Research Article

The Investigation of the Relationship between Math and Philosophical Problems to Improve Teaching of Problem Solving

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Abstract: Teaching math problem solving is one of the most important parts of curricula in schools and universities in most fields of study. There have been numerous studies, which have shown the problems and challenges of this field as well as the advantages of its improvements. Teaching the math problem solving is not merely learning the science of mathematics, but it is learning a set of skills to face different types of problems, which may be encountered for the first time. In addition, numerous studies are showing the role of cognitive and metacognitive processes such as logical reasoning in the improvement of the teaching and learning of mathematics. Many philosophers such as Wittgenstein have considered philosophical research equal to the philosophical problem solving. In the current study, the similarities that, in past and present time, have led people such as Descartes and Pólya to use philosophical problem solving, which is referred to as philosophical research, for math problem solving to improve teaching math problem solving, have been investigated and it has been indicated that how the philosophical problem solving methods lead to development of math problem solving skills. The results of the current study show that the commonality in the use of argumentation, cognitive and metacognitive processes, uncertainty, reasoning, and understanding and use of abstract concepts have led people such as Descartes to try to utilize the philosophical problem solving for math problem solving. Accordingly, the transfer of learning between the philosophical research and math problem solving phenomenon can be used in teaching or in another way, through the combination of the problem solving with the philosophical methods, the teaching of math problem solving can be developed.

Keywords: teaching math problem solving, philosophical research methods, philosophical problem solving

1. Introduction

Mathematics plays an important role in daily life. Among the reason why studying mathematics is essential, the followings can be mentioned: 1) mathematics is a meaning of transparent and logical thinking, 2) it is a meaning of daily life's problems solving, 3) it is a meaning of identification of the conventional relations and experiences, 4) it is a meaning of development of creativity, and finally, it is a meaning of increasing the awareness of the culture. Such ideas are in lines with the general purpose of learning mathematics, which has been also formed by NCTM (National Council of Teachers of Mathematics) as follows: the mathematical relationship, mathematical reasoning, mathematical problem solving, mathematical consistency, and positive tendency to mathematics (Hasiban, Seragie, and Emery, 2018).

Learning of mathematics requires different mental skills among which the problem solving skill is especially important, as a lot of people consider it to be equivalent to the mathematics (Jones, Swan, and Plate, 2014; Surya and Ruhaya, 2017). According to Foster, Wake, and Swan (2014), during the recent years, the problem solving skill has been more emphasized and there have been models designed for the improvement of this skill in several studies (Chapman, 2005; Golnabi, 2015). However, math problem solving is usually difficult for learners. Not only is it necessary that learners recover some information in their minds for problem solving, but also they should write the answer in a new method (Osman, Che Yung, Saleh Abu, 2018; Bryant, 2009; Anderson and Lixel, 2007).

In spite of the subject of problem solving being raised by Pólya, which according to Schoenfield, is the first serious request for dealing with the problem solving teaching, it took years for this subject to be dealt with in the field of mathematics education. From that time until now, the researchers have made numerous efforts to dealt with the issue of problem solving. In addition to mathematics, and teaching it, which includes various fields such as problem solving, in philosophy also, philosophical research includes philosophical problem solving.

Philosophy can be defined as the use of critical and creative thinking about problem-raising questions. In most cases, the philosophical problems are conceptual and require the precise analysis of the words and their meanings. However, they may include exploring practical solutions and the provision of solutions that lead to a better life. The conceptual analysis, researching the meanings and the use of language can be defined as pure philosophy. According to some contemporary philosophers, it is the only type of philosophy (Wittgenstein, 1953). However, there is a more traditional branch, which can be named applied philosophy. This type of philosophy can include the truths that lead to better perception, but it uses this perception for practical problems (Zahedi, 2014; Morton, 1996). According to Socrates, philosophy leads to both better perception and scientific virtues. Life is inherently problematic. In many types of learning or issues related to human life, it can be always asked: what is the problem? Alternatively, it can be asked, what does problem mean? A problem can be defined in at least three ways: 1- what is difficult to understand, finish, or do, 2- a complicated question that needs an answer, and 3- a question which is responded by

computation. The philosophical problems, unlike the mathematical ones, have usually more than one probable solution. In addition, they can be solved by the use of judgment power (Fisher, 2010). The belief in research is rooted in human curiosity and life issues. The curiosity is the inherent tendency to wisdom, and the problem is the real pressure for release. The depth of vision and views is obtained through wisdom attained by research, and it is itself an opportunity for future wisdom (Bagheri, Sajjadiyeh, and Tavassoli, 2010). The research methods are also techniques and means by which the research is done. The research is also a term, which is freely used for any kinds of investigations candidate for revealing new desires and truths. The strength of the thing by which this action is done would be finally reflected in the quality of results (Walliman, 2011).

In Greece, most of the people who were conversant in mathematics and were mathematicians were also philosophers. The historical owners of the math problem solving techniques were also the owners of ancient methods of philosophical problems solving. Descartes' philosophical method is also known for math problems solving.

With the changes in cognitive and scientific fields, and over time, in attribution and analytical mathematics, this science is not merely viewed as an instrumental, contractual and instructional science or a subjective (axiomatic) principle, or an abstract and absolute science (which is more prevalent in an academic level). In the first place, since it is a descriptive and explanatory science whose subject is identification and perception of especial and distinguished faces of the universe, enters the field of education and in a mutual relationship of questioning and answering, require vitality and a different perception of awareness and knowledge. With this introduction, it should be confessed that this approach to mathematics and within it, the teaching of skills such as problem solving exists in the formal environments of the educational system, but it is very faded, pale and faint, being dependent on the creativity, experience, and innovation of the teacher. With this assumption in mind, for promoting this kind of teaching, and perception of mathematics, and for further encouragement of creativities from the childhood and adolescence, which are rooted in a strong nature and constant curiosity of these periods, the starting point of training and the initial composition of it must be a bit reviewed and recreated, so that teaching the math problem solving as learning a science serving the perception, explanation, analysis, and reasoning, would be interpreted proportionate to the audience, and have a more a priori notion of understanding and understanding, and after that, its vast instrumental and applied aspects such as formulation, calculation, and modeling should be focused.

It is not possible to attain such notion by the development of increase and expansion, distribution, propagation of lessons and pamphlets, and educational reinforcement of conventional classrooms, but another way must be searched for. This view and expectation is precisely what puts the mathematics as a merely logical and expressive-cognitive science beside a venerable field as philosophy, since the truth of meaning and purpose in philosophy is the merely logical and analytical explanation and interpretation of the essence, methods, and reasons in the entry of the existence and existed, which includes from being to not being, to life and death, and man and necessity, and possibility and justice, and prosperity, and good and evil, and sadness and happiness (Arian Nejad, 2016).

One of the vital aspects of philosophy is its ability to rearrange the transfer, replacement, and recreation of the ideas and beliefs. Another aspect of it is the effort for presence in the uncertain points of exploration, probability, and dream. Philosophizing requires that the boundaries and freedoms must be questioned. There is more than one way for philosophizing, and there are different ways for recreation of the concepts, which include asking one's self or by being asked by others, logical arguments and challenges, referring to personal experience, and the use of humor. The philosophical problem solving methods provide the teacher with a great opportunity as the provision of a teaching method that neutralizes some disadvantages of descriptive and objective-based curricula most learners are faced with in math problem solving. On the other hand, the philosophical operation not only does teach the students how to cope with problems, but also provides them with new questions and problems for testing, solving, or selecting. In the philosophical operation, the students are helped to identify the essence of a problem through defining the details, making the concepts transparent, comparing, identification of special cases, and specifying the aspects related to the rules (Heinz, Kennedy, and White, 2010). All the cases mentioned are of great importance in teaching math problem solving. Therefore, it should be investigated how these philosophical methods can be used for teaching math problem solving.

Noori, Fayyaz, and Seif (2013) have investigated the effects of a philosophical mindset on the math problem solving capability of students divided by gender. Their findings indicated that besides the confirmation of the main effects of the philosophical mindset on the problem solving capability in students, the amount of such effects is also great. In addition, with the students' philosophical mindset getting stronger, the math problem solving capability of them is also significantly increased.

By the way, the effects of contemplation on math problem solving capability are also significantly higher than the effects of comprehensiveness and flexibility. From one and a half-century onwards, the history of sciences has been carrying philosophical issues which have been easily accepted (Foucault, 2010). The current study was formed on the question that such an idea is stemmed from which factor (or factors)? Since Pólya's idea is backed by a great and famous philosopher such as Descartes, the math problem solving and philosophical problem solving have been

investigated in terms of similarities, based on this foundation. Although the term 'problem' means an obstacle or trouble, its meaning and implication go beyond the common and prevalent problems and difficulties, which can be solved habitually and instantly. In such a state, 'problem' is equivalent to an enigma, issue, difficulty, and obstacle, which cannot be solved without thinking. Accordingly, the problem is neither a merely objective aspect that does not need pondering, nor it is a completely subjective aspect that is manifested in thinking. In this regard, the problem is the sum of both objective and subjective aspects. The objective aspect of problem talks of the causal and external relationships between the affairs while the question is indicative of the subjective and reason aspect. Thus, the problem is placed in the idiomatic position. The problem is the outcome of the current principle in nature and communion, which is the change. If human nature and communion were in absolute stability, neither a change would have happened nor a problem would have raised. The formation and survival of any natural state or entity, i.e. the actual entity and credit entity, depends on the existence of a level of proportion and balance between affairs.

That the unavoidable philosophical problems, several problems, or several classes of problems is an idea on which there is no consensus. However, it can be said that some philosophical problems are unavoidable such as the ultimate values, ultimate causes, and the meaning of life (Palmai, 2015). Wittgenstein defines the philosophical problems as the manifestation of the mental frustration of the philosopher: philosophy is nothing but philosophical problems. Special individual concerns we call them philosophical problems (Wittgenstein, 1996).

2. Methodology

The logic of comparative studies can be understood by comparing to conventional studies in Humanities. This study is placed beyond the quantitative and qualitative studies, uses various approaches, has a descriptive, explanatory, and analytical nature, and seeks to explore the similarities and differences, or orientations of the deductive and inductive methodology, which in practice, shows itself by variable-oriented and case-oriented studies. Here, the concept and descriptive propositions of teaching the math problem solving as well as its process and components, are compared to the concepts and descriptive propositions of the philosophical research and its methods in comparative analysis method, on such a basis, so that their similarities and differences would be revealed.

3. Findings

Similarities between the Philosophical Problem solving and Mathematical Problem solving:

3.1. Use of Reasoning

The reasoning is usually investigated in terms of the rationality of the answers we find for the problems. For a rational person, there are only complete proofs and reasoning. What that wants to be reason must lack any gaps, effacement, or uncertainties from any kinds, and if it is not so, it cannot be a reason. Do we have such complete reasons which conform to a supreme standard in daily life, or courts of law, or the physical sciences? Rarely such reasons can be found. Therefore, to understand that we can hardly find a logic about such complete reason, we may say with a bit of exaggeration that man has learned this logic from a human being or a book. From Euclid and his book of principles. Anyways, reading the principles of plane geometry is still the best chance to attain a correct and firm reason. The Euclidean geometry is not just logical writing, but it is the first and greatest sample of such writings from which the other sciences have tried, and still try, to imitate (Pólya, 1981). Some subjects of the curriculum provide the children with opportunities to expand this skill, more than other subjects. Nevertheless, there are two more formal types of reasoning the children will face. The two reasoning are "induction" and "deduction", which to a high extent reflect each other and both exist in the primary school curriculum. Inductive reasoning occurs more. Understanding the relationship between the two is so important. What provides the highest chances and opportunities for deductive reasoning in its formal meaning is solving the provable problems of mathematics, however, in the primary phases, the inductive reasoning can be generally found more, and increase the child's reasoning in many subjects.

3.2. Thinking

Wisdom is the ability to think. Thinking is what is usually hidden, but always stimulates our daily behaviors (Foucault, 2010). The ancient Greeks were the first society in which the students were taught to think for themselves, discuss and argue, criticize, and not suffice to parrot what teachers say. This issue became the fastest means to foster the thought which existed until then (Heinz, Kennedy, and White, 2010). Isuda and Katagiri (2012) investigated the mathematical domains, approaches, and methods. They have named different types of philosophical thinking such as the deduction, induction, reasoning, monolithic thinking, generalization, developmental thinking, and other cases, in the mathematical method. In addition to these methods, other important spaces have been also elaborated in their work, which overlaps with philosophical thinking.

3.3. Understanding

Understanding is a combination of the sensual data and mental efforts. Aristotle has introduced the sense as the introduction of experience, and the experience as the introduction of science. The science not in its general sense, but a technique or an organized and ordered collection of information. Therefore, philosophy is the human comprehensive knowledge of all creatures of the world. That Aristotle considers the subject of philosophy as the entity or essence is due to the comprehensiveness of the title of entity or essence. The philosophy from the initiation, i.e. Aristotle's time, meant the comprehensive understanding. The distinguished attribute of the philosopher is that he knows what he does not know. Strauss states that in his point of view, the effort for understanding and expansion of information, and deepening and completion would continue. We have an estimative recognition of the whole (supposition). All human beings are in turn philosophers, i.e. they have a comprehensive understanding of the universe. However, this understanding is more based on imagination and supposition. The task of philosophy is to replace our imagination of the whole with the perception of the whole. Wittgenstein (1996) has tried in the [philosophical studies to analyze a concept named understanding. What is so prominent in his efforts is that he, to show that understanding is not a merely mental process, has resorted to examples of understanding in solving the mathematical problems. He, through investigation of the trends processes throughout which understanding occurs, confess that understanding is the main element in philosophical and mathematical problems, which are attached to their own specific linguistic plays, and it, is not a merely mental process. But in many cases, it is attained based on the occasions in which a specific experience occurs.

The students who have a mathematical understanding attain an achievement more than the truth and distinguished procedures. They know why a mathematical idea is important and know the platforms useful for using it. In addition, these students are aware of many of the relationships and ties between the mathematical ideas and in fact, the level of their understanding is related to the depth and vastness of these relationships and connections. The students who learn problem-solving with understanding and perception, do not need to spend much time learning since they see a common pattern between the different situations. The knowledge that is obtained through understanding becomes a basis for recalling or recreating the mathematical truths and methods, and a foundation for solving new and unknown problems, and creation of new science. In addition, understanding helps prevent from basic errors in problem solving (Patrick and Swaford, 2010).

3.4. The Cognitive and Metacognitive Processes

Aghazadeh (2009) has admitted that problem solving is the general and effective form of thinking, i.e. oriented thinking in which the thinker tries to achieve a goal (or goals). Mostly, the words "thinking" and "problem solving" can be used synonymously. The reasoning tasks and well-defined problem can be broken down into cognitive processes, i.e. the cognitive processes needed for problem solving. Some cognitive processes are coding, reasoning, usage, and responding. Metacognition deals with those reasoning processes, which are necessary for problem solving. The development of governing skills is a key but a forgotten element in all levels. The metacognition has been used in different meanings. Some have defined this concept as the person's awareness of the processes of self-recognition or recognition of the others. Moreover, some others have defined it as mentoring the cognitive processes (Joseph, 2010, as cited by Salehi, 2013). The philosophical research addresses the metacognitive context as a part of the philosophical context which has been always the center of attention regarding the essence of thinking, thought, knowledge, and truth and reality subjects (Heinz, Kennedy, White, 2010).

The philosophical research provides the chance for contemplation and thinking, identification of the problems, and the systematic and continuous exploration of the solutions. Also, the thinking and metacognitive discussions provide us with some opportunities. A simple definition of philosophy is that it is the process of thinking about thinking (Fisher, 2010).

3.5. Abstraction:

Abstraction is not specific to mathematics. However, we deal with abstraction in mathematics more than any other fields. Abstraction gives more power to mathematics and prepares it for general conclusions. The mathematics is a reflection of our surrounding, and it is related to life. However, there are mathematical theories that do not find commonalities with practical activities of human after years. Investigation of these real subjects is done through abstraction. The mathematics departs from its material origin, in this abstract state. It loses its ties with this origin. Here, the internal power of mathematics acts and its evolution is independently done as a conclusion of former theories, and it leads to newer abstract theory. These new theories are the result of the natural evolution of abstraction and it only seems that it has departed from their foundations. They also return to the material world they have departed from, and this very world confirms their authenticity (Shahriari, 2013). What has made the logical and inductive methods useful and effective in the logic and mathematics is the subject of these two sciences that is the pure abstract concepts (Russel, 2011). What is sometimes called 'abstract method' in mathematics is the thing obtained from undertaking method of treatment of mathematical objects. This treatment method can be summarized in the following motto: any mathematical object is the work it does. Similar mottos have been repeatedly used in the language philosophy, and these mottos are highly controversial. Two examples of language philosophy are as follows: "there are only differences in the language" and "the meaning of every word is its application", which are

stated by de Saussure and Wittgenstein. In addition, the integrated cry of the logical positivists can be added to these examples (the meaning of each proposition is the research methodology of its authenticity).

3.6. Doubt:

To evaluate the power of mathematics, two points must be taken into consideration: 1) mathematics forms a part of human recognition and thus, it advances in lines with the evolution of recognition. Closer the human recognition of the outside world to reality, more efficient and precise the mathematical rules and theories will be. There would be never a time in which the mathematical theories have an absolute precision and efficiency, 2) mathematics only constitutes an aspect of recognition and not all of it. Like any other sciences, it has a meaning and concept only through the relation with other sciences (Shahriari, 1999). There is unchangeable uncertainty for doubt about the principles and propositions and what is included by philosophy. Besides, there is the doubt element even in the solved philosophical problems. There are also no obstacles on doubting any kinds of reasoning. Moreover, the emergence and manifestation of the doubt and finding new and different answers, or even the rearrangement of the problems is considered to be normal. Some philosophers also have valued the philosophy with this uncertainty. As soon as we deal with the philosophizing (which is philosophical thinking), we would see that even the most ordinary affairs of everyday life refer to problems whose answers are so incomplete and partial. The philosophy, although not being able to answer the doubts it has raised with certainty, induce various ways which expand our mind and release it from the straps of the unmerciful habit. So, although it decreases our certainty about the facts, it increases our knowledge about the possible details of the essences and eliminates the dogma of pride and arrogance which is specific to those who have never undertaken the freeing doubt, and revitalize and sustain our sense of wonder and terror by provision of the unknown aspect of the object and the known affairs (Durant, 2006).

4. Conclusion

In recent decades, the dominant trend of mathematics teaching curriculum has been problem solving. In the 1970s, there were no tracks of problem solving and during those years, the exercise has been the center of attention. Placement of curricula more similar to what has been provided by Pólya is usually used as witty points for solving specific types of problems. Such training is mostly artificial and superficial mathematically. Not only mathematically, but also it is more probable that it is also artificial and superficial cognitively. Schoenfield as one of the well-known theorists has emphasized the followings for an ideal educational environment (alongside the knowledge content): 1) knowledge of context, 2) explorative rules and strategies of problem solving, 3) executive strategies, and 4) learning strategies. In addition, through focusing on the important aspects of thinking and understanding in that field, it helps with the growth and development of the system of beliefs suitable for learning. The training which is merely based on the contextual knowledge and problem solving strategies (which is only a part of the mathematical thinking) would probably lead to ambiguous and ignorable achievement (for example, Dorri, 2018; Karimian and Rafi Pour, 2012; Faramarz Pour and Rafi Pour, 2013). As long as the teaching of mathematics is not moving towards a firm relationship with the vast flow of cognitive and metacognitive subjects, it would remain focused on the skill, instead of fundamental understanding (Rezaeian, 2016). The math problems solving teachers can improve this skill in the school and university students through the capacities of the philosophical problem solving methods. The realistic problems give the students the chance to imitate their mathematical thinking in a world they witness, and provides them with the opportunity to use mathematics in a way they know, and create new strategies. In addition, governing the situations that challenge or measure the students' ideas in math problem solving through philosophical research methods can be useful. Creation of cognitive controversy, which is one of the attributes of problem solving in mathematics and philosophy, leads to significant advancements in students' learning (De Green, e. Nisterm, Di Angel, M. Darly, Di Cohen, 2004; Mogonia and Popescu, 2015). Alongside all the philosophical research methods, today, the philosophical methods for the children have a relatively significant position in training such as different types of reasoning or mental development. Most of the methods used in philosophy for the children such as research communion, controversial teaching, and teaching thoughtful and rational thinking, are methods for which there are equivalents in philosophy. Here, two types of the use of philosophical methods in teaching math problem solving can be named. The first one considers the transfer phenomenon. The other principle considers teaching the philosophical methods as skills for thinking, identification, and distinguishing the concepts, designing argument, designing solution-finder reasoning, efforts for understanding and clarifying the conditions dominant over the problem and probable answers, discussion about the existence of answer (or answers) and the conditions ruling them, generalization of the results and findings from problem solving, learning teamwork such as research communion, and the cognitive and metacognitive processes and skills which are all widely used in philosophical research methods. For transfer, it can be said that the occurrence of this phenomenon is probable if the skills and methods are taught or utilized separately and with awareness. Mehdi Zadeh (1977) has investigated different approaches of learning transfer in philosophical and mathematical thinking and has concluded that if there was a skill in these two types of thinking, the learning transfer would occur. Training the phenomenological method as a research method for arrangement and achievement of the concepts would be used in the form of transfer besides teaching the math problem solving for problems, which need arrangement or

development of the concepts. The opposition of this approach is training with a combination of philosophical and mathematical methods in math problem solving. Designing and solving problems that require analysis of the conditions or arrangement of the concepts with precision is another form of training. For other methods, proper mathematical problems can be also designed by the use of hermeneutics, ascending and descending dialectic, Socrates method, and different types of poststructuralist methods such as deconstruction, and then try to solve them by the help of philosophical methods. Mostly, these methods are used for problems that are in the form of description, and to investigate the possible answers, the howness of answers, the conditions of answers, and the probable answers. Fisher (2010) considers the philosophical discussion in the context of Philosophy for Children (P4C) as the development of cognition and metacognition of the children. In the past, it was believed that the mathematical problems are of two types: problems with findable answers and problems with provable answers. Problems with findable answers require designing solution-finder reasoning and then a precise calculation plan to find the answers. The provable problems require designing the argument or confirmation through conventional methods. Today, the school mathematics problems are not confined to these two approaches, with the advent of new approaches of math problem solving teaching. Open-answer questions or questions which are basically designed for the students to investigate the possibility of probable answer or answers, tests that functionally ask the students to solve the problems with the teamwork and probably, various answers, or get newer results, have all taken a position in the training and testing of the math problem solving skill (Brobcher, Pinier, and Ricket, 1990; Dutch, Groh, and Allen, 2001; Gooya, 2000). The students who get familiar with the philosophical research methods, learn how to face a variety of solutions in problem solving. They also learn to know that in spite of the differences between them, each is correct and based on firm foundations. When they get familiar with the deductive processes and probable reasoning for math problem solving, they would look upon their traditional views with doubt. They learn to hear the agreements and disagreements and defend their proposed solution confidently and finally give the others a chance to decide and respect the decisions made. The public aspect and working in a research communion also affect their beliefs, increase the self-awareness and self-discipline, and decrease the reliance on the external power, which is the teacher, helping them stop waiting for instructions. Obliging the students to say “why” and “how”, and that they explain their ideas, lead them to know that mathematics is not mysterious and incomprehensible.

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