A Survey on Impulse Noise Removal from Color Image

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Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 4 June 2021

Abstract: A broad survey on various filters for removing impulse noise from color images is presented in this paper. Color images are multivariate vector value signals which are non-stationary in nature. Various nonlinear filters are suggested in the literature. The filters are categorized into 11 classes and discussed in details.

Keywords: Adaptive vector median filter, color image, Impulse noise, noise model, nonlinear, vector median filter.

I. INTRODUCTION

Color images are used in various applications like remote sensing, content-based image retrieval, computer aided diagnosis, medical image analysis etc. which lead to a growing importance in color image processing. Most of the applications require some task like feature extraction, segmentation and edge detection [1]. Also color image processing applications like object recognition, image matching, color image compressions, computer vision etc. used color information [2].

But color images are frequently degraded by noise due to failing of sensors, electronic interference, imperfect optics, or fault in the data transmission process. This noise introduces color variation making pixel value different from the original value and thus produces error which complicates the subsequent stages of image processing [3]. The introduction of noise also lowers the visual quality. Thus in color image processing applications deletion of noise is a compulsory preprocessing step.

Filters are frequently used to convert a signal into a form appropriate for specific purpose [5]. Color images are nonstationary in nature due to the presence of edges and fine details and also the human visual system is nonlinear and nonlinear filters are preferred more than linear filters [3,4].

Impulse noise is high energy noise which occurs for short duration. In grayscale image median filters is successfully applied for deletion of this noise. But in a color image each pixel has three components and there is a strong relationship in between them. But the straight application of the median filter also known as the component-wise or marginal median filter is not suitable as it produces color artifacts. But in vector filtering techniques the input pixel is treated as vectors and no different color are introduced [3,4]. As there is no general way to outline ordering in vector space, many nonlinear filters are suggested in the literature for impulse noise elimination. In this study a huge number of nonlinear filters are categorized into 11 families.

The paper is arranged as; Section 2 describes the categories of filters, Section 3 presents a commonly used impulse noise model, Section 4 describes popular filtering performance criteria for evaluating filters and Section 5 contains the conclusion.

II. Category of filters

In this section, the various filters for removing impulse noise are grouped into 11 categories. They are

1. Basic vector filters
2. Weighted vector filters
3. Adaptive vectors filters
4. Hybrid vector filters
5. Peer group vector filters
6. Fuzzy vector filters
7. Vector sigma filters
8. Entropy vector filters
9. Quaternion based vector filters
10. Morphological based vector median filter
11. Miscellaneous

2.1 Basic vector filters

These are the primary filters that are suggested for removing impulse noise from color images. These filters used vector ordering technique, a variation of the reduced sub-ordering principle.

2.1.1 Vector Median Filter (VMF)

VMF and extended vector median filter (EXVMF) [6] are introduced for processing vector valued signals having properties similar with median filters operation such as good smoothing ability and zero impulse response while maintaining sharp edges in the signal. The lowest ranked vector with minimum aggregated distance to the input vector present in the filtering window is the output of VMF.

The vector median is computed as follows: -

Let $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n$ represent the vectors inside the filtering window of size N x N

a) For each vector element $\boldsymbol{x_i}$, the sum of distance S_i to all other vectors inside the filter window is calculated using the Minkowski metric (either the L_1 or L_2 norm) i.e.,

$$S_i = \sum_{j=1}^{N} \left\| \boldsymbol{x}_i - \boldsymbol{x}_j \right\|_\gamma, \; i = 1, 2, \ldots, N \qquad (1)$$

where $\gamma = 1$ for city block distance and $\gamma = 2$ for Euclidean distance.

b) Find min such that S_{min} represents the lowest S_i.

c) Corresponding to S_{min}, $\boldsymbol{x}_{min}$ is the vector median.

An efficient extension of this filter is proposed in [7] known as Robustified vector median filter. This filter works on the trimmed distances between pixels present to the sliding window. The robust vector median filter is defined as

$$\boldsymbol{x}_{RMED} = argmin \sum_{j=1}^{N} \rho\{\boldsymbol{x}, \boldsymbol{x}_j\} \qquad (2)$$

where ρ is a threshold distance defined by

$$\rho\{\boldsymbol{x}, \boldsymbol{x}_j\} = \begin{cases} \|\boldsymbol{x} - \boldsymbol{y}\|_\gamma & \text{if } \|\boldsymbol{x} - \boldsymbol{y}\|_\gamma \le d \\ d & \text{otherwise} \end{cases} \qquad (3)$$

and d is a filter parameter.

'd' is used to replace the distance when the absolute difference between two pixel intensities is greater than *a* threshold value. The effect of noise which contribute mostly to the sum of distances is reduced in this technique.

A modified form of the standard VMF is suggested in [8] known as Fast Modified Vector Median Filter (FMVMF). In this filter, the center pixel is substituted by one of the neighbor pixel when the distance associated with one of the neighboring pixel is less than the center pixel.
Let the distance associated with the center pixel be

$$R_o = -p_o + \sum_{j=1}^{n-1} p(\boldsymbol{F}_o, \boldsymbol{F}_j) \quad (4)$$

P_o is a threshold parameter.
and distance associated with the neighbors of F_o be

$$R_i = \sum_{j=1}^{n-1} p(\boldsymbol{F}_i, \boldsymbol{F}_j),\ i = 1, \ldots, n\text{-}1. \quad (5)$$

Then, for some k, R_k is smaller than R_o

$$R_k = \sum_{j=1}^{n-1} p(\boldsymbol{F}_k, \boldsymbol{F}_j) < R_0 \quad (6)$$

Then $\boldsymbol{F}_o$ is being replaced by $\boldsymbol{F}_k$.

2.1.2 Extended Vector Median Filter (EXVMF)
This filter [6] used vector median operation with an averaging filter. EXVMF denoted as $\boldsymbol{x}_{EVMF}$ is given by

$$\boldsymbol{x}_{EVMF} = \begin{cases} \boldsymbol{x}_{ave}, & \sum_{i=1}^{N} \|\boldsymbol{x}_{ave} - \boldsymbol{x}_i\|_2 < \sum_{i=1}^{N} \|\boldsymbol{x}_{vmf} - \boldsymbol{x}_i\|_2 \\ \boldsymbol{x}_{vmf}, & \text{otherwise} \end{cases} \quad (7)$$

where $\boldsymbol{x}_{ave} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{x}_i$ (8)

and $\boldsymbol{x}_{vmf}$ is the vector median output. It works like VMF near edges and the arithmetic mean filter (AMF) in smooth areas.

2.1.3 Alpha-trimmed Vector Median Filter (α-VMF)
This filter [9] picks the smallest ranked $1+\alpha$ vectors as input to an averaging filter. The output is defined as

$$\boldsymbol{x}_{\alpha-VMF} = \frac{1}{(1+\alpha)} \sum_{i=1}^{1+\alpha} \boldsymbol{x}_i,\ \ \alpha \in [0, n\text{-}1] \quad (9)$$

The trimming operation gives good result for impulse noise and the averaging operator helps to cope with Gaussian noise.

2.1.4 Generalized vector median filter (GVMF)
A generalization of the vector median filter is proposed in [10] which outputs centrally located pixel within a peer group of pixel from the pixels in the filtering window.
Another generalization of VMF called Sharpening Vector Median Filter (SVMF) was also proposed in [11]. In this filter for the ordering of vectors the sum of α closest distances for each pixel to other pixels from the sliding window was computed. The central pixel was replaced by the pixel that minimizes the trimmed cumulated distances. A generalization of SVMF was also proposed in [12].

2.1.5 Basic vector directional filter (BVDF)
This filter [13] used aggregated sum of angles between the vectors in a window. It is a rank ordered filter in which the vectors with atypical directions are regarded as an outlier. The output of this filter is a vector from the input vectors with the lowest sum of angles with the other vectors. It is mathematically defined as

$$\theta_i = \sum_{j=1}^{N} A(\boldsymbol{x}_i, \boldsymbol{x}_j), \quad i = 1, 2, \ldots, N \quad (10)$$

where $A(\boldsymbol{x}_i, \boldsymbol{x}_j) = cos^{-1}\left(\frac{\boldsymbol{x}_i . \boldsymbol{x}_j}{\|x_i\|\|x_j\|}\right)$ (11)

where $A(\boldsymbol{x}_i, \boldsymbol{x}_j)$ is the angle between the vectors $\boldsymbol{x}_i$, and $\boldsymbol{x}_j$. Then

$\theta_{(1)} \leq \theta_{(2)} \leq \cdots \theta_{(r)} \dots \leq \theta_{(N)} \rightarrow \boldsymbol{x}_{(1)} \leq \boldsymbol{x}_{(2)} \leq \cdots \boldsymbol{x}_{(r)} \dots \leq \boldsymbol{x}_{(N)}$ (12)

"Vector's direction corresponds to chromaticity of the image" [14], therefore chromaticity is preserved better than VMF in this filter.

2.1.6 Generalized vector directional filter (GVDF)

This filter [14] considers the magnitude and direction of the input vectors. A set of lowest ranked vectors is selected based on the angular distance criterion as input to another filter that consider the magnitude of the vectors to produce a single output vector like the multistage median filter, AMF (arithmetic mean filter) and various morphological filters.

2.1.7 Directional Distance Filters (DDF)

This filters [15,16] combines VMF and VDF in a unique way. This filter eliminates impulse noise much more effectively than the VMF and also preserves chromaticity. Vectors direction indicates their chromaticity, while their magnitude measures their brightness. It is defined as

$\Omega_i = (\sum_{j=1}^{N} \|\boldsymbol{x}_i - \boldsymbol{x}_j\|_{\gamma})^{1-p} . (\sum_{j=1}^{N} A(\boldsymbol{x}_i, \boldsymbol{x}_j))^{p}$ (13)

where

$p \in (0, 1)$, $\boldsymbol{x}_i$, i =1, 2, …, n is the input set. Ω_i correspond to $\boldsymbol{x}_i$ and the input vector $\boldsymbol{x}_i$ which minimizes Ω_i is the output of DDF.

2.2 Weighted vector filters

These are those filters in which a non-negative weight is allocated to every pixel of the filtering window for removing outliers.

2.2.1 Weighted Vector Median Filter (WVMF)

In this filter, each pixel in the sliding window is assigned a non-negative integer value which is known as weights. This weights offers more flexibility than the median-based filter. It is a generalization of VMF. The output of the weighted vector median for vectors $\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_n$ inside the filter window having corresponding non-negative integer-valued weights $w_1, w_2, \dots, w_N$ is the vector $\boldsymbol{x}_{wvm}$ such that [9, 17]

$\boldsymbol{x}_{wvm} \in \{\boldsymbol{x}_i;\ i = 1, \dots, N\}$ and for all $j = 1, \dots, N$

$\sum_{i=1}^{N} w_i \|\boldsymbol{x}_{wvm} - \boldsymbol{x}_i\|_{\gamma} \leq \sum_{i=1}^{N} w_i \|\boldsymbol{x}_j - \boldsymbol{x}_i\|_{\gamma}$ (14)

2.2.2 Extended Weighted Vector Median Filter (EWVMF)

It is an extension of WVMF. For vectors $\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_n$, having corresponding weights $w_1, w_2, \dots, w_n$, the output of Extended weighted vector median (EWVMF) [9,17] is

$$\boldsymbol{x}_{EWVM} = \begin{cases} \boldsymbol{x}_{wave}, \text{ if} \sum_{i=1}^{N} w_i \|\boldsymbol{x}_{wave} - \boldsymbol{x}_i\| < \sum_{i=1}^{N} w_i \|\boldsymbol{x}_{wvm} - \boldsymbol{x}_i\| \\ \boldsymbol{x}_{wvm}, \quad \text{otherwise} \end{cases} \tag{15}$$

It usually selects the average as output in smooth areas and WVM near edges.

2.2.3 α-Trimmed Weighted Vector Median Filter (α-TWVMF)

This Filter (α-TWVMF) [9] of vectors $\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_n$, having corresponding weights $w_1, \dots, w_n$ is defined as

$$\boldsymbol{x}_{\alpha-twvm} = \begin{cases} \boldsymbol{x}_{\alpha}, if \ \sum_{i=1}^{N} w_i \|\boldsymbol{x}_{\alpha} - \boldsymbol{x}_i\| < \ \sum_{i=1}^{N} \|\boldsymbol{x}_{wvm} - \boldsymbol{x}_i\| \\ \boldsymbol{x}_{wvm}, \quad \text{otherwise} \end{cases} \tag{16}$$

where

$\boldsymbol{x}_{\alpha} = \frac{1}{|s_{\alpha}|} \sum_{\boldsymbol{x}_i \in s_{\alpha}} \boldsymbol{x}_i$

and $s_{\alpha} = \{\ \boldsymbol{x}_i;$ having $s_i \leq s_{(N-\alpha)}\}$, $s_{(i)}$ is the i^{th} smallest of $s_1, \dots, s_N$ and $|s_{\alpha}|$ represents the number of elements in s_{α} and can have values 0, 1, …, N-1.

2.2.4 Weighted Vector Directional Filters (WVDF)
The output of this filter [18,19] reduces the sum of weighted angular distances to other input samples from the sliding window. It is defined as

$$\boldsymbol{x}_{WVDF} = argmin_{\boldsymbol{x}_i \in W} \sum_{j=1}^{N} w_j A(\boldsymbol{x}_i, \boldsymbol{x}_j) \qquad (17)$$

where $A(\boldsymbol{x}_i, \boldsymbol{x}_j)$ denotes the angle between two vectors. Similarly, by using both the magnitude and angular distance criteria weighted directional distance filters (WDDF) [20] is also obtained.

2.2.5 Center weighted vector median filter (CWVMF)
This filter [21, 23] is formed when only weight of center pixel is varied and the others remain fixed. It is defined as

$$\boldsymbol{x}_{CWVMF}k = argmin_{\boldsymbol{x}_i \in w} (\sum_{j=1}^{N} w_j(k). \|\boldsymbol{x}_i - \boldsymbol{x}_j\|) \qquad (18)$$

with $w_j(k)=\begin{cases} N-2k+2, & \text{for } j=(N+1)/2 \\ 1, & \text{otherwise} \end{cases}$ (19)

where k is a smoothed parameter and only the center weight $w_{(N+!)/2}$ is varied with k. Here the weight assign to the center pixel is a non-negative integer. Center weighted vector directional filter (CWVDF) is proposed in [21]. A modification of CWVMF, known as Modified Center Weighted Vector Median Filter (MCWVMF) is proposed in [23,24]. In this filter only the aggregated distance related with the center pixel is weighted and the weight is a real number between 0 and 1.

2.2.6 Rank Order Weighted Vector Median Filter (ROWVMF)
In this filter [25], the distance between a pixel $\boldsymbol{x}_i$ and all other pixels inside the window is calculated and is ordered to obtain $d_{i(r)}$ by assigning a rank r.

$$d_{i1}, d_{i2}, \ldots, d_{iN} \rightarrow d_{i(1)}, d_{i(2)}, \ldots, d_{i(N)} \qquad (20)$$

Then, weighted sum of distance is computed using the distance ranks as follows:

$$\Lambda_i = \sum_{r=1}^{N} f(r). d_{i(r)} \qquad (21)$$

where $f(r)$ denotes a constant function associated with the distance rank r. A new order of vectors is formed by sorting Λ_i

$$\Lambda_{(1)}, \Lambda_{(2)}, \ldots, \Lambda_{(N)} \rightarrow \boldsymbol{x}^*_{(1)}, \boldsymbol{x}^*_{(2)}, \ldots, \boldsymbol{x}^*_{(N)} \qquad (22)$$

where $x^*_{(1)}$ is the output of ROWVMF. Another filter having similar concept called Rank-based Vector Median Filter (RVMF) is also proposed in [26].

2.2.7 Genetic Algorithm based weighted vector directional filter (GAWVDF)
In [27] an optimized WVDF based on Genetic Algorithm is proposed in which the filter weights are adapted in order to match the changing image and noise characteristics. As compared with other optimization techniques, the GA-based methods are able to provide a globally optimal solution as GA-based methods examine the whole solution space.

2.3 Adaptive vector filters
Vector median filters and its variants perform filtering operation on entire pixels regardless of whether the pixel is noisy or not, leading to blurring of edges and fine details. Noise characteristics varies in the image and hence non-adaptive filters have low performance. The adaptive filters implement estimation procedure based on the nature of data on local image statistics, to handle the difficulty of varying noise characteristics.

2.3.1 Adaptive vector median filter (AVMF)
The objective of this filter (AVMF) [28] is to remove the corrupted elements while maintaining desired signal features. It is achieved by using the identity operation and VMF according to the decision rule which is stated as follows

If $val \geq T$ Then $\boldsymbol{x}_{(N+1)/2}$ is impulse
Else $\boldsymbol{x}_{(N+1)/2}$ is not corrupted

where val is the vector distance between the mean of the first r vector order-statistics $\boldsymbol{x}_{(1)}$, $\boldsymbol{x}_{(2)}$, …, $\boldsymbol{x}_{(r)}$ associated with the smallest distance $L_{(1)}$, $L_{(2)}$, …, $L_{(r)}$ and the central pixel $\boldsymbol{x}_{(N+1)/2}$, for r ≤ N.
Other adaptive filters like Adaptive vector directional filter (AVDF) [29] and Adaptive distance directional filter (ADDF) [30] are also proposed.

2.3.2 Rank Conditioned Vector Median Filter (RCVMF)
In this filter [31], every pixel in the filtering window is given a rank according to the ordered distance. If the center pixel's rank is larger than a predefined rank of uncorrupted vector pixels, then the output is vector median. It is stated as

$$\boldsymbol{x}_{RCVMF} = \begin{cases} \boldsymbol{x}_{VMF}, & \text{if } r_{(N+1)/2} > r_k \\ \boldsymbol{x}_{(N+1)/2}, & \text{otherwise} \end{cases} \quad (23)$$

where $r_{(N+1)/2}$ denotes the rank of center pixel and r_k the rank of predefined healthy pixel. An improvement of RCVMF is also defined in this paper known as the Rank Conditioning and Threshold Vector Median Filter (RCTVMF) [31]. Another criterion i.e., distance D between the central pixel and predefined healthy pixel is used for detecting impulse noise. It is mathematically denoted as

$$\boldsymbol{x}_{RCTVMF} = \begin{cases} \boldsymbol{x}_{VMF}, & \text{if } r_{\frac{N+1}{2}} > r_k \text{ and } D > T \\ \boldsymbol{x}_{(N+1)/2}, & \text{otherwise} \end{cases} \quad (24)$$

where T is a pre-defined threshold.
A two-stage impulse detection adaptive filter called Adaptive Rank-Ordered Mean (AROM) filter is also proposed in [32]. In this filter for detection of whether the central pixel contains impulse or not, rank conditioned median (RCM) and center-weighted median (CWM) filters are used. To find more suitable local thresholds CWM is used and to checks if the central pixel is well-within the ordered data set RCM is used.

2.3.3 Rank-Ordered Differences Statistics Based Switching Vector Filter
In this filter [33], the magnitude of Rank-Ordered Differences Statistics (ROD) is used to decide if the central pixel contains impulse noise or not. The ROD is defined as

$$ROD_m(\boldsymbol{x}) = \sum_{i=1}^{m} r_i(\boldsymbol{x}) \quad (25)$$

where $r_i(\boldsymbol{x})$ is the i^{th} smallest distance of the center pixel to the neighborhood pixel of the filter window.
The logic of this filter is that uncorrupted pixels has a significantly lower ROD value than noisy pixels. A pixel $\boldsymbol{x}$ is mark as uncorrupted if ROD ($\boldsymbol{x}$) is smaller than a fixed value else it is corrupted. It is defined as follows

$$\boldsymbol{y}_{RODSAMF} = \begin{cases} \boldsymbol{x}, & \text{if } \boldsymbol{x} \text{ is noise} - \text{free} \\ AMF_{out}, & \text{if } \boldsymbol{x} \text{ is noisy} \end{cases} \quad (26)$$

2.3.3 Rank Weighted Adaptive Switching Filter (RWASF)
This is an adaptive version of the Rank Weighted Vector Median Filter (RWVMF) [34]. In this filter, the difference between Δ_1, cumulated weighted distance of the central pixel and $\Delta_{(1)}$, output of rank weighted vector median filter is used for detection of noisy pixels. If the difference is greater than a threshold value, then output of arithmetic mean filter (AMF) calculated using only the uncorrupted pixels is used for replacing it otherwise it remains unchanged. It is mathematically defined as follows [34]

$$\boldsymbol{y}_1 = \begin{cases} \boldsymbol{x}_{AMF}, & \text{if } \Delta_1 - \Delta_{(1)} > T \\ \boldsymbol{x}_1, & \text{otherwise} \end{cases} \quad (27)$$

where $\boldsymbol{y}_1$ is the output of RWASF and T is a threshold value.

2.3.5 Adaptive Marginal Median Filter (AMMF)

This filter [35] is a variation of the Vector marginal median filter (VMMF) [3], aims at integrating the vector correlation of VMF and noise reduction capability of VMMF. A set of vectors *S* constituted by *m* vectors is selected from the ordered aggregated distance used in VMF which are most similar to the Vector Median $\boldsymbol{y}_{(1)}$ such that $S = \{\boldsymbol{y}_{(1)}, \boldsymbol{y}_{(2)}, \ldots, \boldsymbol{y}_{(m)}\}$ for $m \leq N$. To achieve high noise reduction Vector Marginal Median Filter is applied to this set of vector *S*. The output of AMMF is defined as

$$\boldsymbol{y}_{AMMF} = ((\text{med}(\{ y^R_{(1)}, \ldots, y^R_{(N)} \})),(\text{med}(\{ y^G_{(1)}, \ldots, y^G_{(N)} \})),(\text{med}(\{ y^B_{(1)}, \ldots, y^B_{(N)} \}))) \quad (28)$$

2.3.6 Adaptive vector filters based on Non-causal linear prediction technique.

In this filter, for impulse noise detection non-causal linear prediction is used. Filters proposed in [36,37] utilizes non-causal linear prediction co-efficient to find prediction error. The difference between the original current pixel and the predicted pixel is used as a measure for impulse detection. The predicted pixel value at central location (r, c) is calculated as follows

$$\hat{\boldsymbol{x}}(r, c) = \sum_{(i,j)\in W_2} a(i,j).\boldsymbol{x}(r-i, c-j) = \boldsymbol{X}_\eta \boldsymbol{a}_\eta \quad (29)$$

where W_2 is the non-causal region for linear prediction, $\boldsymbol{X}_\eta$ represents the matrix of vector pixels used for prediction, $\boldsymbol{a}_\eta$ denotes the vector obtained from the prediction coefficients and η is the order of prediction. The output of the non-causal linear prediction based vector filter $\boldsymbol{x}_{NCVF}$ [36] is

$$\boldsymbol{x}_{NCVF} = \begin{cases} \boldsymbol{x}_{VMF}, & if \ \|e(r,c)\| > Th \\ \boldsymbol{x}(r,c), & \text{otherwise} \end{cases} \quad (30)$$

where Th is a predefined threshold values and $\|e(r,c)\| = \max(|\boldsymbol{x}_R(m,n) - \hat{\boldsymbol{x}}_R(m,n)|, |\boldsymbol{x}_G(m,n) - \hat{\boldsymbol{x}}_G(m,n)|, |\boldsymbol{x}_B(m,n) - \hat{\boldsymbol{x}}_B(m,n)|)$. (31)

In [38] filtering is done when the central pixel is found noisy according to the non-causal linear prediction error. In this filter based on the level of error the size of the window is decided. In [39], the window size of a noisy pixel is adapted based on the availability of good pixels. Adaptive VMF output is used for replacing noisy pixel and weighted mean of the good pixels is used for non-noisy pixel.

2.3.7 Adaptive center weighted vector filters

In this filters, detection of impulse is based on user-defined threshold and corrupted pixel is changed by one of the output of VMF, BVDF and DDF forming Adaptive center weighted vector median filter (ACWVMF) [40], Adaptive center weighted basic vector directional filter (ACWBVDF) [41] and Adaptive center weighted directional distance filter (ACWDDF) [42]. The output of these filters are as follows

$$\boldsymbol{x}_{ACWVF} = \begin{cases} \boldsymbol{x}_{VF}, & \text{if } val \geq Tol \\ \boldsymbol{x}_{(N+1)/2}, & \text{otherwise} \end{cases} \quad (32)$$

where $\boldsymbol{x}_{VF} = \boldsymbol{x}_{VMF}$ when $val = \sum_{k=\lambda}^{\lambda+2} \left\| \boldsymbol{x}_{CWVMF} k - \boldsymbol{x}_{(N+1)/2} \right\|$, (33)

$\boldsymbol{x}_{VF} = \boldsymbol{x}_{BVDF}$ if val $= \sum_{k=\lambda}^{\lambda+2} A(\boldsymbol{x}_{CWBVDF} k - \boldsymbol{x}_{(N+1)/2})$ and (34)

$\boldsymbol{x}_{VF} = \boldsymbol{x}_{DDF}$ if val $= \sum_{k=\lambda}^{\lambda+2} A^\gamma(\boldsymbol{x}_{CWDDF} k - \boldsymbol{x}_{(N+1)/2}) \left\| \boldsymbol{x}_{CWDDF} k - \boldsymbol{x}_{(N+1)/2} \right\|^{1-\gamma}$ (35)

where $\lambda \in [1, \frac{N+1}{2} - 1]$.

2.3.8 Robust switching vector filters (RSVF)

This filter uses the robust median statistics to decide if the center pixel is corrupted or not. A pixel is corrupted if the cumulative Minkowski distance is larger than the median cumulative distance in its neighborhood, and is substituted by output of VMF, BVDF or DDF otherwise it remains unaffected. It is defined as

$$\boldsymbol{y}(\mathrm{r,c}) = \begin{cases} \boldsymbol{x}_{(n+1)/2}, \text{if } d_{(n+1)/2} \le \alpha.\,\mathrm{med}(d_1, \dots, d_n) \\ \boldsymbol{x}_F, \qquad \text{otherwise} \end{cases} \tag{36}$$

where med(.) is the robust univariate median operator and α is the filter parameter used for preserving image details and smoothing. If $d_i = \sum_{j=1}^{n} L_p(\boldsymbol{x}_i, \boldsymbol{x}_j)$, then $\boldsymbol{x}_F = \boldsymbol{x}_{VMF}$ and is called Robust Switching Vector Median Filter (RSVMF) [43]. If $d_i = \sum_{j=1}^{n} A(\boldsymbol{x}_i, \boldsymbol{x}_j)$ then $\boldsymbol{x}_F = \boldsymbol{x}_{BVDF}$ and is called Robust Switching Basic Vector Directional Filter (RSVDF) [44]. Finally, Robust switching directional distance filter (RSDDF) [44] is when

$d_i = (\sum_{j=1}^{n} A(\boldsymbol{x}_i, \boldsymbol{x}_j))(\sum_{j=1}^{n} L_p(\boldsymbol{x}_i, \boldsymbol{x}_j))$, then $\boldsymbol{x}_F = \boldsymbol{x}_{DDF}$.

2.3.9 Modified Switching Median Filter (MSMF)

In this filter [45], to identify likely contaminated pixels Adaptive Vector Median Filter (AVMF) is used in the first stage. In the second stage this likely contaminated pixel is tested to detect if it is edge or noise using four one-dimensional Laplacian operator. In this stage the input pixel is convolved with four convolution kernel w_p (p =1-4) respectively and the lowest difference of these four convolutions z_{ij} is used for detection of edge.

i.e., z_{ij} = min $\{f_k(i,j) \times w_p \mid p = 1\text{-}4\}$

$$\boldsymbol{y}_{MSMF} = \begin{cases} \boldsymbol{y}_{VMF}, \ z_{ij} \ge T \\ f_k(i,j), \ \text{otherwise} \end{cases} \tag{37}$$

2.3.10 Adaptive Trimmed Averaging Filter for Impulse Noise Removal

This filter [46] decides the center of a group of most similar pixels and output the average of these pixels. It is the average of pixels enclosed in a sphere of radius h centered at $\boldsymbol{x}_{RVM}$.

$$\boldsymbol{x}_{ATAF} = \frac{1}{\alpha}\sum_{k=1}^{\alpha} \boldsymbol{x}_k \ : \rho\{\boldsymbol{x}_k, \boldsymbol{x}_{VMF}\} \le h \tag{38}$$

where $\boldsymbol{x}_{RVM} = \underset{x, x_j \epsilon\, w}{argmin} \sum_{j=1}^{n} \rho\{\boldsymbol{x}, \boldsymbol{x}_j\}$ and $\rho\{x,y\} = \begin{cases} \|x-y\| & \text{if } \|x-y\| \le h, \\ h & \text{otherwise} \end{cases}$ (39)

2.3.11 Soft switching technique for impulse noise removal

In this paper [47], concept of trimmed cumulated distances is used in the impulse noise detection process. Corrupted pixels are restored by the weighted mean of the sharpening vector median filter output and the central pixel. The output $\boldsymbol{y}_1$ of this filter is expressed as

$$\boldsymbol{y}_1 = \gamma\,.\boldsymbol{a}_1 + (1\text{-}\gamma).\,\boldsymbol{a}^*_{(1)}. \tag{40}$$

where $\boldsymbol{a}^*_{(1)}$ is the sharpening vector median filter (SVMF) output.

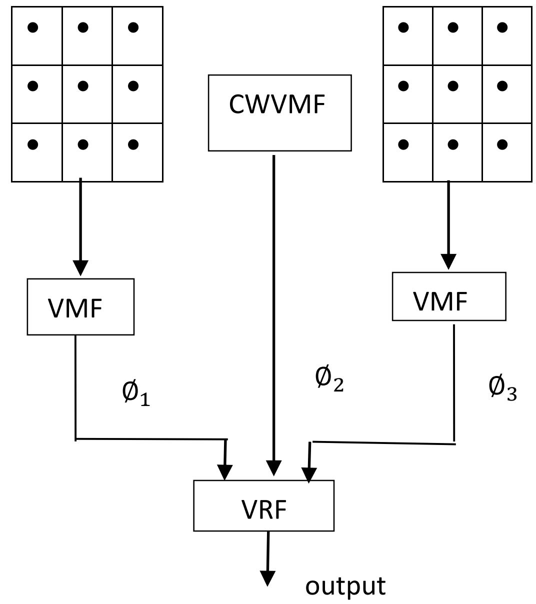
For γ=0, sharpening vector median filter (SVMF) is the output and for γ=1 the central pixel $\boldsymbol{a}_1$ is kept unchanged. A family of filters such as STVMF (Switching Trimmed with VMF output (STVMF), Adaptive Switching Trimmed with VMF output (ASTVMF), Fast Adaptive Switching Trimmed with VMF output (FASTVMF), Switching Trimmed with AMF output (STAMF), Adaptive Switching Trimmed with AMF output (ASTAMF), Fast Adaptive Switching Trimmed with AMF output (FASTAMF) are introduced in [48], in which reduced ordering and trimmed cumulative Euclidean distances to only the most similar pixels of the neighborhood are used in the impulse detection step. Arithmetic mean filter (AMF) is used to replace the center pixel if not corrupted and VMF output if found noisy. A self-tuning version of FASTAMF is proposed in [49] to free parameter selection problem.

2.4 Hybrid vector filters

Hybrid filters uses various sub-filters and gives the output as a combination of the input vectors samples.

2.4.1 Vector Median-Rational Hybrid Filter (VMRHF)

The output vector of VMRHF is the result of a vector rational function operating on three sub-outputs of the three sub-filter i.e., one center weighted vector median filter (CWVMF) and two vector median (VM) sub-filters [21, 23] which is shown in Fig. 1.



2.4.2 Fuzzy vector median rational hybrid filter (FVMRHF)

Fuzzy vector median rational hybrid filter [51] applied fuzzy techniques in the filtering process. In the first stage, one fuzzy center weighted vector magnitude filter and two bidirectional fuzzy vector median sub-filters are used. And the output of these sub-filters act as input to the vector rational function in the next stage.

2.4.3 Adaptive Vector Median Rational Hybrid Filter (AVMRHF)

An adaptive approach of the vector median rational hybrid filter is also proposed in [52]. In this filter, first stage utilized three adaptive vector median filter and the output of these filters serve as input to the rational function in the next stage.

2.5 Peer group vector filters

These vector filters are based on the concept of peer group. A set of pixels in a sliding window which are very close to a pixel according to a specific measure are the peer group of that pixel.

2.5.1 Peer group averaging filter

In this filter (PGA) [54], the center pixel of a sliding window is changed by the weighted average of its peer group members. Color similarity between two color vectors is used to determine the peer group which is measured by Euclidean distance.

2.5.2 Peer group vector filter

In [53], according to the distance between the pixels in the sliding window and the central pixel they are sorted in ascending order for finding peer group as follows

$C_i = \left\| \boldsymbol{x}_{(N+1)/2} - \boldsymbol{x}_i \right\|_\gamma$ for $i = 1, 2, \ldots, N$ (41)

The peer group is computed as *m* pixels in the ordered sequence that rank lowest with *m* given by $m = \frac{(\sqrt{N}+1)}{2}$

First order difference of the peer group is computed to check the presence of impulse.

$\delta_i = C_{i+1} - C_i$ for $i = 1, 2, \ldots, m$. (42)

The central pixel is considered noisy if one of these differences exceeds a pre-specified threshold and will be interchanged by VMF output else it remains unaffected. A variant of PGF is Fast peer group filter (FPGF) [55] in which the central pixel is regarded as corrupted if *m* pixel is not found to be similar. A peer group switching filter is proposed in [56], based on analysis of Fisher's linear discriminant working on the aggregated distances. Replacement of corrupted pixel is done by VMF output. In Fast averaging peer group filter (FAPGF) [57], the center pixel is considered as corrupted if the peer group or number of close pixel is too low otherwise it will be declared as not corrupted.

Fig. 1 Structure of VMRHF

Weighted average of noise-free samples from the local neighborhood is used to replace corrupted pixel. If the distance between two pixels in a given color space does not exceed a predefined threshold value, then they are considered as close. A novel 3D (3 dimensions) directional peer-group filter (3DPGF) is suggested in [58] for deletion of random valued impulse noise. In this filter, directional peer group method is performed in the noise detection stage and a 3D peer group weighted-mean technique is used to remove the noise. A fuzzy approach to peer group concept is also proposed in [59]. In this paper, the peer group concept is adapted to the use of a novel fuzzy metric. A two stage fuzzy based peer group concept called the fuzzy peer group filter is introduced in [60].

2.6 Fuzzy vector filters

Unlike traditional mathematical modelling techniques fuzzy rule based approach permits the integration of human knowledge in the design of signal processing [3]. To adjust to local image features fuzzy based filter used data dependent coefficients [61,62]. Fuzzy membership functions established on different distance functions are used to decide the weights on a nonlinear adaptive filter.

2.6.1 Fuzzy weighted Averaging filter (FWAF)

Fuzzy weighted average filter (FWAF) [61-63] is a special class of the general nonlinear fuzzy algorithm which is of the form

$$\hat{y} = g\left(\frac{\sum_{j=1}^{n} w_i \boldsymbol{x}_i}{\sum_{i=1}^{n} w_i}\right) \quad (43)$$

In this filter, the function g(.) is the identity function and it is defined as

$$\boldsymbol{x}_{fwaf} = \frac{\sum_{i=1}^{n} w_i \boldsymbol{x}_i}{\sum_{i=1}^{n} w_i} \quad (44)$$

Based on the fuzzy transformation and the distance criterion used many fuzzy filters can be obtained.

Fuzzy vector median filter (FVMF) [61-63] is obtained when the distance criterion used is the Minkowski metric L_γ and the exponential form as the membership function

$$\mu_i = \exp\left[-\frac{L_\gamma(i)^a}{\varepsilon}\right] \quad (45)$$

where ε is a distance threshold and a is a positive constant which regulate the quantity of fuzziness in the weights.

Fuzzy vector directional filter (FVDF) [61,62] is obtained when angular distance measure and asigmoidal membership function are used for determining the fuzzy weight. The fuzzy weight associated with the vector $\boldsymbol{x}_i$ is given by

$$\mu_i = \frac{\varepsilon}{(1+\exp(\alpha_i))^r} \quad (46)$$

where α_i is the angular distance measure.

2.6.2 Adaptive Nearest-Neighbor Filter (ANNF)

ANNF [64] is based on nearest neighbor rule in which fuzzy weights are determined by

$$w_i = \frac{b_{(n)} - b_{(i)}}{b_{(n)} - b_{(1)}} \text{ for } i = 1, 2, \ldots, n. \quad (47)$$

where $b_{(n)}$ and $b_{(1)}$ represents the highest and lowest cumulative angular distances respectively inside the filtering window.

2.6.3 Adaptive nearest neighbor multichannel filter (ANNMF)

This filter [65] is a variation of the ANNF. It combines vector magnitude and vector direction filter. The distance measure of vector $\boldsymbol{x}_i$ is given by

$$d_i = \sum_{j=1}^{N}(1 - S(\boldsymbol{x}_i, \boldsymbol{x}_j)) \quad (48)$$

$$S(\boldsymbol{x}_i, \boldsymbol{x}_j) = \left(\frac{\boldsymbol{x}_i \boldsymbol{x}^t}{|\boldsymbol{x}_i||\boldsymbol{x}_j|}\right)\left(1 - \frac{|\|\boldsymbol{x}_i\| - \|\boldsymbol{x}_j\||}{\max(|\boldsymbol{x}_i|, |\boldsymbol{x}_j|)}\right) \quad (49)$$

2.6.4 Fuzzy Ordered Vector Directional Filters (FOVDF)

Fuzzy ordered vector directional filters [61,66] are a fuzzy generalization of the α-trimmed filters. Based on the fuzzy membership strength the input vectors are arranged and contribution to the output vectors is done only by these vectors having the maximum fuzzy weights. It is given as

$$\boldsymbol{x}_{FOVDF}=\frac{1}{z}\sum_{i=1}^{\zeta} w_{(i)}\,\boldsymbol{x}_{(i)} \qquad (50)$$

where $z=\sum_{i=1}^{\zeta} w_{(i)}$

where $w_{(i)}$ denotes the i^{th} ordered fuzzy membership function such that

$w_{(\zeta)} \leq w_{(\zeta-1)} \leq \ldots \leq w_{(1)}$, with $w_{(1)}$ being the fuzzy co-efficient having the largest membership value.

2.6.5 Adaptive Fuzzy Hybrid Multichannel Filter (AFHMF)

Adaptive Fuzzy Hybrid Multichannel Filter (AFHMF) [67] consists of three parts- a hybrid multichannel filter, a fuzzy ruled-based system and a learning algorithm. Hybrid multichannel filter comprises of four components- VMF, BVDF, Identity Filter (IF) and a summation combinatory.

2.6.6 Fuzzy Decision Vector Filter (FDVF)

Fuzzy Decision Vector Filter [68] is a modification of the modified switching median filter (MSMF) [45] that overcome the uncertainty and ambiguity of impulse noise pixel in digital images. Fuzzy membership is used for noise detection and it is expressed as

$$\boldsymbol{z}_{ij}=\begin{cases} d_{max}, & z_{ij} > d_{max} \\ z_{ij}, & \text{otherwise} \end{cases} \qquad (51)$$

$$\mu(i,j)=\frac{d_{max}-z_{ij}}{d_{max}-d_{min}} \qquad (52)$$

$\mu(i,j)$ will classified pixel as noise pixel and noise-free pixel. The output of this filter is

$$\boldsymbol{y}_{FDF}=\begin{cases} f_k(i,j), & \mu(i,j) \geq 0.9 \\ \frac{f_{(k)}(i+u,j+v)\mu(i+u,j+v)}{\sum \mu(i+u,j+v)}, & \text{if } \big(\mu(i,j)\big) \leq 0.9 \text{ and } \mu(\,i+u,j+v) \geq 0.8 \\ \frac{f_{(k)}(i+u,j+v)\mu(i+u,j+v)}{\sum_{\mu\epsilon N,v\epsilon N} \mu(i+u,j+v)}, & (\,0.8 \geq (i+u,j+v) \geq 0.6), \text{ otherwise} \end{cases} \qquad (53)$$

2.6.7 Corrected Fuzzy Averaging Filter (CFAF)

In this filter the rank ordered difference (ROD) [69] statistics is used in the impulse noise detection step. A low value of ROD of the central pixel indicates that the center pixel is expected to be uncorrupted and a higher value of the ROD indicates a higher noise degree for the center pixel of the window. The certainty degree $\delta(F_0)$ for the vague statement "F_0 is noisy" is defined using x = ROD(F_0) as

$$\delta(F_0)=\mathrm{f}(x)=\begin{cases} 0, & x \leq k_1 \\ \frac{x-k_1}{k_1-k_1}, & k_1 < x < k_2 \\ 1, & k_2 \leq x \end{cases} \qquad (54)$$

where F_0 is the central pixel of the sliding window, k_1 and k_2 are constants.

Also a certainty degree of the vague statement "F_i is not noisy" is also assigned to each pixel of the window. A fuzzy averaging between the center pixel and a robust estimator of uncorrupted color vector is computed as follows

$$\overline{F_0}=(1-\delta(F_0))\,F_0+\delta(F_0)\,F_{RVMF} \qquad (55)$$

A correction step is also incorporated to this fuzzy averaging operation for appropriate processing of the noisy image.

2.6.8 Region Adaptive Fuzzy Filter (RAFF)

In Region Adaptive Fuzzy Filter (RAFF) [70] classification of corrupted and uncorrupted pixels is done using improved minimum mean value detection (IMMVD) mechanism. An adaption method is used to select the maximum allowable size of window during fuzzification and filtering and to adapt to local noise densities. In order to preserve more image details this filter performed region selective iteration filtering on highly corrupted regions.

2.7 Vector Sigma Filters

Vector Sigma Filters are extension of the gray scale sigma filters [71]. In this filters, detection of noisy pixels depends on calculation of the multivariate variance of the input sample. To decide an effective switching rule between no filtering (identity operation) and filter output, statistical measures of vector's deviation and robust order-statistic concepts are used in combination with diverse distance measures between multichannel inputs. Output of VMF, BVDF and DDF are used to replace noisy pixel and are referred to as Sigma VMF (SVMF) [73, 74], Sigma BVDF (SBVDF) [72,73,74], Sigma DDF (SDDF) [73,74] respectively. These filters are controlled by a tuning parameter λ to decide the switching threshold. Mathematically, the Sigma Vector Median Filter (SVMF) is defined as

$$\boldsymbol{x}_{SVMF} = \begin{cases} \boldsymbol{y}_{(1)}, \text{for } D_{(N+1)/2} \geq Tol \\ \boldsymbol{y}_{\frac{N+1}{2}}, \quad \text{otherwise} \end{cases} \tag{56}$$

where $\boldsymbol{y}_{(1)}$ is the output of VMF, $D_{(N+1)/2}$ is the distance measure of the center pixel $\boldsymbol{y}_{(N+1)/2}$ and *Tol* is a threshold value given by

$$Tol = D_{(1)} + \lambda\, \Psi_\gamma = \frac{N-1+\lambda}{N-1}\, D_{(1)} \tag{57}$$

where $D_{(1)}$ is the lowest aggregated Minkowski metric associated with the vector median, λ is a tuning parameter that adjust the smoothing properties of the SVMF and Ψ_γ is the approximated multivariate variance of the vector contained in the sliding window and is given by

$$\Psi_\gamma = \frac{D_{(1)}}{N-1}.$$

Sigma Basic Vector directional filter (SBVDF) is stated as follows

$$\boldsymbol{x}_{SBVDF} = \begin{cases} \boldsymbol{y}_{(1)}, \text{for } \alpha_{(N+1)/2} \geq Tol \\ \boldsymbol{y}_{\frac{N+1}{2}}, \quad \text{otherwise} \end{cases} \tag{58}$$

$$Tol = \alpha_{(1)} + \lambda \Psi_A \tag{59}$$

$$\Psi_A = \frac{\alpha_{(1)}}{N-1} \tag{60}$$

where $\alpha_{(1)}$ is the minimum aggregated angular distance, Ψ_A is the approximated variance calculated using the angular distance of multichannel samples inside the sliding window, $\boldsymbol{y}_{(1)}$ is the output of BVDF operation and $\alpha_{(N+1)/2}$ is the aggregated angular distance of the central pixel $\boldsymbol{y}_{(N+1)/2}$.

Similarly, Sigma Directional Distance Filter is given by

$$\boldsymbol{x}_{SDDF} = \begin{cases} \boldsymbol{y}_{(1)}, \text{for } \Omega_{(N+1)/2} \geq Tol \\ \boldsymbol{y}_{\frac{N+1}{2}}, \quad \text{otherwise} \end{cases} \tag{61}$$

$$Tol = \Omega_{(1)} + \lambda \Psi_{\gamma A} \tag{62}$$

$\Psi_{\gamma A}=\frac{\Omega_{(1)}}{N-1}$ (63)

where $\Omega_{(N+1)/2}$ is the aggregated hybrid measure of the central pixel $\boldsymbol{y}_{(N+1)/2}$, $\Omega_{(1)}$ is the smallest hybrid measure, $\Psi_{\gamma A}$ is the approximated variance calculated using the hybrid measure of multichannel samples inside the sliding window, $\boldsymbol{y}_{(1)}$ is the output of DDF operation.
While multivariate variance based on the sample mean or the lowest-ranked vector is used to adaptively determined threshold value in adaptive vector sigma filters (ASVMF, ASBVDF, ASDDF) [75].

2.8 Entropy vector filters
These are the adaptive multichannel extensions of the grayscale local contrast entropy filter introduced in [76]. Let $\{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_N\}$ be the grayscale samples inside the sliding window of size *N*, then the contrast of a pixel $\boldsymbol{y}_i$ is defined by

$C_i = \frac{|\boldsymbol{y}_i-\bar{\boldsymbol{y}}|}{\bar{\boldsymbol{y}}} = \frac{\Delta_i}{\bar{\boldsymbol{y}}}$ (64)

where $\bar{y}$ represents the mean of the input set $\{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_N\}$ and is the gradient level. The local contrast entropy H_i and P_i local contrast probability associated with pixel $\boldsymbol{y}_i$ is defined as

$P_i = \frac{\Delta_i}{\sum_{j=1}^{n}\Delta_j}$ (65)

$H_i = -P_i \log P_i$ (66)

Entropy vector median filter is based on the concept of robust order-statistics theory and local entropy contrast. The output of the entropy vector median filter (EVMF) [77, 79] is given by

$\boldsymbol{x} = \begin{cases} \boldsymbol{y}_{(1)}, & \text{if } P_{(N+1)/2} \geq \beta_{(N+1)/2} \\ \boldsymbol{y}_{(N+1)/2}, & \text{otherwise} \end{cases}$ (67)

where **x** is the filter output, $\boldsymbol{y}_{(1)}$ is the VMF output and $\beta_{(N+1)/2}$ is the adaptive threshold of the central sample given by:

$\beta_i = \frac{-P_i log P_i}{H} = \frac{-P_i \log P_i}{-\sum_{j=1}^{N} P_j log P_j}$ (68)

where *H* is the overall entropy and is defined as

$H = -\sum_{i=1}^{N} P_i log P_i$ (69)

where P_i is the local contrast probability associated with the input vector samples $\boldsymbol{y}_i$ and is given by

$P_i = \frac{(\sum_{k=1}^{m}|y_{ik}-\mu_k|^{\gamma})^{\frac{1}{\gamma}}}{\sum_{j=1}^{N}(\sum_{k=1}^{m}|y_{jk}-\mu_k|^{\gamma})^{\frac{1}{\gamma}}}$ (70)

μ_k represents the k^{th} component of the mean.

Other entropy vector filters such as Entropy Basic Vector Directional Filter (EBVDF) [78] and Entropy Directional Distance Filter (EDDF) [78] are also proposed by using the corresponding angular and hybrid measure.

2.9 Quaternion based vector filters

Quaternion concept is used in finding and deleting impulse noise from color images. A quaternion number is a four dimensional number that consists of a real part and three imaginary parts [80-83]. An RGB color pixel is expressed in quaternion form as

$$q_1 = r_1\hat{\imath} + g_1\hat{\jmath} + b_1\hat{k} \qquad (71)$$

where r_1, g_1 and b_1 are red, green and blue channels respectively.

Quaternion considers both chromaticity and intensity components of the color pixel when used as distance measure. Many switching filters based on quaternions are proposed in the literature. In [84] a two-stage filters using both the quaternion based switching filter and a local mean filter is designed for removing mixture noise. A new two stage filter is proposed in [85] which incorporates the peer group concept along with the quaternion based distance measure for impulse detection. A Quaternion based Switching Vector Median Filter is proposed in [86]. In this filter a modification of median of absolute deviation from median (MAD) is used for detecting impulse and corrupted pixels are substituted by VMF output calculated using quaternion. A two stage noise detection quaternion vector median filter for impulse noise removal from medical image is also proposed in [87].

2.10 Morphological based filter

Morphological filters are non-linear image filter established by the combination of parallel or sequential fundamental morphological operations of opening, closing, dilation and erosion [88]. The famous structures of morphological filters are closing followed by opening (CO) and opening followed by closing (OC). Dilation and Erosion resembles a max/min filtering action for suppression of impulse noise. Erosion distributes the minimum pixel while the maximum pixel within the operation window is distributed by dilation. A learning-based color morphological filter was proposed in [88]. In this filter the morphological operations are learning-based operations, in which color pixel ordering scheme is learned according to the pre-estimation of healthy and contaminated pixels. Support vector machine (SVM) is used for finding a decision values for classifying pixels into uncorrupted and corrupted pixels. An image reconstruction step is carried out after each morphological operation for restoring the original features. In [89] a two stage morphological noise detector is used for identifying noisy pixels from corrupted image. In the first stage erosion and dilation operator are used and in the second stage opening and closing operator are used for identifying uncorrupted pixel from those pixels which are identified as corrupted in the first stage. These morphological operators are applied to each channel separately and inpainting is applied to the noisy pixels for each channel independently. A multivariate extension of the self-dual morphological operator is proposed in [90] which can be utilized for noise removal and segmentation process. To construct multivariate dual morphological operator a pair of symmetric vector orderings (SVO) is introduced in this paper. A hybrid filter is proposed in [91] which combines decision based trimmed median filter for salt and pepper noise suppression and cancellation and mathematical morphology. A hybrid vector ordering which combines both reduced order and the bit mixing order for performing morphological operator is proposed in [92]. A new approach to detection of noisy pixels and deletion of the detected noise using morphological filters is proposed in [93]. In this method, a variation of the noise detection method used in fast peer group filter (FPGF) is proposed. Many complex mathematical tools like principal component analysis (PCA) [94], probabilistic estimation extremas [95], support vector machine (SVM) [96] are also used to develop the multivariate morphological operators.

2.11 Miscellaneous Filters

These are the filters that cannot be included in any of the filter class described above even though they have some common properties.

2.11.1 Machine learning and neural network based filtering approach

Usually machine learning and neural network techniques are applied for high-level image processing task such as image segmentation, object recognition, computer vision etc. because of the strong capability of automatic feature extraction and classification. Support vector machine (SVM) are used for classification of uncorrupted and corrupted impulse pixels in color images. Multiclass support vector machine (SVM) based adaptive filter (MSVMAF) is developed in [97] for deletion of high density impulse noise from color images. This method takes the benefits of both adaptive vector median filtering and multiclass SVM. Prediction error computed using fixed size window is used for classification of corrupted pixels. Recently applications of deep neural network techniques for impulse noise reduction in color images are found in the literature. In [98] deep convolutional neural network is used for both noise identification and image reconstruction. To detect noisy pixels a noise classifier network is trained which not only can identify the corrupted pixels but also can further identify the noisy channel. In [99] Denoising Convolutional Neural Networks (DnCNN) is utilized for reduction of impulse noise in color images. Concept of deep residual learning which is trained to learn the residual image is utilized in this network. A structure-adaptive vector filter is proposed in [100] which employs a deep convolutional neural network as noise classifier. A switching filter based on deep learning is proposed in [101], in which distorted pixels are detected by a deep neural network and restored with the fast adaptive mean filter.

2.11.2 Similarity based Impulsive Noise Removal Filters

A filter is proposed in [102] built on the concept of similarities between the pixels in a predefined window. A convex similarity function is used to calculate the similarity between pixels. The cumulated sum of similarities M for the central pixel $\boldsymbol{x}_1$ and its neighboring pixels are calculated as follows:

$$M_1 = \sum_{j=2}^{N} \mu(\boldsymbol{x}_1, \boldsymbol{x}_j), \qquad (72)$$

$$M_k = \sum_{j=2, j\neq k}^{N} \mu(\boldsymbol{x}_k, \boldsymbol{x}_j) \qquad (73)$$

where $\mu(\boldsymbol{x}_i, \boldsymbol{x}_j)$ is a convex similarity function. If $M_1 < M_k$, $k = 2, \ldots, N$, then central pixel is corrupted and is substituted by that $\boldsymbol{x}_k$ for which $k = \text{argmax } M_i$, $k = 2, \ldots, N$. A similar filter based on this concept is also proposed in [103].

2.11.3 Filters based on Digital Path Approach

The concept of digital paths is utilized in reducing impulse noise from color images. In [104], fuzzy membership functions defined over vectorial inputs linked via geodesic path is utilized. In this approach instead of using a fixed window probable connections between the successive image pixels using the concept of geodesic paths is proposed. In [105], fuzzy measure is applied to image pixels linked by digital paths. This digital path concept is used in [106,107] to calculate the cost of optimal path of the center pixel. Here the path starts from the border of the sliding window and reach its center. A pixel is considered as outlier if the minimum cost is high. A fast technique for suppressing such noise from color image is presented in [108]. This method utilized the concept of digital paths which connect the central pixel with its boundary in a sliding window. Central pixel is considered as outlier if minimum cost of all the path assigned is high.

2.11.4 Vector Signal-Dependent Rank Order Mean Filters

A multichannel extension of the grayscale Signal Dependent Rank Order Mean (SDROM) filter [109] for reducing impulse noise from color image is proposed in [110] known as the Vector Signal Dependent Rank Order Mean (VSDROM) filter. In this filter, the pixels in the window are arranged based on the aggregate distances calculated to all other pixels. Then distances between the lowest ranked four pixels and the central pixel are compared against increasing threshold. If one of these distance is bigger than the respective threshold, then central pixel noisy and will be changed by VMF output.

2.11.5 Vector marginal median filters (VMMF)

Vector marginal median filter [3] computes the median value of each channel separately and the center pixel is changed by the median value of the respective channel.

2.11.6 Directional-magnitude vector filter (DMVF)

Directional-magnitude vector filter [111] is a content-based rank ordered filter in which the similarity between two vectors $\boldsymbol{y}_i$ and $\boldsymbol{y}_j$ is defined as the ratio of some function of what $\boldsymbol{y}_i$ and $\boldsymbol{y}_j$ share (commonality) to what they comprise together (totality). The distance measure between the two vectors used in this filters is

$$D(\boldsymbol{y}_i, \boldsymbol{y}_j) = \frac{(|\boldsymbol{y}_i|^2 + |\boldsymbol{y}_j|^2 - 2|\boldsymbol{y}_i||\boldsymbol{y}_j|\cos(\Theta))^{0.5}}{(|\boldsymbol{y}_i|^2 + |\boldsymbol{y}_j|^2 + 2|\boldsymbol{y}_i||\boldsymbol{y}_j|\cos(\Theta))^{0.5}} \qquad (74)$$

And distance for noisy vector $\boldsymbol{y}_i$ inside the processing window of length *n* is defined as

$$d_i = \sum_{j=1}^{n} D(\boldsymbol{y}_i, \boldsymbol{y}_j) \qquad (75)$$

The output of DMVF is defined as

$$\boldsymbol{y}_{DMVF} = \boldsymbol{y}_{(1)} \qquad (76)$$

with $\boldsymbol{y}_{(1)} \leq \boldsymbol{y}_{(2)} \leq \ldots \leq \boldsymbol{y}_{(n)}$.

III. Impulse Noise Model

Impulse noise can be divided into two types: Uncorrelated Impulse noise and Correlated impulse noise. The uncorrelated impulse noise has the following form [56]

$$q'_k = \begin{cases} n_k & \text{with probability } p \\ q_k & \text{with probability } 1-p \end{cases} \qquad (77)$$

where *p* is corruption probability of the channel; *k* = 1,2,3 represents the three channels in RGB color space; n_k and q_k represent the contaminated and original component respectively. For fixed-valued impulse noise n_k can take either 0 or 255 and for random-valued impulse noise it can have any values in [0,255].

Correlated impulse noise model proposed by [9] has the following form

$$\boldsymbol{x}(n) = \begin{cases} a, & \text{with probability } 1-p \\ (d, a_2, a_3)^T, & \text{with probability } p_1 . p \\ (a_1, d, a_3)^T, & \text{with probability } p_2 . p \\ (a_1, a_2, d)^T, & \text{with probability } p_3 . p \\ (d, d, d)^T, & \text{with probability } p . p\Sigma \end{cases} \qquad (78)$$

where $a = (a_1, a_2, a_3)^T$ is constant noise free vector, *d* is the impulse value, $\boldsymbol{x}$(n) is the noisy signal, $p\Sigma$ = 1-p_1- p_2- p_3 and $\sum_{i=1}^{3} p_i \leq 1$. Impulse *d* can have either negative or positive values but not both. We assume that $d \gg a_1, a_2, a_3$ and thus $d - a_1 \cong d - a_2 \cong d - a_3$.

IV. Evaluation of filter performance

The following parameters are used to assess the performance of filter: -

1. Mean Absolute Error (*MAE*)

$$MAE = \frac{1}{3 \times M \times N} \sum_{k=1}^{3} \sum_{i=1}^{M} \sum_{j=1}^{N} |o_{(i,j)k} - y_{(i,j)k}| \qquad (79)$$

2. Mean Squared Error (*MSE*)

$$MSE = \frac{1}{3 \times M \times N} \sum_{k=1}^{3} \sum_{i=1}^{M} \sum_{j=1}^{N} (o_{(i,j)k} - y_{(i,j)k})^2 \qquad (80)$$

where $o_{(i,j)} = [o_{(i,j)1}, o_{(i,j)2}, o_{(i,j)3}]$, $y_{(i,j)} = [y_{(i,j)1}, y_{(i,j)2}, y_{(i,j)3}]$ are the original and filtered pixel respectively with (*i*, *j*) denoting the spatial position in a $M \times N$ color image and *k* presenting the color channel.

3. Peak Signal to Noise ratio (*PSNR*)

$PSNR = 10\ log_{10}[\frac{I_{max}^2}{MSE}]$ (81)

where I_{max} is the greatest pixel value of the original image.

4. Normalized Color Distance (*NCD*)

The NCD is defined in the Lu*v* color space by

$$NCD = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\sqrt{(L_{(i,j)}^{o}-L_{(i,j)}^{*})^2+(u_{(i,j)}^{o}-u_{(i,j)}^{*})^2+(v_{(i,j)}^{o}-v_{(i,j)}^{*})^2}}{\sum_{i=1}^{M}\sum_{j=i}^{N}\sqrt{(L_{(i,j)}^{o})^2+(u_{(i,j)}^{o})^2+(v_{(i,j)}^{o})^2}} \quad (82)$$

where $L_{(i,j)}^{o}$, $u_{(i,j)}^{o}$, $v_{(i,j)}^{o}$ and $L_{(i,j)}^{*}$, $u_{(i,j)}^{*}$, $v_{(i,j)}^{*}$ are values of the lightness and two chrominance components of the original image sample and filtered image sample respectively.

5. Structural Similarity Index (*SSIM*)

$SSIM = \frac{(2\mu_x\mu_y+c_1)(2\mu_{xy}+c_2)}{(\mu_x^2+\mu_y^2+c_1)(\sigma_x^2+\sigma_y^2+c_2)}$ (83)

where μ_y and μ_x are mean of the filtered and original image, c_1 and c_2 are constants, σ_x^2 and σ_y^2 denote the corresponding covariance and variance of the original and filtered images. SSIM evaluates similarity between two images.

MAE estimate detail preservation, noise suppression capability is evaluated by MSE and PSNR. NCD evaluates perceptual error in the CIELab color space. And for an good filter, it is expected to have MAE, MSE and NCD minimum while PSNR and SSIM to have high value.

V. Conclusion

A broad survey of the various methods for removing impulse noise from color images is presented. In this study the filters are categorized into 11 classes based on the techniques and methods used. Some recently introduced algorithms are also added in this study.

VI. FUTURE SCOPE

In future we would like to work on impulse noise removal from color medical images.

ACKNOWLEDGEMENTS

Authors are much thankful for the financial assistance from UGC, Ministry of Human Resource Development under JRF scheme.

Conflict of Interest: Author Thiyam Romita Chanu declares that she has no conflict of interest. Author Thounaojam Rupachandra declares that he has no conflict of interest. Author Khumanthem Manglem declares that he has no conflict of interest.

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