Research Article

Dynamic Pricing for Perishable Items Considering Substitutes and Advertisement

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Abstract: This study is an attempt to model and solve a pricing model of inventory control model. The literature on this topic is really comprehensive and many different factors including product variety, product perishability, money's temporal value, etc. have been thoroughly studied. Most of these studies are included in literature review section of this paper. The innovation and distinction between this study and those available in the literature is consideration for advertisement as an influential factor which affects demand function. In addition, two different models are studied through considering maintenance costs in two varieties as time-dependent and time-independent. In this model, the demand function will be investigated for two products which are simultaneously on the market. The demand for products is a multi-variate function of products' price and advertisement and timing factors through considering substitution rate. Substitution rate clarifies that a customer can be inclined toward any of the products with different prices. To enhance the applied dimension of this study, advertisement factor (frequency advertisement) has been considered on the one hand and maintenance costs have been considered in two different forms of time-dependent and time-independent on the other. The following section entails a schematic representation of this study. Profit function has been optimized through mathematical modelling of the problem. The main question of this study is selecting an optimal level for the price and advertisement expenses or continuity in pricing problem to maximize the profits obtained by a business in a [0, T] time frame.

Keywords: supply chain, dynamic pricing, perishable items

1. Introduction

Nowadays, the organizations are operating in a highly dynamic and competitive environment. In such circumstances, customer satisfaction is considered as the basic prerequisite of organizational survival and success. The environment governing the firms made them to exclude traditionally-oriented operations. In classic perspective, price has been considered as the only instrument to generate revenue, while nowadays, generating revenue plays a key role in customer satisfaction. Besides, coordination and orientation of pricing and production- and inventory-control related factors are considered as really significant. In an attempt to model most of the inventory control problems, unlimited lifetime of the products is considered only implicitly. That's while, most of the products lose their commercial value through time and they would become useless. For some products, this process is speedier that what you may even conceive. These products are called "perishable". It is clear that conducting inventory control for these products is considered as really significant because of the incurred costs as a result of losing part of the inventory.

Due to increased expenditure, globalization, shorter product life time and the immediacy required in responding the customers attracted ore customers to the supply chain. Accordingly, revenues must be maximized and expenditures must be minimized. One of the expenditures involved in supply chain is inventory control costs. On the other hand, pricing/cost is considered as the best leverage for maximizing the profits gained from supply chain through proper selection of resources and demands. In case pricing wouldn’t be undertaken properly, it will lead to increased profit from supply chain. On the contrary, in case price would get more or less than product's value, it will influence both demand and revenue and it will lead to a reduction of demands for a firm and its failure. In such an environment, customer is regarded as the necessary prerequisite for success and survival. Thus, proper pricing in supply chain is considered as really significant. In the past, pricing and inventory control operations have been done separately by marketing and production departments. Each of these departments were directed toward maximizing their own profits. Marketing section achieved this maximum profit though cutting on costs. Different businesses have understood that customer satisfaction is necessary for profit maximization.

In recent studies, pricing and inventory control have been considered together. Considering the customers' highs sensitivity to pricing, care must be taken in pricing similar products, because in case the customer perceives any difference in the price of two similar products, he would prefer not buy that. Instead, he prefers to buy similar products in a lower price. Therefore, ignoring product substitution may lead to abnormal profits.

Supply chain management is one of the fundamentals in the world of commerce. Supply chain doesn’t concern manufacturers and suppliers only; rather, it includes other different facets such as transportation, warehousing, retail sector, and even the customers themselves. Supply chain is a network composed of facilities and distribution options which concerns raw material procurement, transforming them into intermediate or final products, and finally distributing them among customers. Supply chain management is directed toward coordination in production, inventory (warehouse), locating, and transporting among participants in a particular supply chain in order to achieve the best combination of accountability and efficiency to achieve success in the market (Michael Hug, 2003).

In most of pricing attempts, price is considered as a natural mechanism for revenue management. In most of retail sectors and businesses, dynamic pricing procedure is applied. Some forms of dynamic pricing include personal pricing, discounts, and auctions. Dynamic pricing is just as old as the trade itself and has been common from the beginning. As an example, consider the owners' negotiations on a higher price in marketplace in order to gain higher profits as well as their customers' satisfaction (Mihami & Kamalabadi, 2012).

In recent decades, many developments have been observed with regard to scientific methods applied for dynamic pricing. Dynamic pricing is a favorable tactic for different prices over time and is really suitable for goods such as seasonal clothes. In case, these goods won't be sold in their appropriate season, they may lose their value.

He et al. (2010) studied inventory's optimal policies for perishable items in a manifold market. They suggested that one of the techniques to increase firms' profitability is the existence of different times and geographical locations for products' sale. In other words, the good must have its own specific demand in firm's marketplace and therefore the market would be manifold. In their study, the optimal production policy has been applied for manufacturing final products and the optimal ordering policy has been computes for raw products. In this study, price has been considered as a constant parameters and therefore, its value is pre-determined. In addition, perishability rate has been defined in the model using a constant parameter. Numerical examples and results of sensitivity analysis on different model parameters are included in final section of this paper. The weakness of this paper may lie in its demand function. Demand function has been considered as constant and known; while, the literature has considered demand function in terms of time, price, or inventory level.

Geetha and uthayakumar (2010) offered an inventory model for non-immediate, delayed payment perishable goods. In their work, beneficial analytical and mathematical illustrations have been included to prove the unity and optimism of answers. In addition, different examples have been used to illustrate model's efficiency. Another significant point in this study is the inclusion of managerial advices based on the recommended model to reduce inventory costs. Mihami & Kamalabadi (2012) and Avina (2013) consider single-term problem. Mihami & Kamalabadi suggested that demand is a function of both time and price and they developed the optimization problem to define optimal price, optimal order amount, and optimal substitution timing schedule for non-immediate perishable and depreciable products. Also, Avindava (2013) applied price- and time-dependent demand function and developed the mathematical model to determine the optimal price.

Rabbani et al. (2015) considered demand function as a function of sales price and advertisement factor. In this study, advertisement factor has been included as advertisement frequency in a three-staged supply chain. The impact of substitute product as well as the maintenance costs are time-dependent. Advertisement plays a significant role in enhancing customers' demands, in this study, products' perishability will start right after the product emergence (immediate perishability). Then, optimization would be illustrated through concave revenues.

Cho et al. (2014) considered a manifold problem for perishable items with different life times. They found that demands for different product lifetime depend upon the products' price and the existence of substitute products. Then, the authors attempted optimization and calculation of optimal price through dynamic modelling.

Considering the literature on supply chain pricing and the fact that price plays a basic role in all aforementioned models as the main parameter influencing customer demands, we can conclude that these models contributed to maturation of this field of study through various hypotheses. The relevant studies considered factors such as allowed delays or shortage of products, constant or time-dependent deterioration rate and consideration of one product or multiple products simultaneously.

Demand displacement rate has been considered as a parameter influencing product demand in some of the studies. The basic premise of these studies was that customers' demands may be displaced through price and other product-related features.

Advertisement has been studied as a potentially influential factor in cases which two products are simultaneously available in the market. Besides, the maintenance costs can be considered both constant and time-dependent/independent. This is considered as a distinction with the relevant literature.

In pricing problems considering inventory control, demand is considered as a function of price or time. In this particular study, demand is a function of price and promotion expenses. Promotional expenses can positively impact profits. Consideration for this price and its modelling is considered as one of the innovations of this study. Moreover, maintenance problems, i.e. time-dependent and time-independent will be solved through relevant models.

Considering the literature on perishable products' pricing and the fact that advertisement is a significant factor in increasing demands, demand will be regarded as a function of price and advertisement expenses in this study.

Di=f(pi,ca)

And the main question of this study concerns finding out the optimal price level and advertisement expenditures to maximize the profits obtained from a business in a particular [0, T] time frame.

The author(s) seek to model perishable items' pricing problem and find out the optimal prices and advertisement expenditures through influencing demand function through the variation resulting from advertisement expenditures.

We attempt to illustrate the impact of advertisement on demand through adding a factor to linear demand function (the resulting function is not a linear one anymore). Then through inclusion of advertisement expenditures in cost function and calculation of revenue and profits, the impact of this cost on these functions will be revealed. Finally, the optimal prices and advertisement expenditure will be defined. Then, the study models will be developed through consideration of time-dependent and time-independent maintenance costs.

The optimal value of problem variables will be found through developing a mathematical model and finding its solution.

Solving this model may be applicable for retailers who face substitute goods (e.g. grocery, seasonal clothes, etc.) in sales period.

2. Materials and Methods

The purpose of these models is to define the optimal price policy and the frequency of advertisement is such that profit maximization would be achieved. As it has been previously mentioned in this study, advertisement factor plays a key role in changing and increasing customers' demands which is considered in the model as the advertisement frequency. In this study, the problem will be solved in two different models. First, through considering a substitute product for products under study; and second, through considering maintenance problem in two different forms.

In this section, two different models have been included for definition of an optimal pricing policy for products with a short lifetime.

In the first model, the maintenance costs has been considered as constant for all new and old products. In this model, the old products still persist in the market even after the entrance of the new products. As a simple example, one can consider raisin as an agricultural product. Upon the entrance of the new agricultural product to the market, the older raisin product has a lower price compared with the new one and this lower price will make the customers to put a higher demand for this older product. In other words, the customers would be displaced toward purchasing this new item upon the emergence of the new products (Chu Lee, 2015).

In the second model, maintenance costs are considered for old products. The maintenance cost of these products would be increased linearly as a result of getting older (citing maintenance costs section in Rabbani et al. (2015).

In these models, mathematical computations are used to define the optimal price and advertisement frequency. Due to the similarity of the symbols used in these models, the applied symbols in the models would be elaborated upon first.

Demand displacement coefficient (L): this coefficient expresses the percentage of demand which will be displaced as a result of differences in price and customers' tastes. It is clear that the demand transfer rate at the beginning of the period to the previous one or from the last period to the next period will equal zero.

Time period (T): the whole study term equals a time unit (e.g. a year)

Time period (t): the time when an old product is available in the market and is unavailable afterwards.

Profit Function (TR): this function expresses the existing system's profit which equals the difference between the revenue generated from demand satisfaction and the existing system costs.

Revenue Function (R): the revenue generated from satisfying customers' demands according to products prices.

Demand Functions (D_1 and D_2): these functions express the customers' demands which depend upon the system's price and advertisement. These functions are comprised of two parts, one linear price-dependent part and an advertisement factor.

Model's decision variables

Advertisement frequency value (A): the number of demands being made throughout a period. As it has been mentioned before, the innovative aspect of this study is the same advertisement factor.

The price of products being offered (P_1, P_2): in this mode, two products can be simultaneously available in the market. The customers express their own interest and demand for a product according to the products' prices. The optimal prices are two be found after model generation ad its solution.

System costs

- **Maintenance costs (H):** this type of cost concerns the maintenance of the inventory within the system. Consideration for this cost difference is the departure point of these two models. This cost has been considered as constant and as a linear price ascending function for older products in first and second models respectively.
- **Advertisement costs (G):** the cost incurred on a system for each advertisement iteration. In these models, total advertisement cost is calculated as the multiplication of advertisement frequency by the advertisement costs (A*G=AC) (A displays the frequency/iteration of advertisement in each term/period).
- **Constant ordering cost (K) and Purchase costs (C):** the purpose behind the term "ordering" is attempts made to purchase or supply the required products/goods for an organization. Ordering costs also includes other different costs such as booking administration, transportation, information processing, product delivery costs, and salary of purchase personnel. In case a product piece is manufactured inside a factory, thus its ordering cost will equal the cost of preparing production line, machinery, personnel salary, etc. for producing the intended piece.

Model Assumptions

Nearly, there are some assumptions for pricing and inventory control, some of which are real and some others are used to simplify the models. Model assumptions re as follows:

- Demand function has been considered as a function of price and advertisement
- Shortage is not allowed within the system and doesn't occur at all.
- Procurement costs are nearly zero
- Time horizon is unlimited
- Advertisements are only for new products.
- The price of new products is a coefficient of older ones.
- The maintenance costs function is a linear one.
- There's a transfer of demand from older products to the new ones.
- The amount of older product available within market equals that of the new product at the end of the term/period.

Inventory control through time

We study this model for T time period. In fact, a cycle of the above curve will be studied. In this period, Q_2=I02 of the older product will be available on the market and the new product will enter the system (immediately and one for all). For the simplicity of task, this time will be set equal to the temporal offset and T will be set as a temporal unit equal to one year.

First model: Considering constant maintenance costs:

Maintenance costs for both products will be considered as constant

H1=H2

Demand Functions

Considering the curve, demand functions will be defined as follows:

D1=(b-ap)(1+A)λ+LP(1-α)2 (1)

D2=(b-ap(1-α)2)-LP(1-α)2 (2)

As discussed in parameter setting section, the price for older products is different for these two models and their relation is as follows:

P(1-α)2=P2 (3)

Besides, one of the problem's assumptions is that there's a demand transfer rate from older products to newer ones.

Demand transfer rate is the same demand transfer for any change in price unit.

Q is calculated from the inventory curve and calculating the area under the inventory curve:

$Q=Q_1+Q_2$ (4)

Q1=t1($b-ap$(1$-\alpha$)2$-Lp$(1$-\alpha$)2)+T((1+A)λ($b-ap$)+Lp(1$-\alpha$)2) (5)

Q2=∫$D2t$10dt=t1($b-ap$(1$-\alpha$)2$-Lp$(1$-\alpha$)2) (6)

Revenue Function

Revenue function is in fact the revenue generated from demand supply and is defined as the multiplication of products' demands by the demand itself:

SR=SR1+SR2 (7)

Which equals the sum of revenue generated from supplying the demand for older product and the revenue generated from supplying the demand for the new product.

SR=∫$D2t$10*p(1-α)2dt+∫$D1T$0*Pdt

SR=pT((1+A)λ($b-ap$)+Lp(1$-\alpha$)2)+pt1($b-ap$(1$-\alpha$)2$-Lp$(1$-\alpha$)2)(1$-\alpha$)2 (8)

Profit function

This function is defined as the subtraction of revenue from the costs. At first, system costs will be defined:

TP=SR$-K-C*Q-A*G-H*Q$ (9)

TP=$-$250$-AG-$12bHt12+12aHpt12+12HLpt12$-C$(2t1($b-ap$(1$-\alpha$)2$-Lp$(1$-\alpha$)2)+T((1+A)λ($b-ap$)+Lp(1$-\alpha$)2)) +pT((1+A)λ($b-ap$)+Lp(1$-\alpha$)2)+pt1($b-ap$(1$-\alpha$)2$-Lp$(1$-\alpha$)2)(1$-\alpha$)2$-aHp$t12$\alpha-HLp$t12α+12aHpt12α2+12HLpt 12α2$-H$(12(1+A)λbT2$-$12a(1+A)λpT2+12LpT2+bTt1$-apT$t1$-LpT$t1$-LpT$2α+2apTt1α+2LpTt1α+12LpT2α2$-a$ pTt1α2$-LpT$t1α2)

Solution for the first model

The main purpose of this study was to obtain the optimal values of A, P, and t, such that the target (profit) function would be maximized accordingly.

The mathematical model will be formed through the mentioned functions. Then, the optimization would be illustrated and optimal point would be determined through the use of algorithms and mathematical solutions.

Due to the complexity of the equations offered in this model, the solution applied by Nakha'ee and Mihami would be applied. To this end, it's necessary to assume an optimal and minimum value for a constant value of a bi-variate function. This will be possible only through illustrating the concaveness of partial derivatives computation function. Finally, the optimal value for all three variables will be obtained as a result of function's concaveness for the third parameter (by considering the other two variables as constant).

Due to the use of Hessian matrix as a solution step, it would be clarified first and optimization would be illustrated then.

Hessian Matrix

For f:Rn R function, Hessian function will be defined as:

H=[$\partial A,ATP$ $\partial A,pTP$

$\partial A,pTP$ $\partial p,pTP$]

Now, in case the definite Hessian matrix will be positive, it would be convex; otherwise (negative Hessian matrix), it would be concave.

Iterative algorithm

The following algorithm is used to find the optimal points for numerical values:

Step 1: set $A_1=a_0$

Step 2: find the P_i value from the optimal price equation.

Step 3: calculate t and find A_2 from equation (2).

Step 4: repeat steps 2-3 until the following equation holds.

The difference between two A values will be smaller than a small value.

In case a variable becomes convergent to an optimal value, other variables become convergent as well.

Inventory Control Demand Function

D1=(b-ap)(1+A)λ+LP(1-α)2

D2=(b-ap(1-α)2) -LP(1-α)2

Q1 value is the same inventory at the beginning of a new product's term. It will be obtained from the following equation:

$Q1 = \int_0^{t1} D2dt + \int_0^{T} D1dt$

As it has been set out in problem's assumptions, our main assumption is that new product's unused inventory equals the inventory of the older product which entered this term.

The following equation would result from integer calculations:

Q1=t1$(b-ap(1-\alpha)2-Lp(1-\alpha)2)+T((1+A)\lambda(b-ap)+Lp(1-\alpha)2)$

Besides, the calculations for Q_2 will be as follows:

d Q2=∫$D2t$10t=t1$(b-ap(1-\alpha)2-Lp(1-\alpha)2)$

finally, the following equation is used to calculate total Q:

Qtotal=Q1+Q2

The products inventory at t time would be:

I1(u)=Q1-∫$D1duu$0

t1$(b-ap(1-\alpha)2-Lp(1-\alpha)2)+T((1+A)\lambda(b-ap)+Lp(1-\alpha)2)-u((1+A)\lambda(b-ap)+Lp(1-\alpha)2)$ (10)

I2(u)=Q_2-$\int_0^{u} D2d$

t1$(b-ap(1-\alpha)2-Lp(1-\alpha)2)-u(b-ap(1-\alpha)2-Lp(1-\alpha)2)$ (11)

Cost calculation of the second model

1- Maintenance costs:

HC_1: new products' maintenance costs

$HC_1=H*\int_0^{u} I1du$ 12

$$H(\frac{1}{2}(1+A)^{\lambda}bT^2-\frac{1}{2}\alpha(1+A)^{\lambda}pT^2+\frac{1}{2}LpT^2+bT\mathrm{t}1-apT\mathrm{t}1-LpT\mathrm{t}1-LpT^2\alpha+2apT\mathrm{t}1\alpha+2LpT\mathrm{t}1\alpha+\frac{1}{2}LpT^2\alpha^2-apT\mathrm{t}1\alpha^2-LpT\mathrm{t}1\alpha^2)$$

As it has been discussed, the new product's maintenance cost is constant n this model.

HC_2: older products' maintenance costs

$HC_2=\int_0^{T}(H+UT)*I2du$ 3-15

$$\frac{1}{6}\mathrm{t}1^2(3H+\mathrm{t}1w)(b-(a+L)p(-1+\alpha)^2)$$

2. Revenue calculation (SR):

It is in fact the revenue generated from supplying demands.

SR=$P*(1-\alpha)$^2∫$D2dtt$10+$P*$∫$D1dtT$0

$pT((1+A)\lambda(b-ap)+Lp(1-\alpha)2)+p\mathrm{t}1(b-ap(1-\alpha)2-Lp(1-\alpha)2)(1-\alpha)2$ (13)

3. Take Profit (TP)

TP = SR - K - C*Q - A*G - H1 - H2

$-250-AG-C(2t1(b-ap(1-\alpha)2-Lp(1-\alpha)2)+T((1+A)\lambda(b-ap)+Lp(1-\alpha)2))+pT((1+A)\lambda(b-ap)+Lp(1-\alpha)2)-16t1$ $2(3H+t1w)(b-(a+L)p(-1+\alpha)2)+pt1(b-ap(1-\alpha)2-Lp(1-\alpha)2)(1-\alpha)2-H(12(1+A)\lambda bT2-12a(1+A)\lambda pT2+12LpT2+$ $bTt1-apTt1-LpTt1-LpT2\alpha+2apTt1\alpha+2LpTt1\alpha+12LpT2\alpha2-apTt1\alpha2-LpTt1\alpha2)$ (14)

Our main purpose is to optimize this function. To this end, the existence of optimal points would be illustrated through function's concaveness. Then the optimal points would be calculated to this end, the Hessian matrix will be formed first as a result of computing partial derivatives and then, this matrix's determinant's sign will be studied.

Calculating the second model and studying its optimization

At first, partial derivatives of profit function would be studied and calculated.

$$\partial_p \mathrm{TP} =$$

$$-C(2t1(-a(1-\alpha)^2 - L(1-\alpha)^2) + T(-a(1+A)^\lambda + L(1-\alpha)^2)) + pT(-a(1+A)^\lambda + L(1-\alpha)^2) + T((1+A)^\lambda(b-ap) + Lp(1-\alpha)^2) + pt1(-a(1-\alpha)^2 - L(1-\alpha)^2)(1-\alpha)^2 + t1(b - ap(1-\alpha)^2 - Lp(1-\alpha)^2)(1-\alpha)^2 + \frac{1}{6}(a+L)t1^2(3H+t1w)(-1+\alpha)^2 - H(-\frac{1}{2}a(1+A)^\lambda T^2 + \frac{LT^2}{2} - aTt1 - LTt1 - LT^2\alpha + 2aTt1\alpha + 2LTt1\alpha + \frac{1}{2}LT^2\alpha^2 - aTt1\alpha^2 - LTt1\alpha^2)$$

$$\partial_{\mathrm{t1}} \mathrm{TP} =$$

$$-2C(b - ap(1-\alpha)^2 - Lp(1-\alpha)^2) - \frac{1}{6}\mathrm{t1}^2 w(b-(a+L)p(-1+\alpha)^2) - \frac{1}{3}\mathrm{t1}(3H + \mathrm{t1}w)(b - (a + L)p(-1+\alpha)^2) + p(b - ap(1-\alpha)^2 - Lp(1-\alpha)^2)(1-\alpha)^2 - H(bT - apT - LpT + 2apT\alpha + 2LpT\alpha - apT\alpha^2 - LpT\alpha^2)$$

$$\partial_A \mathrm{TP} =$$

$$-G - (1+A)^{-1+\lambda}C(b-ap)T\lambda + (1+A)^{-1+\lambda}p(b-ap)T\lambda - H\left(\frac{1}{2}(1+A)^{-1+\lambda}bT^2\lambda - \frac{1}{2}a(1+A)^{-1+\lambda}pT^2\lambda\right)$$

$$\partial_{\mathrm{t1,t1}} \mathrm{TP} =$$

$$-\frac{2}{3}\mathrm{t1}w(b-(a+L)p(-1+\alpha)^2) - \frac{1}{3}(3H + \mathrm{t1}w)(b-(a+L)p(-1+\alpha)^2)$$

$$\partial_{A,A} \mathrm{TP} =$$

$$-(1+A)^{-2+\lambda}C(b-ap)T(-1+\lambda)\lambda + (1+A)^{-2+\lambda}p(b-ap)T(-1+\lambda)\lambda - H(\frac{1}{2}(1+A)^{-2+\lambda}bT^2(-1+\lambda)\lambda - \frac{1}{2}a(1+A)^{-2+\lambda}pT^2(-1+\lambda)\lambda)$$

$$\partial_{A,p} \mathrm{TP} =$$

$$a(1+A)^{-1+\lambda}CT\lambda - a(1+A)^{-1+\lambda}pT\lambda + (1+A)^{-1+\lambda}(b-ap)T\lambda + \frac{1}{2}a(1+A)^{-1+\lambda}HT^2\lambda$$

$$\partial_{A,\mathrm{t1}} \mathrm{TP} = 0$$

$$\partial_{p,\mathrm{t1}} \mathrm{TP} =$$

$$-2C(-a(1-\alpha)^2 - L(1-\alpha)^2) + p(-a(1-\alpha)^2 - L(1-\alpha)^2)(1-\alpha)^2 + (b - ap(1-\alpha)^2 - Lp(1-\alpha)^2)(1-\alpha)^2 + \frac{1}{6}(a+L)t1^2w(-1+\alpha)^2 + \frac{1}{3}(a+L)t1(3H+t1w)(-1+\alpha)^2 - H(-aT - LT + 2aT\alpha + 2LT\alpha - aT\alpha^2 - LT\alpha^2)$$

3. Results

Numerical examples have been provided in this section in order to study the recommended model and the developed solution. Accordingly, the algorithm's performance will be studied through implementing it and studying its results under different sample problems. In the following, sensitivity analyses are conducted on the parameters. Then, the differences are illustrated through illustrations. It is noteworthy that all the computations are conducted through substitution in Mathematica 9.0 software.

Numerical examples and results analysis

The study models are developed through the constraints existing in the problem. These constraints are:

a>L a>=0 b>0 1 > w,λ,α>0

First model

In this model, the maintenance costs equal the constant value for both products. The numerical examples and sensitivity analysis is provided in this section (most of the model parameters are selected according to Rabbani et al. (2015)).

In this case, a simple algorithm will be offered for calculation of optimal points:

1. The primary value for A_1 will be set as : A_1=100
2. The primary price value will be set as t_1=0.01
3. The optimal p value will be obtained through the previous equations.
4. Then, A_2 will be calculated using t and P_2.
5. Repeat the algorithm until: $[Ai - Ai] < 0.001$.

The optimal values will be obtained in the 5th iteration.

A^*=2.182

P^*=16.407

T^*=0.822

The determinant value of Hessian function in this point equals 81144.1 (As evident, it is a positive value).

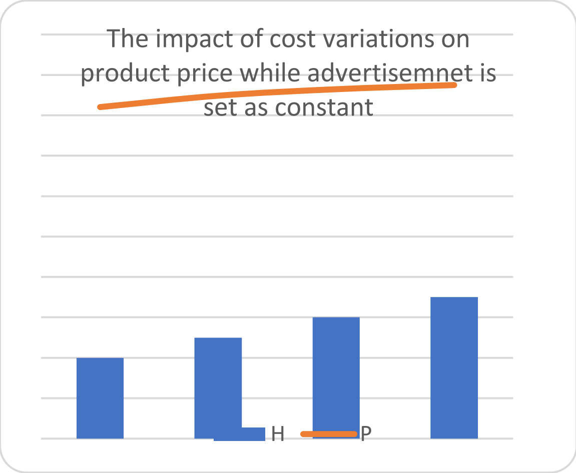
Now, the impact of variable parameters on optimal value will be studied (Table 1).

Table 1. the impact of variable parameters on optimal answers

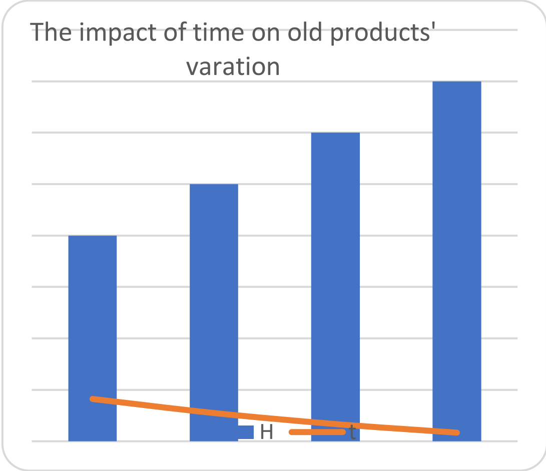
L	λ	H	P	A	T	SR	TP
75	0.03	4	16.4077	2.182	0.822562	32334.3	19899
		5	16.9705	2.182	0.549221	29801.2	18762.4
		6	17.2916	2.182	0.334361	17822.1	29152
		7	17.4965	2.182	0.167449	16974.3	27910.3
	0.04	4	16.3758	2.182	0.816095	32536.9	19998.1
		5	16.9358	2.182	0.543606	29992	19842.4
		6	17.2071	2.182	0.322956	17899.6	28806.6
		7	17.4375	2.182	0.160622	17861.6	28095
	0.05	4	16.2559	2.182	0.791822	32754.5	19994.5
		5	16.8766	2.182	0.534011	30230.8	19947.6
		6	17.1547	2.182	0.316252	29016	17980
		7	17.3833	2.182	0.154348	28274.7	17114.3

As evident from Table 1, any increase in maintenance costs will result in reduction of revenues for a constant value of λ. Thus, the resultant profit will be reduced as well. Besides, any increase in λ parameter will result in reduction of optimal price and therefore the generated revenue will be increased.

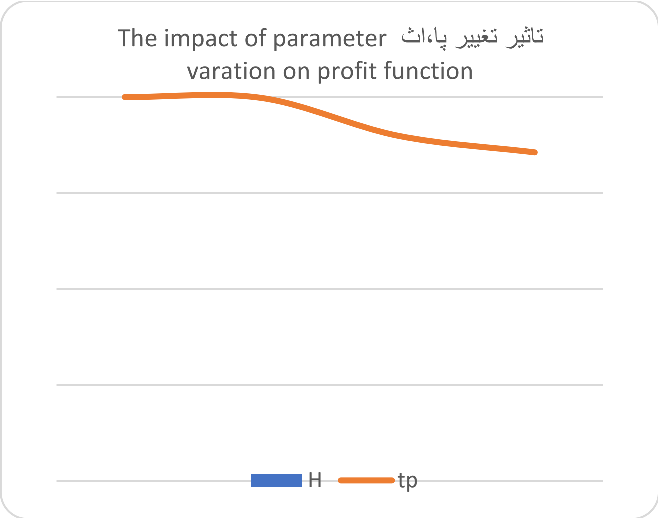
In addition, for any variation in λ optimal price parameter (i.e. increase), the time period of maintaining an old product in the market will be decreased (Figure 1).



A



B



C

Figure 1. a) products' price will be increased as a result of any increase in maintenance costs; b) as it is evident from this figure, the required time for the old product staying in the market will be increased as a result of increases in maintenance costs.

Then, the same problem would be repeated using the substitution coefficient. At first, the optimal value for advertisement will be calculated using the iterative algorithm (Table 2).

Table 2. second iteration of iterative algorithm

Iteration	A	T	P	Ai-Ai
1	100	0.01	P_0	
2	3.73395	0.730658	15.9539	
3	3.38383	0.813656	16.3637	
4	3.3514	0.821313	16.4015	
5	3.3514	0.822.98	16.4054	0.0001

A*=3.3514

P*=16.405

T*=0.822

As it can be found from a comparison of tales, any increase in transfer coefficient will result in the required time and cost for older products to remain constant; however, advertisement frequency/continuity will be increased. In other words, higher advertisement frequencies are required.

Second model

As it has been noted previously, the maintenance cost in this model is a function of time.

H2(t)=H+Wt

(Shah et al., 2013).

Figure 2 displays the behavior of maintenance cost function through the time passage.

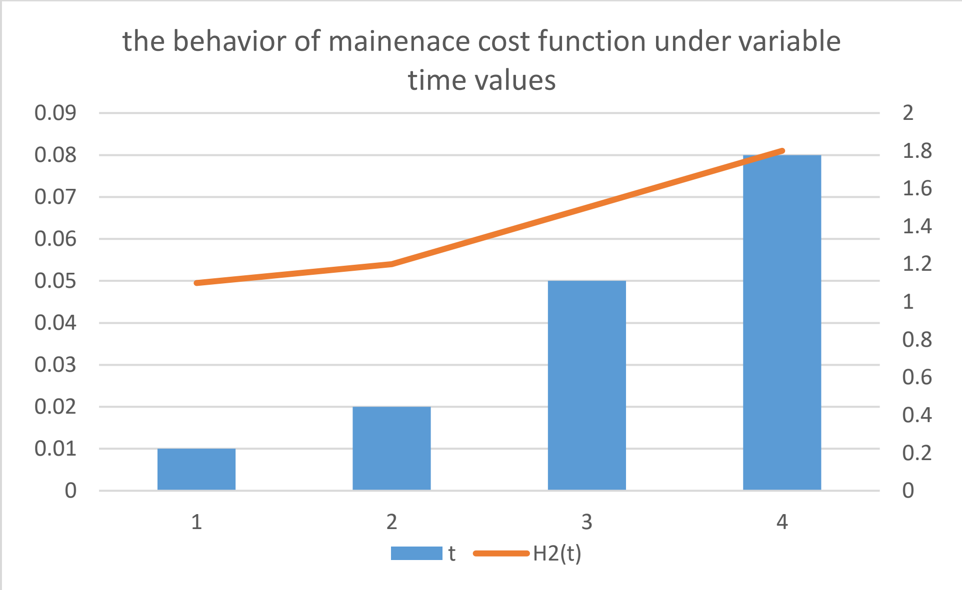


Figure 2. maintenance cost function behavior

As the figure suggests, maintenance cost function will be increased as a result of any increase in time.

Now, time-dependent maintenance cost problem will be solved (second model's parameters have been also selected according to Rabbani et al. (2015)).

In this model, iterative algorithm has been used for calculation of optimal point as well. Optimal point will be obtained in 4th iteration.

Second model's solution algorithm m and optimal point

Step 1: set A_1=100

Step 2: obtain P_i from optimal point equation.

Step 3: calculate the t value and obtain A_2 using two values.

Step 4: repeat 2nd and 3rd steps until the following equation holds:

$$\left|Ai - A^{i-1}\right| < 10^{-2}$$

Table 3. iterative algorithm to find the optimal points

Iteration	A_i	P*	T*	A_i-A_i+1	TR
1	100	12.9785	0.743845		4373
2	143.142	12.9538	.741738	43.142	43723.4
3	143.296	12.9523	0.74159	0.15	43723.4
4	143.307	12.9523	0.741589	0.00	43723.4

A*=143.307

P*=12.9523

T*=0.741589

TR=43723.4

The following points are noteworthy as a result of studying optimal values:

- In this model, the optimal price will be decreased because maintenance cost function will increase upon any increase in time. It is logical that the seller attempts increased sales rate and inventory reduction as a result of decreased prices. Higher maintenance durations is not to the system's benefit.
- Advertisement factor will also increase rapidly, such that maintenance time will be decreased as a result of higher advertisement and higher demands for the products.
- The optimal lingering duration of older products in the market has been reduced compared with the case in which maintenance costs has been kept constant.

Sensitivity analysis and studying parameters' variations

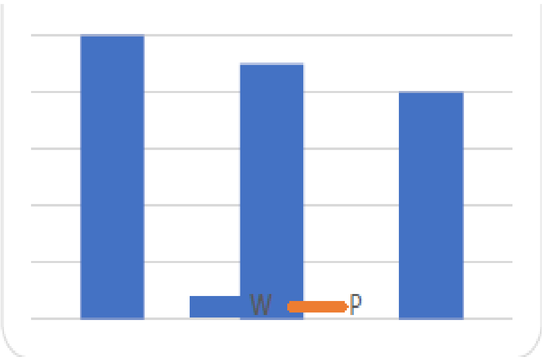
Just as with the first model, the variations in problem's parameters on the response are studied in the following table. Table 4 includes the results of studying variations in maintenance costs' parameter.

Table 4. studying λ and W variations

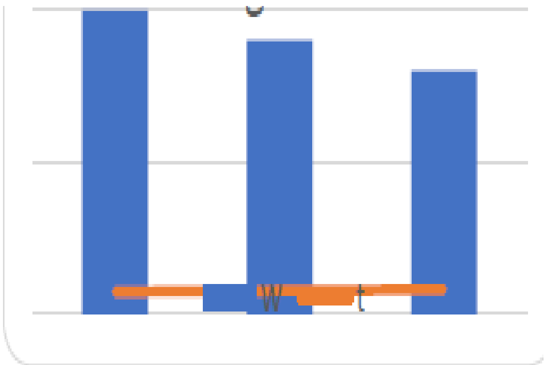
λ	W	H	P	A	T	SR	TP
0.25	10	1	12.9523	143.307	0.741589	82142.6	43723.4
	9	1	12.9424	143.307	0.775695	82584.9	43804.4
	8	1	12.9325	143.307	0.815458	82628.1	43891.1
	11	1	12.9622	143.307	0.711907	81752.7	43652.1
0.4	10	1	12.3504	143.307	0.681517	331652	143302
	9	1	12.3455	143.307	0.71289	332136	143113
	8	1	12.3313	143.307	0.748335	332843	143226
	11	1	12.3445	143.307	0.653097	331407	143157
0.2	10	1	13.233	143.307	0.768158	62206.1	34799.5
	9	1	13.2219	143.307	0.80362	62641.2	34882
	8	1	13.2109	143.307	0.844963	63144.1	34977.5
	11	1		143.307			

Table analysis and description

As evident from table 4, for any constant λ, the price will decrease upon any reduction in W and the revenue and profit will be increased as a result. Besides, the duration of lingering in the market will be increased (Figure 3).



A



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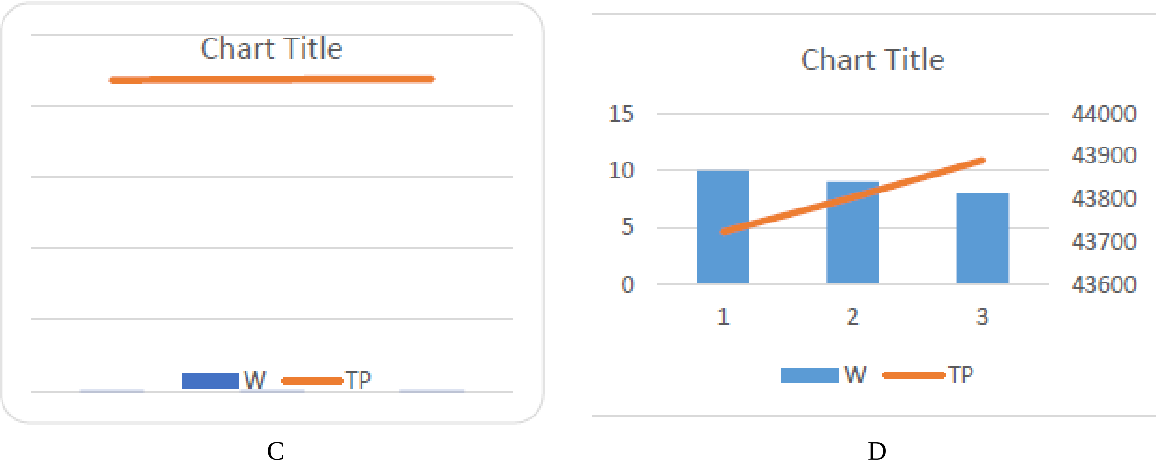

Figure 3. a) the impact of variable w on t; b) the impact of w on p; c) the impact of w on profitability; d) the impact of w on profit function

Moreover, lower λ will result in price increase.

4. Discussion and Conclusion

Considering the variations in products' lifetime and the role played by pricing on demand and revenues and the role played by perishable and seasonal products in different inventory control models and the literature on this issue, this paper attempted to study the impact of pricing through influencing advertisement as a key factor in customer demand, revenue generation and firm's profitability.

This model assumed the existence of two products with different lifetimes (new and old products) such as raisin as an agricultural product. Two different products have been assumed, one of which has been previously present in the market, which is old but surviving the market at the same time; while the new product is being supplied to the market with a different and higher price.

We attempted to model the problem and find the decision variables (i.e. price and advertisement) during the existence of both products in the market then, profit function has been optimized (i.e. maximized) through its mathematical modelling. In fact, this has been made possible through reviewing the literature on pricing perishable items and consideration for advertisement as a significant factor.

The literature on this topic is really rich indeed. From preliminary EOQ model in inventory control model to pricing models with different assumptions. The literature on inventory control and pricing are divided in three categories:

Simultaneous pricing and inventory control for immediately perishable goods:

Immediate perishability includes those products, the decay process of which starts right after entering the warehouse. Most of the studies investigated perishability rate through different assumptions including constant perishability rate, variable perishability rate, or perishability as a function of time. Moreover, in many of the studies, demand function has been assumed as absolute or probable varying with probability density function. In addition, the multiple products and the assumption regarding the existence of substitute products is another assumption of this study. In some of the studies, allowed shortage has been studied in tow forms, i.e. compensable and irrecoverable.

Simultaneous pricing and inventory control for non-immediate perishable goods

Non-immediate perishability concerns a subcategory of perishable items, the decay process of which doesn't start right after entering the warehouse; rather, their decay process starts some while after entering the warehouse.

Simultaneous pricing and inventory control for immediately perishable items while considering other factors

Considering other factors including temporal value, delays, etc.

As it has been previously discussed about this model, we're looking for optimizing a model with an absolute price- and demand- dependent demand function (which is considered as demand continuity).

To this end, problem-related parameters are defined and the absolute demand function will be presented. Accordingly, revenue and profit functions will be defined.

The model costs include purchase costs, maintenance costs, advertisement costs, and ordering costs. Then profit function will be calculated as the difference of revenue and total costs. Finally, the objective of this study is maximizing profit function and calculating price and advertisement costs.

System maintenance costs: in this problem, maintenance costs will be assumed in two different forms, i.e. time-dependent and time-independent. Two different problems are developed through this assumption and the relevant problems are solved then.

Through inclusion of advertisement in the model, customer's demand will change, which distinguishing this model from others, which will result in higher demand and revenue. Moreover, the dependency of maintenance costs to time will reduce the lingering duration of older products in the market. Besides, advertisement will significantly increase such that old products won't need to be maintained anymore.

Recommendations for Future Research

- Studying multiple products simultaneously
- Consideration of non-linear and probability cases affecting demand
- Consideration for shortage costs
- Consideration for a parameter as customer's inclination for the new product
- Consideration of the model with exponential maintenance costs
- Model solution with dynamic modelling and using probability demand function.

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