

Effect of FRP Sheets Length on the Ultimate Loading Capacity of CFRP and GFRP Strengthened Hollow-Core Slabs by the Finite Element Method

Muhammed Maleknia ^{a*}, Morteza Biklaryan ^b, Mostafa Radmehr ^c

^a PhD Candidate, Chalous Branch, Islamic Azad University, Chalous, Iran.

^b Department of Civil Engineering, Chalous Branch, Islamic Azad University, Chalous, Iran.

^c Lecturer, Damavand Branch, Islamic Azad University, Damavand, Iran.

Article History: Received: 5 April 2021; Accepted: 14 May 2021; Published online: 22 June 2021

Abstract: Numerous studies have been conducted on the strengthening of reinforced concrete building components such as beams and columns, which are under shear, bending, or torsion, using the fiber-reinforced polymer (FRP) in the form of external coverage. However, a few studies have addressed the strengthening of concrete slabs, especially hollow-core slabs. Moreover, the calculation of the optimal fiber length to reduce the cost and time spent on project implementation is a basic issue. Therefore, this study scrutinized the effect of FRP sheets length on the ultimate loading capacity of CFRP and GFRP strengthened hollow-core slabs by the finite element method. In this regard, the concrete hollow-core slab was modeled in ANSYS software and compared with the available experimental model for verification. Then, the bending reinforcement of the aforementioned slab by CFRP and GFRP fibers was investigated at different distances from the face of support. These changes were inserted into hollow-core slabs with lengths of 1.5 m and 9 m. It was indicated that in the 1.5 m slab, the optimal CFRP fiber length was about one-tenth from the face of support and this value was one-fifth for GFRP fibers. Also, For the 9 m hollow-core slab, the optimal CFRP fiber length was about one-fifth from the face of support and this value was one-third for GFRP fibers. Finally, it was concluded that FRP sheet installation in lengths more than the optimal length has no impact on the final capacity of the mentioned slabs. Therefore, it is better to install FRP sheets in the calculated distances (optimal fiber length) to save time and costs.

Keywords: Hollow-core slab, CFRP and GFRP sheets, Finite element, FRP sheets length, Strengthening

1. Introduction

Existing statistics show that billions of dollars are spent annually in the world to compensate for corrosion damage in various structures along the sea. The need to reduce the cost of repairing and maintaining large and numerous marine and offshore structures has led engineers and builders to use new retrofitting materials.

Reinforced concrete structures undergo premature decay in the vicinity of a corrosive marine environment. On the one hand, corrosion causes loss of strength and an increase in the volume of rebars inside the structure, which ultimately leads to internal cracking and failure. On the other hand, the concrete texture loses its cohesion due to contact with ambient humidity and begins to crack and break. Changes in the temperature of marine environments, in turn, cause fatigue and failure of the structure by shrinkage and expansion.

Unlike traditional materials, FRP sheets are extremely resistant to highly corrosive environments such as sea saltwater, chemical fluids, and oil and gas. These materials are considered as a new and durable solution in the reinforcement of coastal, marine, and offshore structures that have been made of concrete, steel, and even wood.

As FRP sheets are highly resistant to alkaline and saline environments, they have been the subject of extensive research in the last two decades to reinforce marine and offshore concrete structures. Such use is especially suitable in corrosive environments such as offshore platforms, piers, and reservoirs.

Many of the present buildings are made of hollow-core slabs. The application of reinforced concrete hollow-core slabs is very important to reduce the concrete volume, cost, and weight of the structure. Some of these buildings have been severely damaged due to natural disasters such as earthquake, wind or material fatigue, and alkaline or acidic corrosive factors [1]. In this regard, repair and strengthening of the mentioned structures are crucial and economical.

In recent decades, the use of fiber-reinforced polymers has become one of the most important reinforcement materials in the world. This method has been widely used in various parts of reinforced concrete buildings such as beams, columns, and slabs. For instance, Triantafillou (1998) conducted a study on the shear retrofit of reinforced concrete beams applying CFRP sheets. He concluded that the effectiveness of FRP was enhanced when the fibers come closer to the perpendicular to diagonal cracks [2]. Another study was conducted by Vasques and Karbahari (2003) who investigated the flexural behavior of the single-sided carbon fiber sheet-reinforced slabs under point loading. The results showed that carbon fiber ribbons, in the form of external coverage, can enhance the load-bearing capacity of the reinforced slabs. [3]. Mosallam (2003) explored the flexural behavior of two-way free-opening slabs reinforced with FRP under uniform loading. This study revealed that FRP systems are suitable for increasing the load-bearing capacity of the two-way slabs [4]. Zhang, Masmoudi, and Benmokrane (2004) studied the behavior of one-way reinforced concrete slabs strengthened with a network of CFRP. The findings revealed that FRP reinforced slabs had smaller displacement in comparison with non-reinforced slabs. This technique also enhanced the hardness

and strength of the reinforced slabs [5]. Rusinowski and Olofsson (2006) researched the reinforcement of two-way slabs with the opening. It was indicated that the strength of reinforced slabs increased in comparison with control and free-opening slabs. It was also concluded that the slab with the opening is far stronger than the slab without an opening [6].

Further, a few studies have been conducted recently on reinforced concrete hollow-core slabs with different types of FRP. For instance, Foubert, Mahmoud, and El-Salakawy (2016) explored the behavior of prestressed hollow-core slabs which were strengthened with near-surface mounted (NSM) carbon fiber-reinforced polymer (CFRP) retrofit in flexure. The findings indicated that this strengthening technique significantly increased not only the flexural but also shear capacities of prestressed hollow-core slabs. Besides, the ultimate capacity improvement was decreased in slabs failing as the result of shear before their full flexural capacity attainment [7]. Also, Pachalla and Prakash (2017) aimed at understanding the performance of GFRP reinforced hollow-core slabs with openings using the external bonding method. In order to evaluate the impact of openings and GFRP reinforcement effectiveness, ten hollow-core slabs with and without openings were tested. The findings showed that opening provision decreased the final strength by approximately 43% and GFRP reinforcement increased the strength by about 87%. Overall, the GFRP reinforcement was proved to be an effective technique for restoring and enhancing the strength of prefabricated prestressed hollow-core slabs with openings [8]. Moreover, Kankeri, Prakash and Pachalla (2018) gave insight into the impact of hybrid strengthening on the performance of hollow-core slabs that were pre-cracked. Also, the behavior of the pre-cracked slabs which were strengthened using near-surface mounted CFRP sheets was investigated. It was revealed that the ultimate capacity of the strengthened slabs increased by 50% applying the NSM technique. That is while hybrid strengthening enhanced by 130% when pre-cracked strengthened slabs were compared with the pre-cracked control slab [9]. Meng, Cheng, and Ragaby (2019) also investigated the effect of using CFRP sheets which were externally bonded on the shear reinforcement of prestressed concrete hollow-core slabs. The results proved that this new shear strengthening technique led to a considerable increase in the prestressed concrete hollow-core slabs shear capacity. In addition, an improvement was observed in the ductility of some strengthened slabs before failure [10]. Li, Wu, Wang, and Yu (2019) strengthened hollow-core slabs with prestressed basalt FRP grid to scrutinize their flexural behavior. In this regard, four strengthened slabs and one control slab were tested which indicated that the flexural performance of the strengthened hollow-core slabs with prestressed BFRP grids improved sizably. Furthermore, prestressing BFRP grids resulted in the improvement of slabs cracking behavior as a considerable decrease was observed in the crack width of the strengthened slabs with the enhancement of the BFRP grid prestressing force. However, the ductility of the mentioned slabs was reduced with the BFRP grid prestressing force increase [11]. Also, Nguyen, Hai Tan, and Kanda (2020) conducted a study on deep concrete hollow-core slabs to find out the impact of polypropylene and steel fibers on their web-shear resistance. It was revealed that while moderate increases were observed in shear strength using polypropylene fibers, there were significant increases in shear resistance using steel fibers [12].

Although a limited number of studies have investigated reinforced concrete hollow-core slabs strengthened with FRP, scant attention has been paid to the effect of FRP sheets length on the ultimate loading capacity. This issue was addressed in the current study. In fact, the sections of the Hollow-core slabs used in this paper have been modeled using the ANSYS software.

This software is one of the best and most powerful analytical software in the field of engineering and structural reinforcement, which uses the finite element method for modeling and analysis [13].

2. Methodology

2.1. Modeling

The present study investigated the effect of FRP sheets length on the ultimate loading capacity of CFRP and GFRP strengthened hollow-core slabs. First, to identify the validity of model behavior and the gained results of the current paper, a sample beam as the control sample was modeled by ANSYS software whose geometric and physical features will be discussed in what follows. Further, the loading- deformation diagram of the middle of the beam, the load resulting in initial cracking of the slab, the final load, and the cracking pattern at the final moment were scrutinized. Finally, the findings were compared to the experimental sample presented in Salehi Yanehsari, Modanloo, Beygi and Navayi Neya (2013) [14]. Research in which bending behavior of the hollow core armed concrete slabs reinforced by different types of FRP and steel sheet was investigated by experimental models. In this regard, a hollow core armed concrete slab with the dimensions of 150×45×20 cm and the ratio of balanced tension rebar was fabricated according to the ABA regulation. In order to fabricate this slab, 6 No 12 rebars as the tension rebar and 4 No 8 rebars as the compression rebar were used. Shear rebars with a diameter of 6 mm were located in the center to center distances of 10 cm (in one-third of the lateral slab) and 20 cm (in three-fifth of the middle of the slab) from one another. Also, No 6 bars were applied in the center to center distances of 15 cm as the temperature bars at the top and bottom of the slab and perpendicular to the main rebars. Besides, two openings with a diameter

of 10 cm were created along the length of the slab. Properties and the method of slab reinforcement are illustrated in figure 1.

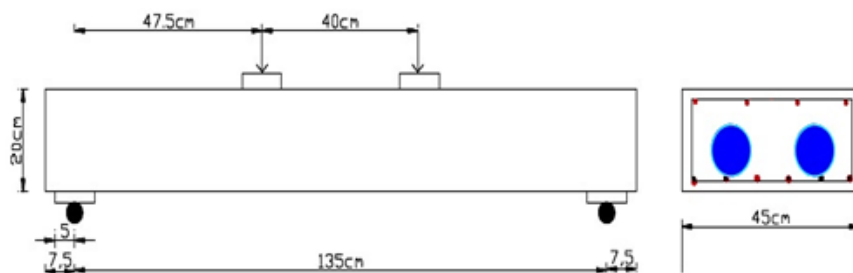


Figure 1. Hollow-core slabs specifications [14]

2.2. Reinforcement pattern and applied elements

In ANSYS software, the solid and 8-node element called Solid65 was used for concrete modeling. Also, the bar and 2-node element named Link180 was applied for rebar modeling. Besides, Shell43 element was employed for FRP modeling due to its insignificant thickness. Moreover, Solid45 was used for modeling of metal plates under the support and the concentrated load (to lower the impact of stress concentration) and to model stirrup, beam 188 element has been used.

2.3. Physical properties

2.3.1. Physical properties of FRP fibers are listed in table 1

Table 1. FRP fibers properties

Sheet thickness mm (in)	Shear modulus Mpa (ksi)	Tensile strength Mpa (ksi)	Poisson coefficient	Elasticity modulus Mpa (ksi)	FRP
1.00 (0.040)	Gxy=3270 (474)	958 (138)	Vxy=0.22 Vxz=0.22 Vyz=0.3	Ex=6200	CFRP
	Gxz=3270 (474)			(9000)	
	Gyz=1860 (270)			Ey=4800 (700) Ez=4800 (700)	
1.3 (0.050)	Gxy=1520 (220)	600 (987)	Vxy=0.26 Vxz=0.26 Vyz=0.3	Ex=2100 (3000)	GFRP
	Gxy=1520 (220)			Ey=7000 (1000)	
	Gxy=2650 (385)			Ez=7000 (1000)	

2.3.2. Concrete properties for control hollow-core slab, are shown in table 2

Table 2. Properties of concrete in hollow-core control slab

Control slab	Ecu (MPa)	Fc (MPa)
	25000	25

2.3.3. Steel properties are listed in table 3 (it is supposed that the steel is a complete elastoplastic material)

Table 3. Properties of steel in hollow-core control slab

Control slab	Elasticity modulus (Es)	Yielding stress of steel (fy)	Poisson coefficient (v)
	200000Mpa	400Mpa	0.3

2.4. Beam geometry and the method of reinforcement

To investigate the behavior of the Hollow-core slab, a slab with dimensions of $150 \times 45 \times 20$ cm is created and 2 holes with a diameter of 10 cm in the middle and along the length of the slab are considered to be all around, as seen in Figures 2 and 3.



Figure 2. Hollow-core slabs specifications

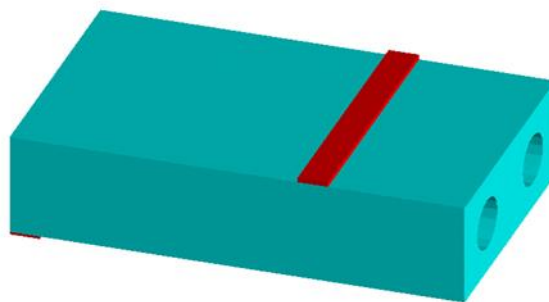


Figure 3. A-A section

3. Validation of the experimental and numerical model

In this section, we compare the results obtained from the finite element method analysis and experimental results. The comparisons performed are load-displacement diagram, investigation of crack distribution during loading, and investigation of stress values in the Hollow-core slab.

3.1. Load-Displacement diagram in the experimental model

In order to validate the model designed in ANSYS software, the findings were compared with the results of a study Salehi Yanehsari et al. (2013) [14].

Figure 4 illustrates that there is a high agreement between the load-displacement diagram developed based on the finite element method with the results of the experimental model. Also, according to table 4 and investigation of the numerical results obtained from the comparison of the experimental model and the model constructed in ANSYS software, an acceptable error percentage with the difference of 3% was achieved.

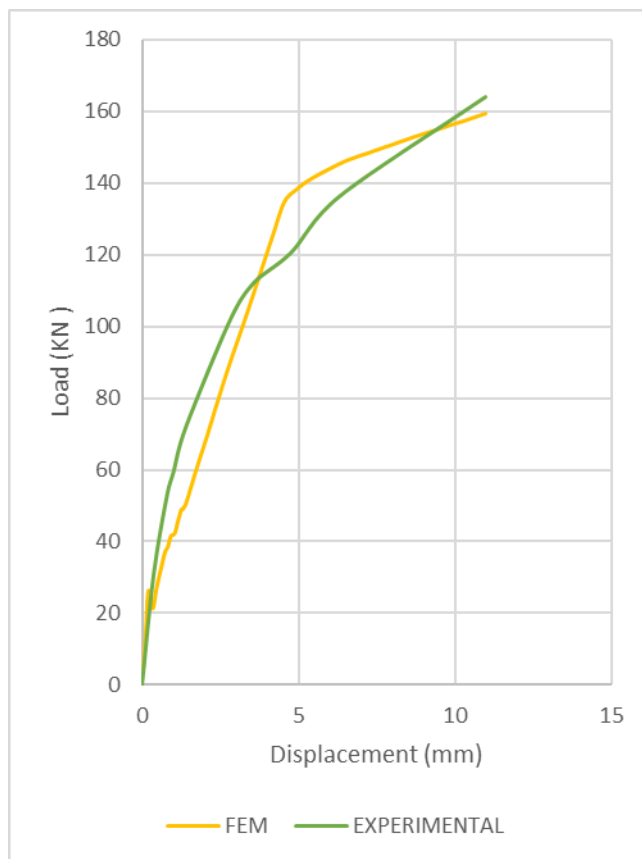


Figure 4. Load – Displacement diagram

Table 4. Comparison of the Results from Control Slab

Ultimate load (KN)	Control slab
165	Experimental sample
160	Numerical analysis
3%	Error percentage

3.2. The Investigation of the Crack Distribution during the Loading

The cracked model of the experimental sample was illustrated in Figure 5 and the evolution of the crack pattern in different steps of loading in the hollow core slab was shown in Figure 6.

As it is obvious in the figures, the cracks have been of a flexural (vertical) type in the crack pattern of the control slab. Furthermore, these cracks have been increased as the result of the load increase and the slab underwent flexural failure due to emerged cracks. Regarding the experimental model shown in figure 5, a good correspondence was observed between the finite element cracked model and the experimental model.



Figure 5. Cracking and failure of experimental specimens

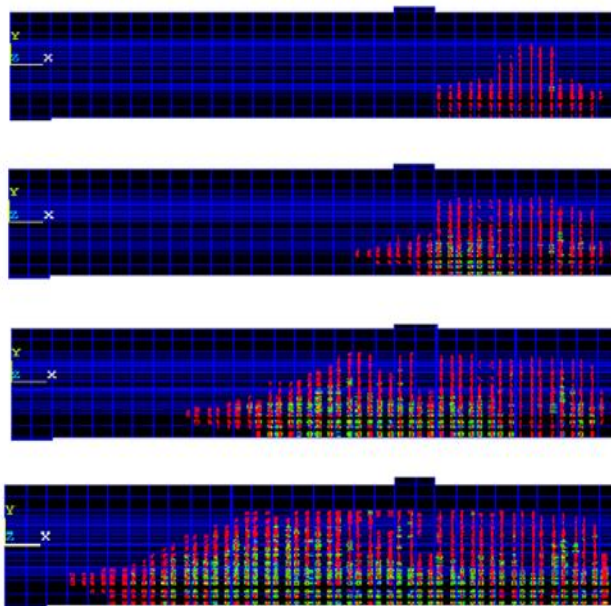


Figure 6. Evolution of the crack pattern in the control slab

3.3. The investigation of stress values in the slab

According to Figure 7, which shows the Von Mises stress in the final steps of loading the slab. in the compression part, the stress reaches its final value of compressive strength, and also in Figure 8, it can be seen that the values of stress in the rebar The tensile loads (6 rebars of grade 12 at the bottom) have reached their yielding values in the final loading step.

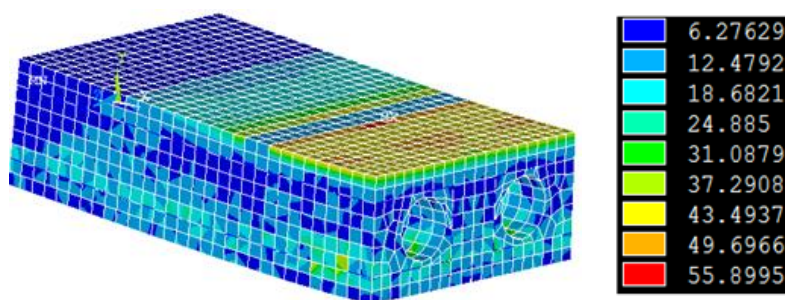


Figure 7. Von mises stress in hollow-core concrete slab

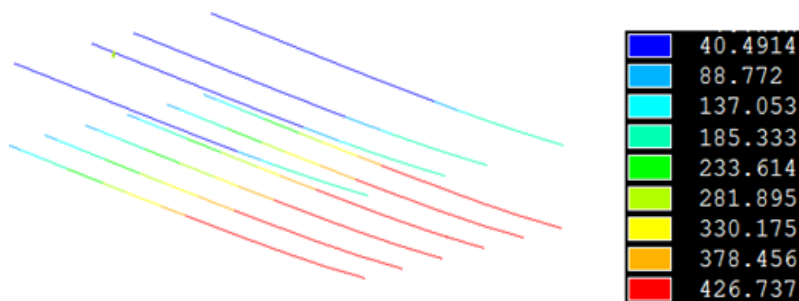


Figure 8. Yielding stress in hollow-core concrete slab tension reinforcements

4. The hollow-core slab strengthening with changing the FRP fiber length

After the validation of the experimental model, the numerical study was conducted to calculate the ultimate loading capacity of CFRP and GFRP strengthened hollow-core slabs with changing the FRP fiber length. In this regard, after modeling the targeted slab in ANSYS software, the capacity of hollow-core slabs with changing the FRP sheets length (5, 15, 30 and 50 cm from the face of support for the slab with the length of 1.5 m and 60, 120, 150, 200, 300 and 400 from the face of support for the slab with the length of 9 m) was examined. The situation of the FRP sheet beneath the hollow-core slab is shown by the violet color in figure 9.

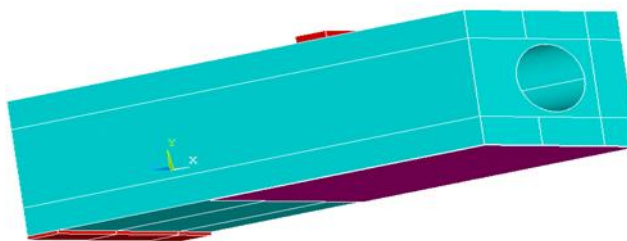


Figure 9. ¼ of the modeled hollow core slab by considering FRP layer

4.1. Investigating the capacity of the hollow-core slab with changing the FRP sheets length in the model with the length of 1.5 m

To investigate the effect of the FRP sheet length on the hollow-core slabs reinforcement, first, a slab with dimensions of 150×45×20 cm and two holes with the diameter of 10 cm was constructed in the middle and along the length of the slab. Then, this slab was reinforced with different lengths of CFRP and GFRP fibers with distances of 5, 15, 30 and 50 cm from the face of support. Next, to investigate the capacity of the hollow-core slab reinforced with different lengths, the targeted models were compared in terms of the changes in the final loading capacity, the type of failure and the type of crack in the hollow-core slab

(L in the following line graphs refers to the distance of CFRP and GFRP fibers from the face of support).

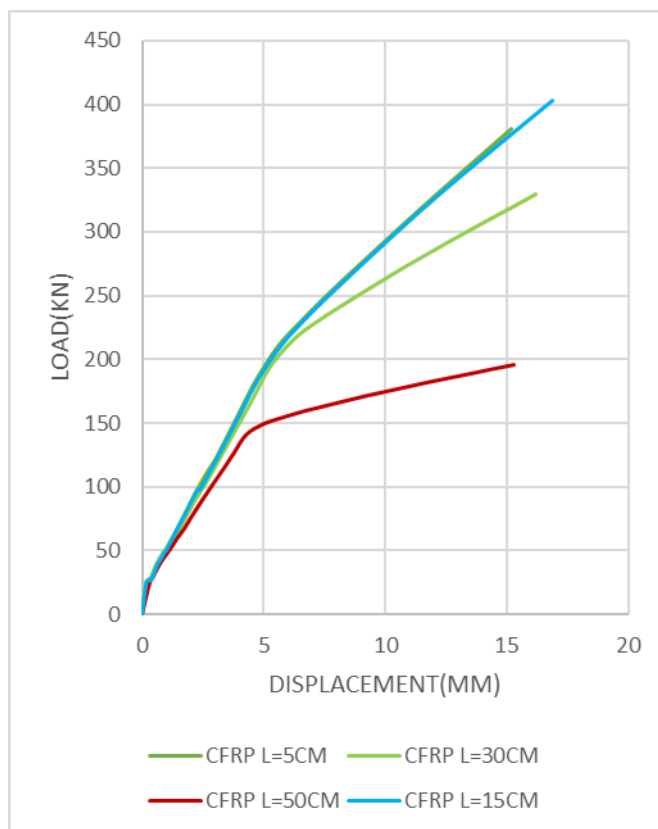


Figure 10. Load-displacement in control model (L=1.5 m) with CFRP fibers

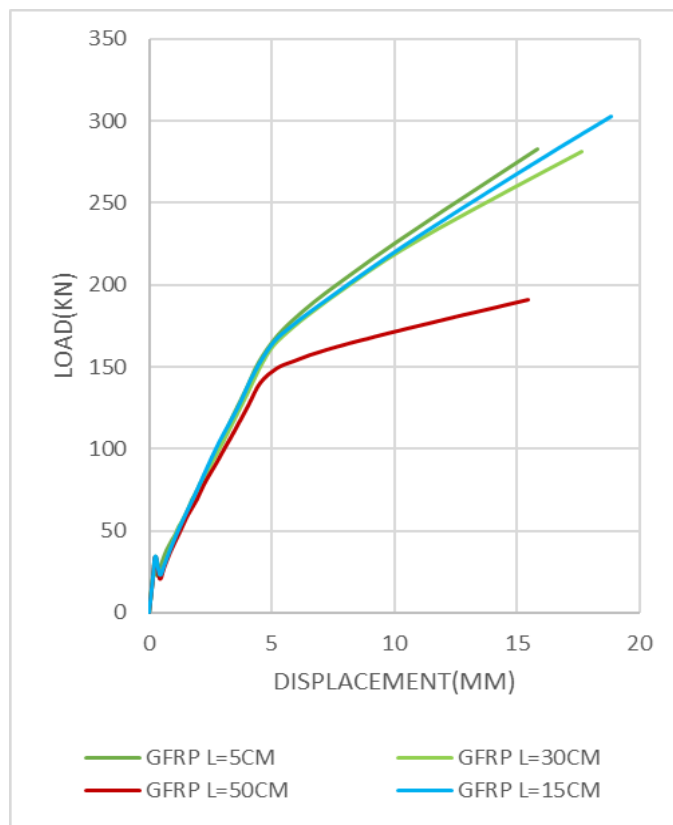


Figure 11. Load-displacement in control model (L=1.5 m) with GFRP fibers

Looking more closely at figures 10 and 11 indicates that no considerable changes were observed in the final loading capacity of CFRP fiber-reinforced slabs in 5 and 15 cm distances from the face of support and GFRP fiber-reinforced slabs in 5, 15 and 30 cm distances from the face of support. After passing these ranges and by decrease in the fiber length, the final capacity of the hollow-core slab decreased gradually.

Table 5. Difference of final load values and type of failure in Hollow core slab with length of 1.5 m

Type and thickness of fibers	Type of failure	Final load (KN)	Decrease of final load after variation of fiber length
CFRP model with length of 5 cm	Bending rupture	376	-
CFRP model with length of 15 cm	Bending rupture	374.35	Almost 0
CFRP model with length of 30 cm	Bending rupture	320.63	15
CFRP model with length of 50 cm	Bending rupture	195.75	48
GFRP model with length of 5 cm	Bending rupture	289	-
GFRP model with length of 15 cm	Bending rupture	278	Almost 0
GFRP model with length of 30 cm	Bending rupture	276	4
GFRP model with length of 50 cm	Bending rupture	191	34

Table 5 shows the final load and the type of rapture differences in the hollow-core slab with the length of 1.5 m.

According to figures 10 and 11 and table 5, calculating the optimal length of FRP fibers for hollow-core slabs reinforcement to reduce the costs and time is inevitable.

4.2. Investigating the capacity of the hollow-core slab with changing the FRP sheets length in the model with the length of 9 m

To investigate the effect of FRP sheet length on the hollow-core slabs reinforcement, first, a slab with dimensions of 900×67.5×30 cm and two holes with the diameter of 10 cm was constructed in the middle and along the length of the slab. Then, this slab was reinforced with different lengths of CFRP and GFRP fibers with distances of 60,

120, 200 and 300 cm from the face of support for CFRP fibers and 60, 120, 200, 300 and 400 cm from the face of support for GFRP fibers. Next, to investigate the capacity of the hollow-core slab reinforced with different lengths, the targeted models were compared in terms of the changes in the final loading capacity, the type of failure and the type of crack in the hollow-core slab. To calculate the final capacity of the concrete hollow-core slab, displacement push was applied and Von Mises stress values were observed. In final steps of loading, stress reached its final values in the compression part and also the values of stress in tension rebars reached their yielding values in the final steps of loading. The noticeable point in this model is that the stress in FRP fibers remained in its linear stage and did not reach its final limit which proves the flexible behavior of the slab. Moreover, the amount of created force in GFRP is lower than that in CFRP which could be attributed to the lower hardness of this layer.

(L in the following line graphs refers to the distance of CFRP and GFRP fibers from the face of support).

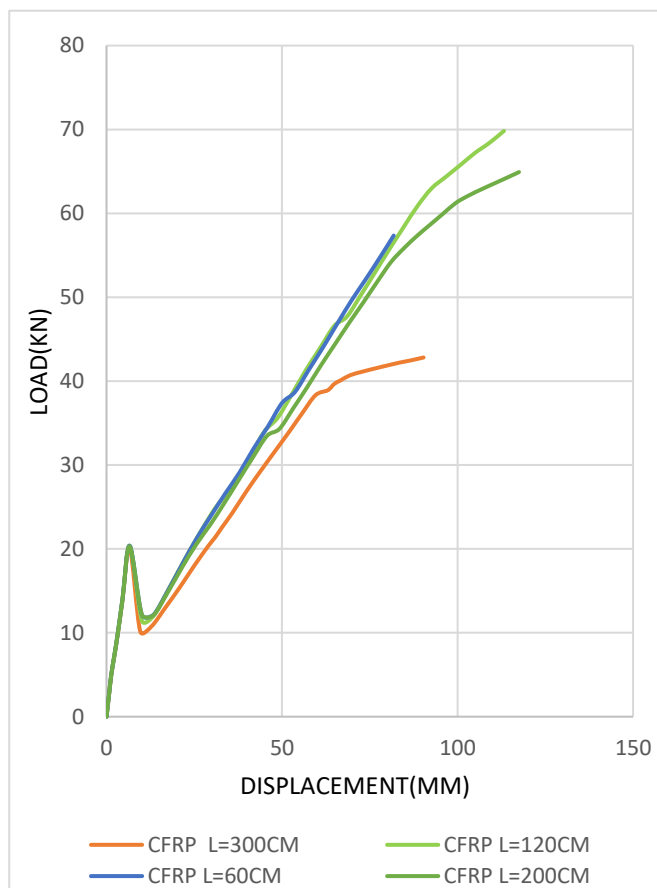


Figure 12. Load-displacement in 9-meter long hollow core slab with CFRP fibers

Looking more closely at figures 12 indicates that no considerable changes were observed in the final loading capacity of CFRP fiber-reinforced slabs at distances of 60, 120 and 200 cm from the face of support. After passing these ranges, the final capacity of the slab underwent a considerable drop.

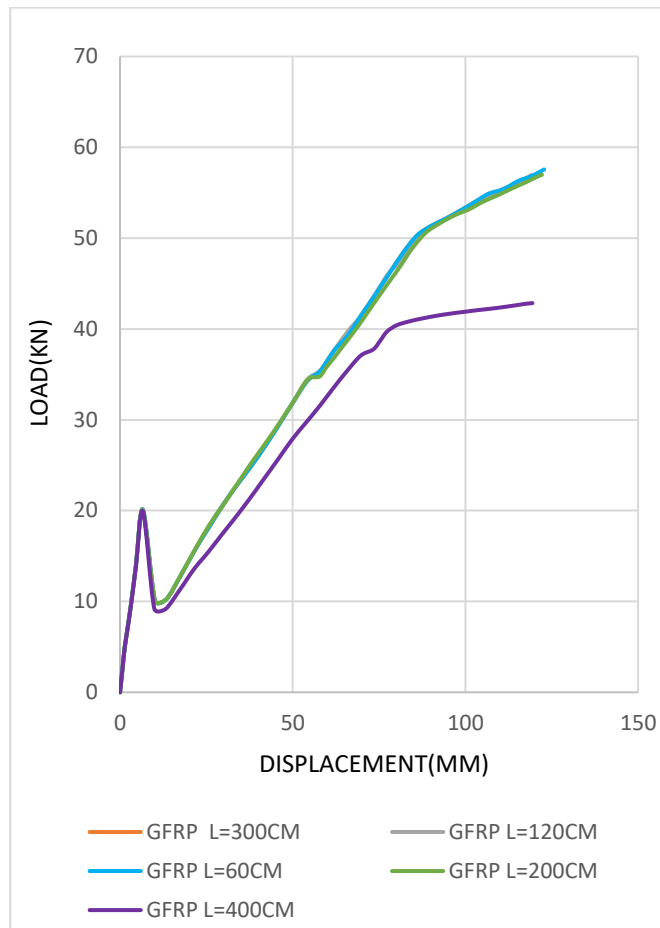


Figure 13. Load-displacement in 9-meter long hollow core slab with GFRP fibers

Looking more closely at figures 13 indicates that no considerable changes were observed in the final loading capacity of GFRP fiber-reinforced slabs at distances of 60, 120, 200, and 300 cm from the face of support. After passing these ranges and by decrease in the fiber length, the final capacity of the hollow-core slab decreased gradually.

Table 5. Difference of final load values and type of failure in Hollow core slab with length of 1.5 m

Type and thickness of fibers	Type of failure	Final load (KN)	Decrease of final load after variation of fiber length
CFRP model with length of 60 cm	Bending rupture	56	-
CFRP model with length of 120 cm	Bending rupture	54	Almost 0
CFRP model with length of 200 cm	Bending rupture	53	5
CFRP model with length of 300 cm	Bending rupture	42	25
GFRP model with length of 60 cm	Bending rupture	52	-
GFRP model with length of 120 cm	Bending rupture	51	Almost 0
GFRP model with length of 200 cm	Bending rupture	51	2
GFRP model with length of 300 cm	Bending rupture	50	4

Table 6 shows the final load and the type of rapture differences in the hollow-core slab with the length of 9m.

According to figures 12 and 13 and table 6, calculating the optimal length of FRP fibers for hollow-core slabs reinforcement to reduce the costs and time is inevitable.

5. Conclusion

FRP technology plays a crucial role in the reinforcement of the structural bearing components due to its prominent features such as the proportion of the high resistance to the weight ratio and durability against corrosion. Consequently, several structures, particularly marine structures and structures exposed to alkaline corrosive agents,

have been reinforced and strengthened by this technique all over the world. Moreover, FRP sheets are used for reinforced concrete slabs retrofit. Numerous bridges and buildings are made of hollow-core slabs which have been used to reduce the concrete volume, costs and also the structure weight. Some of these buildings and bridges are now seriously damaged due to natural disasters such as earthquake, wind, erosion, material fatigue, alkaline and acidic corrosive agents. Therefore, the repair, reinforcement and increase in the strength of these structures are essential.

This study scrutinized the effect of FRP sheets' length on the ultimate loading capacity of CFRP and GFRP strengthened hollow-core slabs by the finite element method. In this regard, the concrete hollow-core slab was modeled in ANSYS software and compared with the available experimental model for verification. Then, the bending reinforcement of the aforementioned slab by CFRP and GFRP fibers was investigated in different distances from the face of support. These changes were inserted into hollow-core slabs with lengths of 1.5 m and 9 m. It was indicated that in the 1.5 m slab, the optimal CFRP fiber length was about one-tenth from the face of support and this value was one-fifth for GFRP fibers. Also, For the 9 m hollow-core slab, the optimal CFRP fiber length was about one-fifth from the face of support and this value was one-third for GFRP fibers. It can be concluded that that if these numbers are exceeded, no significant changes will occur in the final capacity of hollow-core slabs; however, if the FRP length is less than this value, the ultimate capacity of hollow-core slabs will decrease dramatically.

References

1. Taljsten, B., (1997), Strengthening of Existing Concrete Structures with Epoxy Bonded Plates of Steel or Fiber Reinforcement Plastics, *J Mater Civil Eng*, Vol.26(4), p. 206-212.
2. Triantafilou, T.C., (1998), Shear Strengthening of Reinforced Concrete Beams Using Epoxy- Bonded FRP Composites, *ACI Structure Journal*, Vol.95(2), p. 107-115.
3. Vasques, A. and Karbahari, V.M., (2003), Fiber Reinforced Polymer Composite Strengthening Concrete Slabs with Cutouts, *ACI Structure Journal*, Vol.100(5), p.58-70.
4. Mosallam, A.S. and Mosalamr, K.A., (2003), Strengthening Two Way Concrete Slabs with FRP Composite Laminates, *Constr Build Mater*, Vol.17, p.43-54.
5. Zhang, B., Masmoudi, R. and Benmokrane, B., (2004), Behaviour of one-way concrete slabs reinforced with CFRP grid reinforcements, *Construction and Building Materials*, Vol.18, p.625-635.
6. Enochsson, O., Lundqvist, J., Taljsten, B., Rusinowski, P. and Olofsson, T., (2007), CFRP strengthened opening in two way concrete slabs - An experimental and numerical study, *Construction and Building Materials*, Vol.21, p.810-826.
7. Foubert, S., Mahmoud, K. and El-Salakawy, E., (2016), Behavior of Prestressed Hollow-Core Slabs Strengthened in Flexure with Near-Surface Mounted Carbon Fiber-Reinforced Polymer Reinforcement, *Journal of Composites for Construction*, Vol.20(6), p.1-10.
8. Pachalla, S.K.S. and Prakash, S.S., (2017), Load resistance and failure modes of GFRP composite Strengthened hollow core slabs with openings, *Materials and Structures*, Vol.50(3), p.1-14.
9. Kankeri, P., Prakash, S.S. and Pachalla, S.K.S., (2018), Experimental and Numerical Studies on Efficiency of Hybrid Overlay and Near Surface Mounted FRP Strengthening of Pre-cracked Hollow Core Slabs, *Structures*, Vol.15, p.1-12.
10. Meng, X., Cheng, S. and El Ragaby, A., (2019), Shear strengthening of prestressed concrete hollow core slabs using externally bonded carbon-fiber-reinforced polymer sheets, *Precast/Prestressed Concrete Institute (PCI) Journal*, Vol.64(5), p.77-94.
11. Li, X., Wu, G., Wang, S. and Yu, X., (2019), Flexural Behavior of Hollow-Core Slabs Strengthened with Prestressed Basalt FRP Grids, *Journal of Composites for Construction*, Vol.23(3), p.1-12.
12. Nguyen, T.N., Tan, K.H. and Kanda, T., (2020), Effect of polypropylene and steel fibers on web-shear resistance of deep concrete hollow-core slabs, *Engineering Structures*, Vol.210, p.1-13.
13. Shabanali, M.R., (2001), Finite element analysis by ANSYS, Fifth edition, Nas publisher. (In Persian)
14. Salehi Yanehsari, M.M., Modanloo, A., Beygi, M. and Navayi Neya, B., (2013), The Flexural Strength and Flexural - Shear void Concrete Slabs with CFRP and GFRP and Steel Sheets, *Journal of Ferdowsi Civil Engineering*, Vol.24(2), p.57-68. (In Persian).