHG $\mathcal{H}$ is an ordered triple $\mathcal{H} = (V, E, \psi)$ where

(i) $V = \{v_1, v_2 \cdots, v_n\}$ is a finite set of vertices,

(ii) $E = \{E_1, E_2, \cdots, E_m\}$ is a family of intuitionistic fuzzy subsets of V ,

(iii) $E_j = \left\{\left(v_i, \mu_j(v_i), \nu_j(v_i)\right): \mu_j(v_i), \nu_j(v_i) \geq 0 \; and \; \mu_j(v_i) + \nu_j(v_i) \leq 1\right\}, \; j = 1, 2, \cdots, m,$

(iv) $E_j \neq \emptyset, \; j = 1, 2, \cdots, m,$

(v) $\bigcup_j supp(E_j) = V, j = 1, 2, \cdots, m,$

(vi) For all $v_i \in E$ there exists k - disjoint sets $\psi_i, i = 1, 2, \cdots, k \ni$ no two vertices in the same set are adjacent where $E = \bigcap_{i=1}^{k} \psi_i = \emptyset$.

3. Notations

Throughout this chapter the following notations were considered.

(i)$< \mu_{k_i}, \nu_{k_i} >$ denotes the degrees of membership and non-membership of the vertex $v_i \in V$ such that $0 \leq \mu_{k_i} + \nu_{k_i} \leq 1$.

(ii) $< \mu_{k_{ij}}, \nu_{k_{ij}} >$ denotes the degrees of membership and non-membership of the edge $(v_i, v_j) \in V \times V$ such that $0 \leq \mu_{k_{ij}} + \nu_{k_{ij}} \leq 1$. That is, $\mu_{k_{ij}}$ and $\nu_{k_{ij}}$ are the degrees of membership and non-membership of i^{th} vertex in j^{th}edge.

(iii) Let $\mathcal{H}_1 = (V_1, E_1, \psi_1, \langle\mu_{k_i}, \nu_{k_i}\rangle, \langle \mu_{k_{ij}}, \nu_{k_{ij}}\rangle)$ and

$\mathcal{H}_2 = (V_2, E_2, \psi_2, \langle\mu_{k'_i}, \nu_{k'_i}\rangle, \langle\mu_{k'_{ij}}, \nu_{k'_{ij}}\rangle)$

be two IFk-PHGs where $\langle\mu_{k_i}, \nu_{k_i}\rangle$, $\langle\mu_{k'_i}, \nu_{k'_i}\rangle$ are the degrees of membership and non-membership of the vertex v_i and $\langle \mu_{k_{ij}}, \nu_{k_{ij}}\rangle$, $\langle\mu_{k'_{ij}}, \nu_{k'_{ij}}\rangle$ are the degrees of membership and non-membership of the edge v_{ij}.

4. Some basic Properties on IFk-PHGs

Definition 4.1 An IFk-PHG, $\mathcal{H} = (V, E, \psi)$ is said to be a *semi*-μ_k *strong* intuitionistic fuzzy k-partite hypergraph, if $\mu_{k_{ij}} = min\left(\mu_{k_i}, \mu_{k_j}\right)$ for every i and j.

Definition 4.2 An IFk-PHG, $\mathcal{H} = (V, E, \psi)$ is said to be a *semi*-ν_k $strong$ intuitionistic fuzzy k-partite hypergraph, if $\nu_{k_{ij}} = max\left(\nu_{k_i}, \nu_{k_j}\right)$ for every i and j.

Definition 4.3 An IFk-PHG, $\mathcal{H} = (V, E, \psi)$ is said to be a *strong* IFk-PHG, if $\mu_{k_{ij}} = min\left(\mu_{k_i}, \mu_{k_j}\right)$ and $\nu_{k_{ij}} = max\,(\nu_{k_i}, \nu_{k_j})$ for all $(v_i, v_j) \in \psi$.

Definition 4.4 An IFk-PHG, $\mathcal{H} = (V, E, \psi)$ is said to be a *complete*-μ_k *strong* IFk-PHG, if $\mu_{k_{ij}} = min\left(\mu_{k_i}, \mu_{k_j}\right)$ and $\nu_{k_{ij}} \leq max\left(\nu_{k_i}, \nu_{k_j}\right)$ for all i and j.

Definition 4.5 An IFk-PHG, $\mathcal{H} = (V, E, \psi)$ is said to be a $complete$-ν_k $strong$ IFk-PHG, if $\mu_{k_{ij}} \leq min\left(\mu_{k_i}, \mu_{k_j}\right)$ and $\nu_{k_{ij}} = max\left(\nu_{k_i}, \nu_{k_j}\right)$ for all i and j.

Definition 4.6 An IFk-PHG, $\mathcal{H} = (V, E, \psi)$ is said to be a *complete* IFk-PHG, if $\mu_{k_{ij}} = min\left(\mu_{k_i}, \mu_{k_j}\right)$ and $\nu_{k_{ij}} = max\left(\nu_{k_i}, \nu_{k_j}\right)$ for every $v_i, v_j \in V$.

Definition 4.7 The *complement* of an IFk-PHG, $\mathcal{H} = (V, E, \psi)$ is $\bar{\mathcal{H}} = (\bar{V}, \bar{E}, \bar{\psi})$ where

$(i)\; \bar{V} = V$

$(ii)\; \bar{\mu}_{k_i} = \mu_{k_i}$ and $\bar{\nu}_{k_i} = \nu_{k_i}$ for all $i = 1, 2, \cdots, n$.

$$(iii)\bar{\mu}_{k_{ij}} = \begin{cases} min\left(\mu_{k_i}, \mu_{k_j}\right) - \mu_{k_{ij}} & if\; \mu_{k_{ij}} \neq 0 \\ min\left(\mu_{k_i}, \mu_{k_j}\right) & if\; \mu_{k_{ij}} = 0 \end{cases}$$

and

$$\overline{\nu}_{k_{ij}} = \begin{cases} max\left(\nu_{k_i}, \nu_{k_j}\right) - \nu_{k_{ij}} & if\ \nu_{k_{ij}} \neq 0 \\ max\left(\nu_{k_i}, \nu_{k_j}\right) & if\ \nu_{k_{ij}} = 0 \end{cases}$$

for all $i, j = 1,2, \ldots n$.

Theorem 4.1

(i) The complement of a semi-μ_k strong IFk-PHG is a semi-μ_k strong IFk-PHG.

(ii) The complement of a semi-ν_k strong IFk-PHG is a semi-ν_k strong IFk-PHG.

Proof

(i) Let $\mathcal{H}$ be a semi-μ_k strong IFk-PHG and let $\overline{\mathcal{H}}$ be its complement. Since $\mathcal{H}$ is a semi-μ_k strong IFk-PHG, $\mu_{k_{ij}} = \{min\left(\mu_{k_i}, \mu_{k_j}\right)\}$for every $\left(v_i, v_j\right) \in \psi$ where ψ is the disjoint set.

Then for every $\left(v_i, v_j\right) \in \bar{\psi}$,

$$\bar{\mu}_{k_{ij}} = \begin{cases} \min\left(\mu_{k_i}, \mu_{k_j}\right) & if\ \mu_{k_{ij}} = 0, (v_i, v_j) \notin \psi \\ 0 & if\ \mu_{k_{ij}} \neq 0, (v_i, v_j) \in \psi \end{cases}$$

Then $\bar{\mu}_{k_{ij}} = \{min\left(\bar{\mu}_{k_i}, \bar{\mu}_{k_j}\right)\}$ for every $\left(v_i, v_j\right) \in \bar{\psi}$.

This shows that $\overline{\mathcal{H}}$ is a semi-μ_k strong IFk-PHG.

(ii) Similarly, let $\mathcal{H}$ be a semi-ν_k strong IFk-PHG and let $\overline{\mathcal{H}}$ be its complement. Since $\mathcal{H}$ is a semi-ν_k strong IFk-PHG, $\nu_{k_{ij}} = \{max\left(\nu_{k_i}, \nu_{k_j}\right)\}$for every $\left(v_i, v_j\right) \in \psi$.

Then for every $\left(v_i, v_j\right) \in \bar{\psi}$,

$$\bar{\nu}_{k_{ij}} = \begin{cases} \max\left(\nu_{k_i}, \nu_{k_j}\right) & if\ \nu_{k_{ij}} = 0, (v_i, v_j) \notin \psi \\ 0 & if\ \nu_{k_{ij}} \neq 0, (v_i, v_j) \in \psi \end{cases}$$

Then $\bar{\nu}_{k_{ij}} = \{max(\bar{\nu}_{k_i}, \bar{\nu}_{k_j})\}$ for every $\left(v_i, v_j\right) \in \bar{\psi}$.

This shows that $\overline{\mathcal{H}}$ is a semi-ν_k strong IFk-PHG.

Theorem 4.2

If an IFk-PHG be a strong intuitionistic fuzzy k-partite hypergraph then its complement is also a strong IFk-PHG.

Proof

Let $\mathcal{H}$ be a strong IFk-PHG and let $\overline{\mathcal{H}}$ be its complement. Since $\mathcal{H}$ is strong, $\mu_{k_{ij}} = \{min\left(\mu_{k_i}, \mu_{k_j}\right)\}$ and $\nu_{k_{ij}} = \{max\left(\nu_{k_i}, \nu_{k_j}\right)\}$ for every $\left(v_i, v_j\right) \in \psi$ where ψ is the disjoint set. Then

(i) $\bar{\mu}_{k_i} = \mu_{k_i}, \bar{\nu}_{k_i} = \nu_{k_i}$ for every $v_i \in V$

(ii)

$$\bar{\mu}_{k_{ij}} = \begin{cases} \min\left(\mu_{k_i}, \mu_{k_j}\right) & if\ \mu_{k_{ij}} = 0, (v_i, v_j) \notin \psi \\ 0 & if\ \mu_{k_{ij}} \neq 0, (v_i, v_j) \in \psi \end{cases}$$

$$\overline{\nu}_{k_{ij}} = \begin{cases} \max\left(\nu_{k_i}, \nu_{k_j}\right) & if\ \nu_{k_{ij}} = 0, \left(v_i, v_j\right) \notin \psi \\ 0 & if\ \nu_{k_{ij}} \neq 0, (v_i, v_j) \in \psi \end{cases}$$

That is $\bar{\mu}_{k_{ij}} = \{min\left(\mu_{k_i}, \mu_{k_j}\right)\}$ and $\overline{\nu}_{k_{ij}} = \{max\left(\nu_{k_i}, \nu_{k_j}\right)\}$ for every $(v_i, v_j) \in \bar{\psi}$ where $\bar{\psi}$ is the complement of ψ. Thus $\overline{\mathcal{H}}$ is a strong IFk-PHG.

Theorem 4.3

The complement of a complete IFk-PHG is a complete IFk-PHG.

Proof

An IFk-PHG $\mathcal{H} = (V, E, \psi)$ is said to be a *complete* IFk-PHG if $\mu_{k_{ij}} = min\left(\mu_{k_i}, \mu_{k_j}\right)$ and $\nu_{k_{ij}} = \max\left(\nu_{k_i}, \nu_{k_j}\right)$ for every $v_i, v_j \in V$.

By the definition of complement for a membership function,

For all $i, j = 1, 2, \cdots, n$, $\bar{\mu}_{k_{ij}} = min\left(\mu_{k_i}, \mu_{k_j}\right) - \mu_{k_{ij}}$

$$\bar{\mu}_{k_{ij}} = \begin{cases} 0 & if\ \mu_{k_{ij}} \neq 0 \\ min\left(\mu_{k_i}, \mu_{k_j}\right) & if\ \mu_{k_{ij}} = 0 \end{cases}$$

By the definition of complement for a non-membership function,
For all $i, j = 1, 2, \cdots, n$, $\bar{\nu}_{k_{ij}} = max\left(\nu_{k_i}, \nu_{k_j}\right) - \nu_{k_{ij}}$

$$\bar{\nu}_{k_{ij}} = \begin{cases} 0 & if\ \nu_{k_{ij}} \neq 0 \\ max\left(\nu_{k_i}, \nu_{k_j}\right) & if\ \nu_{k_{ij}} = 0 \end{cases}$$

Hence, $\bar{\mu}_{k_{ij}} = min\left(\mu_{k_i}, \mu_{k_j}\right)$ when $\mu_{k_{ij}} = 0$, $\bar{\mu}_{k_{ij}} = 0$ when $\mu_{k_{ij}} \neq 0$ and $\bar{\nu}_{k_{ij}} = \max\left(\nu_{k_i}, \nu_{k_j}\right)$ when $\nu_{k_{ij}} = 0$, $\bar{\nu}_{k_{ij}} = 0$ when $\nu_{k_{ij}} \neq 0$ for every $(v_i, v_j) \in \psi$ where v_i, v_j denote the edge for all $v_i, v_j \in \bar{V}$.
Thus, the complement of a complete IFk-PHG is a complete IFk-PHG.

5. Operations on Intuitionistic fuzzy k-partite Hypergraphs

Definition 5.1 The union of $\mathcal{H}_1$ and $\mathcal{H}_2$ denoted by $\mathcal{H}_1 \cup \mathcal{H}_2$ is defined as

$$\mathcal{H} = \mathcal{H}_1 \cup \mathcal{H}_2$$
$$= \{V_1 \cup V_2, \psi_1 \cup \psi_2, \langle \mu_{k_r} = \mu_{k_i \cup k_i'}, \nu_{k_r} = \nu_{k_i \cup k_i'} \rangle, \langle \mu_{k_{rs}} = \mu_{k_{ij} \cup k_{ij}'}, \nu_{k_{rs}} = \nu_{k_{ij} \cup k_{ij}'} \rangle\}$$ and defined by

$$\langle \mu_{k_r}, \nu_{k_r} \rangle = \begin{cases} \langle \mu_{k_i}, \nu_{k_i} \rangle & if\ v_i \in V_1 \setminus V_2 \\ \langle \mu_{k_i'}, \nu_{k_i'} \rangle & if\ v_i \in V_2 \setminus V_1 \\ \langle \max\left(\mu_{k_i}, \mu_{k_i'}\right), \min\left(\nu_{k_i}, \nu_{k_i'}\right) \rangle & if\ v_i \in V_1 \cap V_2 \end{cases}$$

$$\langle \mu_{k_{rs}}, \nu_{k_{rs}} \rangle = \begin{cases} \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle & if\ v_{ij} \in \psi_1 \setminus \psi_2 \\ \langle \mu_{k_{ij}'}, \nu_{k_{ij}'} \rangle & if\ v_{ij} \in \psi_2 \setminus \psi_1 \\ \langle \max\left(\mu_{k_{ij}}, \mu_{k_{ij}'}\right), \min\left(\nu_{k_{ij}}, \nu_{k_{ij}'}\right) \rangle & if\ v_{ij} \in \psi_1 \cap \psi_2 \\ \langle 0,1 \rangle & otherwise \end{cases}$$

Definition 5.2 The intersection of $\mathcal{H}_1$ and $\mathcal{H}_2$ denoted by $\mathcal{H}_1 \cap \mathcal{H}_2$ is defined as

$$\mathcal{H} = \mathcal{H}_1 \cap \mathcal{H}_2$$
$$= \{V_1 \cap V_2, \psi_1 \cup \psi_2, \langle \mu_{k_r} = \mu_{k_i \cap k_i'}, \nu_{k_r} = \nu_{k_i \cap k_i'} \rangle, \langle \mu_{k_{rs}} = \mu_{k_{ij} \cap k_{ij}'}, \nu_{k_{rs}} = \nu_{k_{ij} \cap k_{ij}'} \rangle\}$$ and defined by

$$\langle \mu_{k_r}, \nu_{k_r} \rangle = \begin{cases} \langle \mu_{k_i}, \nu_{k_i} \rangle & if\ v_i \in V_1 \setminus V_2 \\ \langle \mu_{k_i'}, \nu_{k_i'} \rangle & if\ v_i \in V_2 \setminus V_1 \\ \langle \min\left(\mu_{k_i}, \mu_{k_i'}\right), \max\left(\nu_{k_i}, \nu_{k_i'}\right) \rangle & if\ v_i \in V_1 \cap V_2 \end{cases}$$

$$\langle \mu_{k_{rs}}, \nu_{k_{rs}} \rangle = \begin{cases} \langle \mu_{k_{ij}}, \nu_{k_{ij}} \rangle & if\ v_{ij} \in \psi_1 \setminus \psi_2 \\ \langle \mu_{k_{ij}'}, \nu_{k_{ij}'} \rangle & if\ v_{ij} \in \psi_2 \setminus \psi_1 \\ \langle \min\left(\mu_{k_{ij}}, \mu_{k_{ij}'}\right), \max\left(\nu_{k_{ij}}, \nu_{k_{ij}'}\right) \rangle & if\ v_{ij} \in \psi_1 \cap \psi_2 \\ \langle 0,1 \rangle & otherwise \end{cases}$$

Definition 5.3 The *join* of $\mathcal{H}_1$ and $\mathcal{H}_2$ denoted by $\mathcal{H}_1 + \mathcal{H}_2$ is defined as

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2$$
$$= \{V_1 \cup V_2, \psi_1 \cup \psi_2 \cup \psi', \langle \mu_{k_i + k_i'}, \nu_{k_i + k_i'} \rangle, \langle \mu_{k_{ij} + k_{ij}'}, \nu_{k_{ij} + k_{ij}'} \rangle\}$$ and defined by

$\left(\mu_{k_i + k_i'}\right)(v_i) = \left(\mu_{k_i} \wedge \mu_{k_i'}\right)(v_i)$ if $v_i \in V_1 \cup V_2$

$\left(\nu_{k_i + k_i'}\right)(v_i) = \left(\nu_{k_i} \vee \nu_{k_i'}\right)(v_i)$ if $v_i \in V_1 \cup V_2$

$\left(\mu_{k_{ij} + k_{ij}'}\right)(v_i v_j) = \left(\mu_{k_{ij}} \wedge \mu_{k_{ij}'}\right)(v_i v_j)$ if $v_i v_j \in \psi_1 \cup \psi_2$

$= \left(\mu_{k_{ij}}(v_i). \mu_{k_{ij}'}(v_j)\right)$ if $v_i v_j \in \psi'$

$\left(\nu_{k_{ij} + k_{ij}'}\right)(v_i v_j) = \left(\nu_{k_{ij}} \vee \nu_{k_{ij}'}\right)(v_i v_j)$ if $v_i v_j \in \psi_1 \cup \psi_2$

$= \left(\nu_{k_{ij}}(v_i).\nu_{k'_{ij}}(v_j)\right)$ if $v_iv_j \in \psi'$

Definition 5.4 The *structural subtraction* of $\mathcal{H}_1$ and $\mathcal{H}_2$ denoted by $\mathcal{H}_1 \ominus \mathcal{H}_2$ and is defined as $\mathcal{H} = \mathcal{H}_1 \ominus \mathcal{H}_2 = \{V_1 \setminus V_2, \langle\mu_{k_r}, \nu_{k_r}\rangle, \langle\mu_{k_{rs}}, \nu_{k_{rs}}\rangle\}$ where $'\setminus'$ is the set theoretical difference operation and

$$\langle\mu_{k_r}, \nu_{k_r}\rangle = \begin{cases} \langle\mu_{k_i}, \nu_{k_i}\rangle & if\ v_i \in V_1 \\ \langle\mu_{k_j}, \nu_{k_j}\rangle & if\ v_j \in V_2 \\ \langle 0,1\rangle & otherwise \end{cases}$$

$$\langle\mu_{k_{rs}}, \nu_{k_{rs}}\rangle = \begin{cases} \langle\mu_{k_{ij}}, \nu_{k_{ij}}\rangle & for\ \ v_r = v_i \in V_1 \setminus V_2 \\ & v_s = v_j \in V_1 \setminus V_2 \end{cases}$$

where $V_1 \setminus V_2 = \emptyset$.

Definition 5.5 The *Ringsum* of two IFk-PHGs $\mathcal{H}_1$ and $\mathcal{H}_2$ denoted by $\mathcal{H}_1 \oplus \mathcal{H}_2$ and is defined as $\mathcal{H}_1 \oplus \mathcal{H}_2 = (\mathcal{H}_1 \cup \mathcal{H}_2) \setminus (\mathcal{H}_1 \cap \mathcal{H}_2)$ where $V_1 \cap V_2 \neq \emptyset$.

Definition 5.6 The *product* of two IFk-PHGs $\mathcal{H}_1$ and $\mathcal{H}_2$ denoted by $\mathcal{H}_1 \otimes \mathcal{H}_2$ and is defined as $V(\mathcal{H}_1 \otimes \mathcal{H}_2) = V(\mathcal{H}_1) \otimes V(\mathcal{H}_2)$

$\psi(\mathcal{H}_1 \otimes \mathcal{H}_2) = V(\mathcal{H}_1) \otimes \psi(\mathcal{H}_2) \cup V(\mathcal{H}_2) \otimes \psi(\mathcal{H}_1)$ and

$\left(\mu_{k_i \otimes k'_i}\right)(u_i, u_j) = (\mu_{k_i} \otimes \mu_{k'_i})(u_i, u_j) = \mu_{k_i}(u_i) \wedge \mu_{k'_i}(u_j)$ for every (u_i, u_j) in V and

$\left(\nu_{k_i \otimes k'_i}\right)(u_i, u_j) = (\nu_{k_i} \otimes \nu_{k'_i})(u_i, u_j) = \nu_{k_i}(u_i) \vee \nu_{k'_i}(u_j)$ for every (u_i, u_j) in V

$\left(\mu_{k_{ij} \otimes k'_{ij}}\right)\left((u, u_j)(u, v_j)\right) = (\mu_{k_{ij}} \otimes \mu_{k'_{ij}})\left((u, u_j)(u, v_j)\right) = \mu_{k_i}(u) \wedge \mu_{k'_i}(u_jv_j)$ for every $u \in V_1$ and $u_jv_j \in \psi_2$

$\left(\nu_{k_{ij} \otimes k'_{ij}}\right)\left((u, u_j)(u, v_j)\right) = (\nu_{k_{ij}} \otimes \nu_{k'_{ij}})\left((u, u_j)(u, v_j)\right) = \nu_{k_i}(u) \vee \nu_{k'_{ij}}(u_jv_j)$ for every $u \in V_1$ and $u_jv_j \in \psi_2$

$\left(\mu_{k_{ij} \otimes k'_{ij}}\right)((u_i, w)(v_i, w)) = (\mu_{k_{ij}} \otimes \mu_{k'_{ij}})((u_i, w)(v_i, w)) = \mu_{k'_i}(w) \wedge \mu_{k_{ij}}(u_iv_i)$ for every $w \in V_2$ and $u_iv_i \in \psi_1$

$\left(\nu_{k_{ij} \otimes k'_{ij}}\right)((u_i, w)(v_i, w)) = (\nu_{k_{ij}} \otimes \nu_{k'_{ij}})((u_i, w)(v_i, w)) = \nu_{k'_i}(w) \vee \nu_{k_{ij}}(u_iv_i)$ for every $w \in V_2$ and $u_iv_i \in \psi_1$

Note: The product of two IFk-PHGs is not an IFk-PHG.

Definition 5.7 The *cartesian product* of $\mathcal{H}_1$ and $\mathcal{H}_2$ denoted by $\mathcal{H}_1 \times \mathcal{H}_2$ and is defined as $\mathcal{H} = \mathcal{H}_1 \times \mathcal{H}_2 = (V, \psi')$ where $V = V_1 \times V_2$ and

$\psi' = \{(u, u_j)(u, v_j) : u \in V_1, u_jv_j \in \psi_2\} \cup \{(u_i, w)(v_i, w) : w \in V_2, u_iv_i \in \psi_1\}$. Then

$\left(\mu_{k_i \times k'_i}\right)(u_i, u_j) = (\mu_{k_i} \times \mu_{k'_i})(u_i, u_j) = \mu_{k_i}(u_i) \wedge \mu_{k'_i}(u_j)$ for every (u_i, u_j) in V and

$\left(\nu_{k_i \times k'_i}\right)(u_i, u_j) = (\nu_{k_i} \times \nu_{k'_i})(u_i, u_j) = \nu_{k_i}(u_i) \vee \nu_{k'_i}(u_j)$ for every (u_i, u_j) in V

$\left(\mu_{k_{ij} \times k'_{ij}}\right)\left((u, u_j)(u, v_j)\right) = (\mu_{k_{ij}} \times \mu_{k'_{ij}})\left((u, u_j)(u, v_j)\right) = \mu_{k_i}(u) \wedge \mu_{k'_{ij}}(u_jv_j)$ for every $u \in V_1$ and $u_jv_j \in \psi_2$

$\left(\nu_{k_{ij} \times k'_{ij}}\right)\left((u, u_j)(u, v_j)\right) = (\nu_{k_{ij}} \times \nu_{k'_{ij}})\left((u, u_j)(u, v_j)\right) = \nu_{k_i}(u) \vee \nu_{k'_{ij}}(u_jv_j)$ for every $u \in V_1$ and $u_jv_j \in \psi_2$

$\left(\mu_{k_{ij} \times k'_{ij}}\right)((u_i, w)(v_i, w)) = (\mu_{k_{ij}} \times \mu_{k'_{ij}})((u_i, w)(v_i, w)) = \mu_{k'_i}(w) \wedge \mu_{k_{ij}}(u_iv_i)$ for every $w \in V_2$ and $u_iv_i \in \psi_1$

$\left(\nu_{k_{ij}\times k'_{ij}}\right)((u_i,w)(v_i,w)) = (\nu_{k_{ij}} \times \nu_{k'_{ij}})((u_i,w)(v_i,w)) = \nu_{k'_i}(w) \vee \nu_{k_{ij}}(u_iv_i)$ for every $w \in V_2$ and $u_iv_i \in \psi_1$

Definition 5.8 The *composition* of $\mathcal{H}_1$ and $\mathcal{H}_2$ denoted by $\mathcal{H}_1 \circ \mathcal{H}_2$ and is defined as
$\mathcal{H} = \mathcal{H}_1 \circ \mathcal{H}_2 = (V_1 \times V_2, \psi)$ where $V = V_1 \times V_2$ and
$\psi = \{(u,u_j)(u,v_j): u \in V_1, u_jv_j \in \psi_2\} \cup \{(u_i,w)(v_i,w): w \in V_2, u_iv_i \in \psi_1\} \cup \{(u_i,u_j)(v_i,v_j) : u_iv_i \in \psi_1, u_j \neq v_j\}$.
Then
$\left(\mu_{k_i\circ k'_i}\right)(u_i,u_j) = (\mu_{k_i} \circ \mu_{k'_i})(u_i,u_j) = \mu_{k_i}(u_i) \wedge \mu_{k'_i}(u_j)$ for every (u_i,u_j) in $V_1 \times V_2$
$\left(\nu_{k_i\circ k'_i}\right)(u_i,u_j) = (\nu_{k_i} \circ \nu_{k'_i})(u_i,u_j) = \nu_{k_i}(u_i) \vee \nu_{k'_i}(u_j)$ for every (u_i,u_j) in $V_1 \times V_2$
$\left(\mu_{k_{ij}\circ k'_{ij}}\right)((u,u_j)(u,v_j)) = (\mu_{k_{ij}} \circ \mu_{k'_{ij}})((u,u_j)(u,v_j)) = \mu_{k_i}(u) \wedge \mu_{k'_{ij}}(u_jv_j)$ for every $u \in V_1$ and $u_jv_j \in \psi_2$
$\left(\nu_{k_{ij}\circ k'_{ij}}\right)((u,u_j)(u,v_j)) = (\nu_{k_{ij}} \circ \nu_{k'_{ij}})((u,u_j)(u,v_j)) = \nu_{k_i}(u) \vee \nu_{k'_{ij}}(u_jv_j)$ for every $u \in V_1$ and $u_jv_j \in \psi_2$
$\left(\mu_{k_{ij}\circ k'_{ij}}\right)((u_i,w)(v_i,w)) = (\mu_{k_{ij}} \circ \mu_{k'_{ij}})((u_i,w)(v_i,w)) = \mu_{k'_i}(w) \wedge \mu_{k_{ij}}(u_iv_i)$
for every $w \in V_2$ and $u_iv_i \in \psi_1$
$\left(\nu_{k_{ij}\circ k'_{ij}}\right)((u_i,w)(v_i,w)) = (\nu_{k_{ij}} \circ \nu_{k'_{ij}})((u_i,w)(v_i,w)) = \nu_{k'_i}(w) \vee \nu_{k_{ij}}(u_iv_i)$ for every $w \in V_2$ and $u_iv_i \in \psi_1$
$\left(\mu_{k_{ij}\circ k'_{ij}}\right)((u_i,u_j)(v_i,v_j)) = (\mu_{k_{ij}} \circ \mu_{k'_{ij}})((u_i,u_j)(v_i,v_j)) = \mu_{k'_i}(u_j) \wedge \mu_{k'_i}(v_j) \wedge \mu_{k_{ij}}(u_iv_i)$
for every $(u_i,u_j),(v_i,v_j) \in \psi \setminus \psi'$
$\left(\nu_{k_{ij}\circ k'_{ij}}\right)((u_i,u_j)(v_i,v_j)) = (\nu_{k_{ij}} \circ \nu_{k'_{ij}})((u_i,u_j)(v_i,v_j)) =$
$\nu_{k'_i}(u_j) \vee \nu_{k'_i}(v_j) \vee \nu_{k_{ij}}(u_i,v_i)$ for every $(u_i,u_j),(v_i,v_j) \in \psi \setminus \psi'$
where $\psi' = \{(u,u_j)(u,v_j): u \in V_1, u_jv_j \in \psi_2\} \cup \{(u_i,w)(v_i,w): w \in V_2, u_iv_i \in \psi_1\}$.

Theorem 5.1 Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two IFk-PHGs with vertex sets V_1, V_2 then their union $\mathcal{H} = \mathcal{H}_1 \cup \mathcal{H}_2$ is also an IFk-PHG.
Proof:

Assume $\mathcal{H}_1$ and $\mathcal{H}_2$ be two IFk-PHGs with $\mathcal{H}_1 = \{V_1, \psi_1, \langle\mu_{k_i}, \nu_{k_i}\rangle, \langle\mu_{k_{ij}}, \nu_{k_{ij}}\rangle\}$ and
$\mathcal{H}_2 = \{V_2, \psi_2, \langle\mu_{k_p}, \nu_{k_p}\rangle, \langle\mu_{k_{pq}}, \nu_{k_{pq}}\rangle\} \forall\, i,j = 1,2,\cdots,m$ and $p,q = 1,2,\cdots,n$ vertices respectively.
Then by definition,
$\mathcal{H}_1 \cup \mathcal{H}_2 = \{V_1 \cup V_2, \psi_1 \cup \psi_2, \langle\mu_{k_{i\cup p}}, \nu_{k_{i\cup p}}\rangle, \langle\mu_{k_{ij\cup pq}}, \nu_{k_{ij\cup pq}}\rangle\}$ where
$\langle\mu_{k_{i\cup p}}, \nu_{k_{i\cup p}}\rangle = \langle\min(\mu_{k_i}, \mu_{k_p}), \max(\nu_{k_i}, \nu_{k_p})\rangle$ and
$\langle\mu_{k_{ij\cup pq}}, \nu_{k_{ij\cup pq}}\rangle = \langle\min(\mu_{k_{ij}}, \mu_{k_{pq}}), \max(\nu_{k_{ij}}, \nu_{k_{pq}})\rangle$
Therefore, $\mathcal{H}_1 \cup \mathcal{H}_2 = \{V_1 \cup V_2, \psi_1 \cup \psi_2, \langle\mu_{k_r}, \nu_{k_r}\rangle, \langle\mu_{k_{rs}}, \nu_{k_{rs}}\rangle\}$
which shows that $\mathcal{H} = \mathcal{H}_1 \cup \mathcal{H}_2$ is also an intuitionistic fuzzy k-partite hypergraph.

Theorem 5.2 The Ringsum of two IFk-PHGs is also an intuitionistic fuzzy k-partite hypergraph.
Proof:

The proof is obvious from the definition of Ringsum.

6. Conclusion

In this paper, the operations on IFk-PHGs are defined and discussed. Also, some interesting properties likeunion, intersection, join, structural subtraction, ringsum, cartesian product, composition and complement are dealt with. Since an intuitionistic fuzzy set has shown more advantages in handling vagueness and uncertainty than fuzzy set, we have applied the concept of intuitionistic fuzzy sets in k-partite IFHG. This can be applied in textile engineering. In future, the authors has planned to extend the concept of k-partite IFHG in isomorphism and in transversal of IFk-PHG.

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