Research Article

Hesitant Fuzzy Project Planning and Scheduling using Critical path Technique

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Abstract: Hesitant fuzzy set is extremely useful performance to express people's uncertainty. But there are delicate points in conventional hesitant fuzzy set, which articulated the membershipdegrees of an element to a given set only by some crisp numbers .Hesitant fuzzy set to Triangular Fuzzy Hesitant Fuzzy Set (THFS), in which the membership degreeof an element to a given set is expressed by several possible triangular fuzzy numbers. A new plan to determine the critical path in the project network using Triangular Hesitant Fuzzy set (THF).In this project network Each activity time is THFS.One of multicriteriadecision making Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is used for the project network to identify the best path in the project.Finally a numerical exemplar is furnished to elucidate the critical path of the project network.

Keywords:

1. Introduction

Critical path method (CPM)` is one of the project network based method to evaluating the best project performance in the project network. The flourishing execution of CPM requires the accessibility of a apparent determined time duration for each activity. Nevertheless, in practical situations this very difficult to complete because number of activities are performing using vague data. So in advance the activity times of each activity is very difficult to know. As a result the unknowns or vagueness concerning the time duration of activities in network planning, has led to the improvement of fuzzy CPM. Interactive fuzzy subtraction on a ten point network was proposed by Nasution [21] ,Where In which each activity time represents a trapezoidal fuzzy numbers to determine newest allowable and slack for activities, Also discussed the $\alpha -$cut level of the slack fuzzy slack availability in critical path in the project network. Lin, etal [18],proposed the Critical path method in activity networks with Fuzzy activities duration times. In the project network membership grade project could be completed within the stipulated time was proposed by Liang et al. [19] as well as an algorithm to study critical path in a fuzzy environment. Tian et al. [27] delivered a method to find fuzzy critical path using probability theory and fuzzy-timing high level Petri nets. This way resource arrangement, time management and conflicts in shared work processes. Chen [4] discussed critical path in the project network in fuzzy environment. In this each activity time as a fuzzy number and also adapting the Yager's method of ranking. This method based on the principle of extension and linear programming formulation. A simple approach is developed to identify fuzzy critical path problem using Linear Programming and fuzzy number ranking method was proposed by app Chen et al.[5]. A Novel technique for finding earliest and latest events time of the project network using Linear Programming and Zadeh's principle of extension proposed by Zareei et al.[34]. Finding a critical path in the project network using lexicographic ordering method to rank the a trapezoidal fuzzy number. Mean while each activity in a project network is represented by a trapezoidal fuzzy number. Elizabeth et al. [10] presented Novel ranking methods to classify the fuzzy critical path in the project network using fuzzy membership function. Samayan[25], discussed the importance of hexagonal fuzzy numbers in Fuzzy critical path method based on ranking methods. Rajendran,C[22] discussed the comparison of Critical Path Problem under Fuzzy Environment with predictable process. Now the decision-making is the method of discovered the best alternative in a set of feasible alternatives. In this, several criteria's are taken into account called as multi-criteria decision making (MCDM) problems. MCDM refers to ordering the set of alternatives under usually conflicting criteria. These problems usually effect the delicacy of the tilted data being present, which makes the decision-making procedure complex. So in this decision-making often happens in fuzzy environment because the available date is vague. Therefore, in this state one of best method MCDM(Multi criteria decision making) to prioritizing the set of alternatives under usually inconsistent criteria. Now one of the MCDM method named as TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is presented. Initially Hwang and Yoon[13] presented TOPSIS method. This method discussed about the concept of ideal alternative, which is the optimum level for all considered attributes. Furthermore, the negative ideal is the one of the worst attribute value. In fuzzy environment extended TOPSIS method was proposed by Chen [6]. Linguistic terms are used for alternative's rating, weights criterion. Vertex method is adopting to Find the distance between triangular fuzzy numbers.

Yong [31],discussed place selection location in linguistic environment based on TOPSIS method. The product dispersal in a competitive automobile market and product adoption process in fuzzy TOPSIS and Agent-based diffusion model for an automobile market was discussed by Kim et al.[16].Ashrafzadeh et al.[2] presented a method for selecting the warehouse location under uncertain data in two steps .First identify the criteria and experts supply linguistic ratings to the promising alternatives against the preferred criteria. Fuzzy TOPSIS is used for choosing the best alternatives. An extension to fuzzy TOPSIS using distances of the alternatives method proposed by Dymova et al.[9].Also discussed generalization of particular aggregation modes method, local criteria aggregation in TOPSIS. Finally discussed critical path in the project network. Zammori et al[35] focused on complex critical path problems.The main aim is to determine time, cost,risk by using Fuzzy logic. TOPSIS and finally attain critical path in a project network .Amiri [1] proposed an algorithm, To establish the fuzzy critical path in the project network based on fuzzy TOPSIS. Also consisting of time, cost, risk and quality criteria and using linguistic triangular fuzzy numbers

Interval-valued intuitionistic hesitant fuzzy Choquet integral based TOPSIS method for multi-criteria group decision making was presented by Joshi, Deepa, [15] . Zyoud, [36] discussed A bibliometric-based survey on AHP and TOPSIS techniques. Kumar etal., [17] presented. "TOPSIS method based on the connection number of set pair analysis under interval-valued intuitionistic fuzzy set environment. Dos Santos[8] presented the Performance assessment of green suppliers using entropy-TOPSIS-F. Dorfeshan.Y etal. [7]],discussed a new group TOPSIS-COPRAS methodology with Pythagorean fuzzy sets taking into consideration weights of experts for project critical path problem.

A dual hesitant fuzzy set (DHFS) has been comprehensive form of HFS(Hesitant fuzzy set) and disagree with the effects of uncertainty in the collected data. Tyagi [29] derived The correlation coefficient between two DHFS. The shortcoming the HFS and DHFS is crisp numbers between 0 and 1 are used to represent the membership degree and non-membership degree of an element to a particular set. In addition, in quantitative atmosphere the fuzzy set theoretic tools were discussed. Suppose the decision in qualitative atmosphere, In this situation better to choose fuzzy linguistic Zadeh[33] to get more apparent result by Wu[30]. On the other hand, fuzzy sets, and fuzzy linguistic draw near to have a few limitations [20]. Rodriguez et al [23] observed that the decision maker apprehensive, the process of decision making be vague in providing a particular linguistic term to express their observation based on their ability in the anxious region. For example , in a decision making problem with high degree of uncertainty the decision makers desire to use linguistic terms such as “medium and “high” , “very high” to state their observation. To overcome such limitations he designed the concepts of hesitant fuzzy linguistic term sets(HFLTS).Modify the relative linguistic expressions into HFLTS and introduced an envelope of HFLTS by Rodriguez et al., [24]. But, these envelops of HFLTS are again intervals and these envelops of HFLTS convey the loss of original information. To conquer this problem Yu .D[32]introduced the idea of HTFS, wherein the membership degrees of an element to a given set are expressed as triangular fuzzy number TFN. TFN have been working to differentiate unusual linguistic terms used by decision maker for weight of attributes and to the linguistic terms used for the rating of the obtainable alternatives based on the given set of attributes. Since the HTFS has the dominance on both the TFN and HFS, it will be the most appropriate tool to symbolize the semantic of linguistic terms used in many real life situations of decision making. Garg, [12]. An advanced study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision making presented by Garg[11]. Kakati, Pankaj, et al.[14] presented "Interval neutrosophic hesitant fuzzy Einstein Choquet integral operator for multicriteria decision making. a novel methodology for time planning of resource-constrained software projects with hesitant fuzzy durations: a case study has been presented by Basar, Ayfer [3]. Now in this section- 2 recalls the prelimeners of Hesitant fuzzy sets , Triangular Hesitant Fuzzy set. section- 3 discussed the new distance measure using Triangular Hesitant Fuzzy set(THFS) ,section- 4 discussed the new fuzzy TOPSIS using using Triangular Hesitant Fuzzy set(THFS). section- 5 discussed proposed method for the critical path selection.A numarical exemplar is has been furnished for the proposed method.

2.Preliminaries

This segment deals with basic definations of Hesitant fuzzy sets, Triangular Hesitant Fuzzy set(THFS)

2.1. Hesitant fuzzy sets(HFS)

For a indication set X, Torra [28] defined a hesistant fuzzy set H on x in terms of function $h(x)$ when applied to $x \in X$ returns a subset of $[0,1]$

$H = \{\langle x, h(x) : x \in X \rangle\}$, Where $h(x)$ is a set of some different values in $[0,1]$,representing the possible membership degree of the element $x \in X$ to H and termed as hesitant fuzzy element(HFE)

2.2. Triangular Hesitant Fuzzy set[32]

This section deals with the concept of Triangular Fuzzy Hesitant Fuzzy set(THFS). The member ship degrees of an elements is to a set are represented by several possible triangular fuzzy numbers.Basic definitions of triangular fuzzy set,triangular fuzzy number and Triangular Fuzzy Hesitant Fuzzy sets are discussed.

Definition 1[26]

If R be the set of real numbers , Define a fuzzy number γ on R is a triangular fuzzy number such that the membership function $\mu_\gamma : R \to [0,1]$ defined by

$$\mu_\gamma(x) = \begin{cases} \dfrac{x-\tilde{\gamma}^L}{\tilde{\gamma}^M - \tilde{\gamma}^L}, & x \in \left[\tilde{\gamma}^L, \tilde{\gamma}^M\right] \\ \dfrac{x-\tilde{\gamma}^U}{\tilde{\gamma}^M - \tilde{\gamma}^U}, & x \in \left[\tilde{\gamma}^M, \tilde{\gamma}^U\right] \\ 0 & \text{otherwise} \end{cases}$$

Where $\tilde{\gamma}^L \le \tilde{\gamma}^M \le \tilde{\gamma}^U$ and $\tilde{\gamma}^L, \tilde{\gamma}^U$ are minimum and maximum values of $\tilde{\gamma}$ respectively and $\tilde{\gamma}^M$ is the model value. The triangle fuzzy number is represented as $\left(\tilde{\gamma}^L, \tilde{\gamma}^M, \tilde{\gamma}^U\right)$

The set $\left\{x / \tilde{\gamma}^L < x < \tilde{\gamma}^U\right\}$ is the support of $\tilde{\gamma}$ .If $\gamma^L = \gamma^M = \gamma^R$ then $\tilde{\gamma}$ reduces to a crisp number

Consider the triangular fuzzy numbers $\tilde{\gamma}_1 = \left(\tilde{\gamma}_1^L, \tilde{\gamma}_1^M, \tilde{\gamma}_1^U\right)$ and $\tilde{\gamma}_2 = \left(\tilde{\gamma}_2^L, \tilde{\gamma}_2^M, \tilde{\gamma}_2^U\right)$ *then*

$$\tilde{\gamma}_1 \oplus \tilde{\gamma}_2 = \left(\tilde{\gamma}_1^L, \tilde{\gamma}_1^M, \tilde{\gamma}_1^U\right) \oplus \left(\tilde{\gamma}_2^L, \tilde{\gamma}_2^M, \tilde{\gamma}_2^U\right) = \left(\tilde{\gamma}_1^L + \tilde{\gamma}_2^L, \tilde{\gamma}_1^M + \tilde{\gamma}_2^M, \tilde{\gamma}_1^U + \tilde{\gamma}_2^U\right)$$

$$\tilde{\gamma}_1 \otimes \tilde{\gamma}_2 = \left(\tilde{\gamma}_1^L, \tilde{\gamma}_1^M, \tilde{\gamma}_1^U\right) \otimes \left(\tilde{\gamma}_2^L, \tilde{\gamma}_2^M, \tilde{\gamma}_2^U\right) = \left(\tilde{\gamma}_1^L \tilde{\gamma}_2^L, \tilde{\gamma}_1^M \tilde{\gamma}_2^M, \tilde{\gamma}_1^U \tilde{\gamma}_2^U\right)$$

$$\tilde{\gamma}_1 = \tilde{\gamma}_2 \Leftrightarrow \tilde{\gamma}_1^L = \tilde{\gamma}_2^L, \tilde{\gamma}_1^M = \tilde{\gamma}_2^M, \tilde{\gamma}_1^U = \tilde{\gamma}_2^U$$

$$\tilde{\gamma}_1 \le \tilde{\gamma}_2 \Leftrightarrow \tilde{\gamma}_1^L \le \tilde{\gamma}_2^L, \tilde{\gamma}_1^M \le \tilde{\gamma}_2^M, \tilde{\gamma}_1^U \le \tilde{\gamma}_2^U$$

$$\lambda \square\ \tilde{\gamma}_1 = (\lambda, \lambda, \lambda)\left(\tilde{\gamma}_1^L, \tilde{\gamma}_1^M, \tilde{\gamma}_1^U\right) = \left(\lambda\tilde{\gamma}_1^L, \lambda\tilde{\gamma}_1^M, \lambda\tilde{\gamma}_1^U\right), \lambda > 0, \lambda \in R$$

$$\tilde{\gamma}_1^{-1} = \left(\tilde{\gamma}_1^L, \tilde{\gamma}_1^M, \tilde{\gamma}_1^U\right)^{-1} = \left(\frac{1}{\tilde{\gamma}_1^U}, \frac{1}{\tilde{\gamma}_1^M}, \frac{1}{\tilde{\gamma}_1^L,}\right)$$

Definition 2[26]

Consider the triangular fuzzy number $\tilde{\gamma} = \left(\tilde{\gamma}_1^L, \tilde{\gamma}_1^M, \tilde{\gamma}_1^U\right)$ then the centriod of centroid[1] [502]is $\varsigma(\tilde{\gamma}) = \dfrac{\tilde{\gamma}_1^L + 7\tilde{\gamma}_1^M + \tilde{\gamma}_1^U}{9}$. *Assume that the* $\tilde{\gamma} = \left(\tilde{\gamma}_1^L, \tilde{\gamma}_1^M, \tilde{\gamma}_1^U\right)$ and $\tilde{\gamma}_2 = \left(\tilde{\gamma}_2^L, \tilde{\gamma}_2^M, \tilde{\gamma}_2^U\right)$ are two *triangular fuzzy numbersthen.Ranking of triangular fuzzy number* $\varsigma(\tilde{\gamma}_1) > \varsigma(\tilde{\gamma}_2)$ then $\tilde{\gamma}_1 > \tilde{\gamma}_2$, $\varsigma(\tilde{\gamma}_1) < \varsigma(\tilde{\gamma}_2)$ then $\tilde{\gamma}_1 < \tilde{\gamma}_2$, $\varsigma(\tilde{\gamma}_1) = \varsigma(\tilde{\gamma}_2)$ then $\tilde{\gamma}_1 = \tilde{\gamma}_2$. In such case, define the index associated with the ranking as $I_{\alpha,\beta}(\tilde{\gamma}) = \beta S_M(\tilde{\gamma}) + (1-\beta) I_\alpha(\tilde{\gamma})$ where $\beta \in [0,1]$. $S_M(\tilde{\gamma})$ is the mode associated with the fuzzy number which is equal to '$\tilde{\gamma}_1^M$' for a triangular fuzzy number $\tilde{\gamma} = \left(\tilde{\gamma}_1^L, \tilde{\gamma}_1^M, \tilde{\gamma}_1^U\right)$ and the average value of the area of stability for a triangular fuzzy number and we define the index associated with the ranking as

$I_{\alpha}\left(\tilde{\gamma}\right)=\alpha+\left(1-\alpha\right)\left(\frac{\tilde{\gamma}_1^L+7\tilde{\gamma}_1^M+\tilde{\gamma}_1^U}{9}\right)$ where $\alpha\in\left[0,1\right]$ is the index of optimism which represents the degree of optimism of a decision maker. The ranking is defined as

If $I_{\alpha,\beta}\left(\tilde{\gamma}_i\right)>I_{\alpha,\beta}\left(\tilde{\gamma}_j\right)\Rightarrow\tilde{\gamma}_i>\tilde{\gamma}_j$

If $I_{\alpha,\beta}\left(\tilde{\gamma}_i\right)<I_{\alpha,\beta}\left(\tilde{\gamma}_j\right)\Rightarrow\tilde{\gamma}_i<\tilde{\gamma}_j$

Definition 3[32].*Assume that the* X be a fixed set ,a Triangle fuzzy Hesitant Fuzzy set (THFS) $\tilde{F}$ *on* X *is a function* $\tilde{g}_E\left(x\right)$ *that takings several triangular fuzzy values represented as*

$$\tilde{F}=\left\{<x,\tilde{g}_{\tilde{F}}\left(x\right)>/x\in X\right\}$$

Here $\tilde{g}_{\tilde{F}}\left(x\right)$ contains numeroustriangular fuzzy number, the possible membership degrees of the elements $x\in X$. $\tilde{g}_{\tilde{F}}\left(x\right)$ is a Triangle fuzzy Hesitant Fuzzy set (THFS) and

$\tilde{g}_{\tilde{F}}\left(x_i\right)=\left\{\left(\tilde{\gamma}_1^L,\tilde{\gamma}_1^M,\tilde{\gamma}_1^U\right)/\tilde{\gamma}\in\tilde{g}_{\tilde{F}}\left(x_i\right)\right\}$ here $\tilde{\gamma}$ is the triangular fuzzy number. $\tilde{\gamma}^L,\tilde{\gamma}^U$ are minimum and maximum values of $\tilde{\gamma}$ respectively and $\tilde{\gamma}^M$ is the model value.

Triangular fuzzy hesistant fuzzy set is a very useful tool to express uncertainty .Let us consider the following case.

Case1: Three reviewers are decided to estimate the degrees that the members acceptability the criterionof honest.First time they had met each other and then evealuate under veage condition .The first reviewer believes the minimum possible of the members satisfies the criterion of honest is 0.1,the maximum possible is 0.5 and most possible is 0.3.Therefore evaluation about the candidate given by the first reviewer could be expressed by the triangular fuzzy number $\left(0.1,0.3,0.5\right)$.Similarly thr second reviewer and the third reviewer estimate and uttered by Triangular fuzzy number $\left(0.5,0.7,0.9\right)$ and $\left(0.4,0.9,1.0\right)$ respectively. If three reviewers have equal impact on this evaluation ,they can't be convinced by each other.In this case the complete evaluation about the members on honest can be expressed by a THFE $\left\{\left(0.1,0.3,0.5\right),\left(0.5,0.7,0.9\right),\left(0.4,0.9,1.0\right)\right\}$

Example- 1 . *Consider* $X=\left\{x_1,x_2\right\}$ *be a finite set,* $\tilde{g}_{\tilde{F}}\left(x_1\right)=\left\{\left(0.1,0.2,0.4\right)\left(0.2,0.5,0.6\right)\right\}$ and

$\tilde{g}_{\tilde{F}}\left(x_2\right)=\left\{\left(0.1,0.3,0.5\right)\left(0.4,0.6,0.8\right)\left(0.5,0.6,0.9\right)\right\}$ be the TFHFS of x_1,x_2 to the set $\tilde{F}$ and

$$\tilde{F}=\left\{<x_1,\left\{\left(0.1,0.2,0.4\right)\left(0.2,0.5,0.6\right)\right\}>,<x_2,\left\{\left(0.1,0.3,0.5\right)\left(0.4,0.6,0.8\right)\left(0.5,0.6,0.9\right)\right\}>\right\}$$

Definition 4[32]

Consider Triangular Hesitant Fuzzy sets(THFS), the sum of the HFS is presented as follows

$$\tilde{F}=\left\{\begin{array}{l}<x_1,\left\{\left(\delta_{11}^{L^1},\delta_{12}^{M^1},\delta_{13}^{U^1}\right)\left(\delta_{11}^{L^2},\delta_{12}^{M^2},\delta_{13}^{U^2}\right)\ldots\ldots\ldots\ldots\left(\delta_{11}^{L^p},\delta_{12}^{M^p},\delta_{13}^{U^p}\right)\right\}>,\\ <x_2,\left\{\left(\delta_{21}^{L^1},\delta_{22}^{M^1},\delta_{23}^{U^1}\right)\left(\delta_{21}^{L^2},\delta_{22}^{M^2},\delta_{23}^{U^2}\right)\ldots\ldots\ldots\ldots\left(\delta_{21}^{L^q},\delta_{22}^{M^q},\delta_{23}^{U^q}\right)\right\}>,\\ \ldots\\ <x_n,\left\{\left(\delta_{n1}^{L^1},\delta_{n2}^{M^1},\delta_{n3}^{U^1}\right)\left(\delta_{n1}^{L^2},\delta_{n2}^{M^2},\delta_{n3}^{U^2}\right)\ldots\ldots\ldots\ldots\left(\delta_{n1}^{L^r},\delta_{n2}^{M^r},\delta_{n3}^{U^r}\right)\right\}\end{array}\right\}$$

Where $\tilde{g}_{\tilde{F}}(x_i)=\left\{\left(\delta_{i1}^{L^1},\delta_{i2}^{M^1},\delta_{i3}^{U^1}\right)\left(\delta_{i1}^{L^2},\delta_{i2}^{M^2},\delta_{i3}^{U^2}\right).............\left(\delta_{i1}^{L^\rho},\delta_{i2}^{M^\rho},\delta_{i3}^{U^\rho}\right)/\, i=1,2,3....,\rho\right\}$

$$\tilde{g}_{\tilde{F}}(x_i)=\left\{\min\left(\delta_{i1}^{L^1},\delta_{i1}^{L^2},...,\delta_{i1}^{L^\rho}\right),\text{mean}\left(\delta_{i2}^{M^1},\delta_{i2}^{M^2},...,\delta_{i2}^{M^\rho}\right),\max\left(\delta_{i3}^{U^1},\delta_{i3}^{U^2},...,\delta_{i3}^{U^\rho}\right)/\, i=1,2,3....,\rho\right\}$$

$$\tilde{g}_{\tilde{F}}(x_i)=\left\{\left(\nu_i^L,\nu_i^M,\nu_i^U\right)/\, i=1,2,3....,\rho\right\} \quad \text{Where}$$

$$\nu_i^L=\min\left(\delta_{i2}^{M^1},\delta_{i2}^{M^2},...,\delta_{i2}^{M^\rho}\right),\nu_i^M=\text{mean}\left(\delta_{i2}^{M^1},\delta_{i2}^{M^2},...,\delta_{i2}^{M^\rho}\right),\nu_i^U=\max\left(\delta_{i3}^{U^1},\delta_{i3}^{U^2},...,\delta_{i3}^{U^\rho}\right)$$

Suppose that $\tilde{g}_{\tilde{F}}(x_i),\tilde{g}_{\tilde{F}}(x_j)$ *are HFS in* $\tilde{F}$ *then*

$$\tilde{g}_{\tilde{F}}(x_i)\oplus\tilde{g}_{\tilde{F}}(x_j)=\left\{\left\{\left(\nu_i^L,\nu_i^M,\nu_i^U\right)/\, i=1,2,3....,\rho\right\}\right\}\oplus\left\{\left(\nu_j^L,\nu_j^M,\nu_j^U\right)/=1,2,3....,\rho'\right\}$$

$$\tilde{g}_{\tilde{F}}(x_i)\oplus\tilde{g}_{\tilde{F}}(x_j)=\left\{\left(\min\left\{\nu_i^L,\nu_j^L\right\},\text{mean}\left(\nu_i^M,\nu_j^M\right),\max\left\{\nu_i^U,\nu_j^U\right\}\right)\right\}$$

$$\tilde{g}_{\tilde{F}}(x_i)\oplus\tilde{g}_{\tilde{F}}(x_j)=\left\{\left(\Delta_1^L,\Delta_2^M,\Delta_3^U\right)\right\}\ \tilde{g}_1\otimes\tilde{g}_2=\left\{\left(\tilde{\gamma}_1^L\cdot\tilde{\gamma}_2^L,\tilde{\gamma}_1^M\cdot\tilde{\gamma}_2^M,\tilde{\gamma}_1^U.\tilde{\gamma}_2^U\right)/\,\tilde{\gamma}_1\in\tilde{g}_1,\tilde{\gamma}_2\in\tilde{g}_2\right\}$$

Also,

$$\tilde{g}^\lambda=\left\{\left(\left(\tilde{\gamma}_1^L\right)^\lambda,\left(\tilde{\gamma}_1^M\right)^\lambda\cdot\left(\tilde{\gamma}_1^U\right)^\lambda\right)/\,\tilde{\gamma}_1\in\tilde{g}\right\}$$

$$\lambda\tilde{g}=\left\{\left(1-\left(1-\tilde{\gamma}^L\right)^\lambda,1-\left(1-\tilde{\gamma}^M\right)^\lambda,1-\left(1-\tilde{\gamma}^U\right)^\lambda\right)/\,\tilde{\gamma}\in\tilde{g}\right\},\lambda>0$$

3.New Distance measure using Triangular Hesitant Fuzzy set(THFS)

In this segment a Novel distance measure with sutable illustration is introduced using THFS. *Consider a* Triangular Hesitant Fuzzy set and add by using following process, finally found the new distance measure with the assist of centroid of centroid point of Triangular fuzzy number.Now consider THFS

$$\tilde{F}=\left\{\begin{array}{l}<x_1,\left\{\left(\delta_{11}^{L^1},\delta_{12}^{M^1},\delta_{13}^{U^1}\right)\left(\delta_{11}^{L^2},\delta_{12}^{M^2},\delta_{13}^{U^2}\right).............\left(\delta_{11}^{L^p},\delta_{12}^{M^p},\delta_{13}^{U^p}\right)\right\}>,\\ <x_2,\left\{\left(\delta_{21}^{L^1},\delta_{22}^{M^1},\delta_{23}^{U^1}\right)\left(\delta_{21}^{L^2},\delta_{22}^{M^2},\delta_{23}^{U^2}\right).............\left(\delta_{21}^{L^q},\delta_{22}^{M^q},\delta_{23}^{U^q}\right)\right\}>,\\ ...\\ <x_n,\left\{\left(\delta_{n1}^{L^1},\delta_{n2}^{M^1},\delta_{n3}^{U^1}\right)\left(\delta_{n1}^{L^2},\delta_{n2}^{M^2},\delta_{n3}^{U^2}\right).............\left(\delta_{n1}^{L^r},\delta_{n2}^{M^r},\delta_{n3}^{U^r}\right)\right\}\end{array}\right\}$$

Where $\tilde{g}_{\tilde{F}}(x_i)=\left\{\left(\delta_{i1}^{L^1},\delta_{i2}^{M^1},\delta_{i3}^{U^1}\right)\left(\delta_{i1}^{L^2},\delta_{i2}^{M^2},\delta_{i3}^{U^2}\right).............\left(\delta_{i1}^{L^\rho},\delta_{i2}^{M^\rho},\delta_{i3}^{U^\rho}\right)/\, i=1,2,3....,\rho\right\}$

$$\tilde{g}_{\tilde{F}}(x_i)=\left\{\min\left(\delta_{i1}^{L^1},\delta_{i1}^{L^2},...,\delta_{i1}^{L^\rho}\right),\text{mean}\left(\delta_{i2}^{M^1},\delta_{i2}^{M^2},...,\delta_{i2}^{M^\rho}\right),\max\left(\delta_{i3}^{U^1},\delta_{i3}^{U^2},...,\delta_{i3}^{U^\rho}\right)/\, i=1,2,3....,\rho\right\}$$

$$\tilde{g}_{\tilde{F}}(x_i)=\left\{\left(\nu_i^L,\nu_i^M,\nu_i^U\right)/\, i=1,2,3....,\rho\right\} \quad \text{Where}$$

$$\nu_i^L=\text{mean}\left(\delta_{i2}^{M^1},\delta_{i2}^{M^2},...,\delta_{i2}^{M^\rho}\right),\nu_i^M=\text{mean}\left(\delta_{i2}^{M^1},\delta_{i2}^{M^2},...,\delta_{i2}^{M^\rho}\right),\nu_i^U=\max\left(\delta_{i3}^{U^1},\delta_{i3}^{U^2},...,\delta_{i3}^{U^\rho}\right)$$

$$\tilde{g}_{\tilde{F}}(x_i)\oplus\tilde{g}_{\tilde{F}}(x_j)=\left\{\left\{\left(\nu_i^L,\nu_i^M,\nu_i^U\right)/\, i=1,2,3....,\rho\right\}\right\}\oplus\left\{\left(\nu_j^L,\nu_j^M,\nu_j^U\right)/=1,2,3....,\rho'\right\}$$

$$\tilde{g}_{\tilde{F}}(x_i)\oplus\tilde{g}_{\tilde{F}}(x_j)=\left\{\left(\min\left\{\nu_i^L,\nu_j^L\right\},\text{mean}\left(\nu_i^M,\nu_j^M\right),\max\left\{\nu_i^U,\nu_j^U\right\}\right)\right\}$$

$$\tilde{g}_{\tilde{F}}(x_i)\oplus\tilde{g}_{\tilde{F}}(x_j)=\left\{\left(\Delta_1^L,\Delta_2^M,\Delta_3^U\right)\right\} \quad ... \quad (1)$$

Where $\Delta_1^L = \min\{\nu_i^L, \nu_j^L\}$, $\Delta_3^U = \max\{\nu_i^U, \nu_j^U\}$, $\Delta_2^M = \text{mean}\{\nu_i^M, \nu_j^M\}$, $A = \{(\Delta_1^L, \Delta_2^M, \Delta_3^U)\}$ *the centroid of centroids points* $A = (\Delta_1^L, \Delta_2^M, \Delta_3^U)$ and $\in_{\tilde{A}} = \left(\frac{\Delta_1^{L^1} + 7\Delta_2^{M^1} + \Delta_3^{U^1}}{9}\right)$ *and left and right speards* $\in_{\tilde{A}} - \Delta_1^L, \Delta_3^U - \in_{\tilde{A}}$,

The Distance measure using Triangular Hesitant Fuzzy set is

$$d(\tilde{g}_{\tilde{F}}(x_i), \tilde{g}_{\tilde{F}}(x_j)) = \max\{|\in_{\tilde{A}} - \Delta_1^L|, |\Delta_3^U - \in_{\tilde{A}}|\} \quad \ldots \quad (2)$$

Example- 2

Consider $X = \{x_1, x_2\}$ *be a finite set,* $\tilde{g}_{\tilde{F}}(x_1) = \{(0.1, 0.2, 0.4)(0.2, 0.5, 0.6)\}$ and

$\tilde{g}_{\tilde{F}}(x_2) = \{(0.1, 0.3, 0.5)(0.4, 0.6, 0.8)(0.5, 0.6, 0.9)\}$ be the THFS of x_1, x_2 to the set $\tilde{F}$ and

$$\tilde{F} = \{< x_1, \{(0.1, 0.2, 0.4)(0.2, 0.5, 0.6)\} >, < x_2, \{(0.1, 0.3, 0.5)(0.4, 0.6, 0.8)(0.5, 0.6, 0.9)\} >\}$$

$$\tilde{g}_{\tilde{F}}(x_1) = \left\{\left(\min(0.1, 0.2), \left(\frac{0.2 + 0.5}{2}\right), \max(0.4, 0.6)\right)\right\}$$

$$\tilde{g}_{\tilde{F}}(x_1) = \{(0.1, 0.35, 0.6)\}$$

$$\tilde{g}_{\tilde{F}}(x_2) = \{(0.1, 0.3, 0.5)(0.4, 0.6, 0.8)(0.5, 0.6, 0.9)\}$$

$$\tilde{g}_{\tilde{F}}(x_2) = \left\{\left(\min(0.1, 0.4, 0.5), \left(\frac{0.3 + 0.6 + 0.6}{3}\right), \max(0.5, 0.8, 0.9)\right)\right\}$$

$$\tilde{g}_{\tilde{F}}(x_2) = \{(0.1, 0.5, 0.9)\}$$

$$\tilde{g}_{\tilde{F}}(x_i) \oplus \tilde{g}_{\tilde{F}}(x_j) = \{(\min\{0.1, 0.1\}, 0.425, \max\{0.6, 0.9\})\}$$

$$\tilde{g}_{\tilde{F}}(x_i) \oplus \tilde{g}_{\tilde{F}}(x_j) = \{(0.1, 0.425, 0.9)\}$$

$$\in_{\tilde{A}} = \frac{0.1 + 7(0.425) + 0.9}{9} = 0.442$$

$$d(\tilde{g}_{\tilde{F}}(x_i), \tilde{g}_{\tilde{F}}(x_j)) = \max\{|0.442 - 0.1|, |0.9 - 0.442|\} = \max\{0.342, 0.458\} = 0.458$$

4.*New Fuzzy TOPSIS using*Triangular Hesitant Fuzzy set(THFS)

The process of finding the best alternative in a set of feasible alternatives is called as decision-making. In this, the problems which are taken into account from several criteria are named as multi-criteria decision making (MCDM) problems. MCDM refers to prioritizing the set of alternatives under usually inconsistent criteria. These problems usually effect the preciseness of the skewed data being present, which makes the decision-making process complex. In other words, decision-making frequently happens in a fuzzy environment where the available information is uncertain. So, in such situations, one of the well known MCDM method is presented named as TOPSIS (Technique for Order Preference by Similarity to Ideal Solution). TOPSIS method was first presented by Hwang and Yoon[37]. This method depends on the concept of ideal alternative, which is the finest level for all considered attributes. A new TOPSIS Method is introduced on THFS.In this, the activity times are taken as THFS.The procedure of this method is presented as follows

Step-1

In this the feasible alternative are to be generated. Suppose that the number of alternatives and evaluation criteria are m, n *respectivelythen the* Decision matrix using THFS is

$$D = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix}_{m\times n} \quad \textit{where}$$

$$y_{11} =< x_{11}, \left\{ \left(\delta_{11}^{L^1}, \delta_{11}^{M^1}, \delta_{11}^{U^1} \right) \left(\delta_{11}^{L^2}, \delta_{11}^{M^2}, \delta_{11}^{U^2} \right) .... \left(\delta_{11}^{L^p}, \delta_{11}^{M^p}, \delta_{11}^{U^p} \right) \right\} >$$

$$y_{12} =< x_{12}, \left\{ \left(\delta_{12}^{L^1}, \delta_{12}^{M^1}, \delta_{12}^{U^1} \right) \left(\delta_{12}^{L^2}, \delta_{12}^{M^2}, \delta_{12}^{U^2} \right) .... \left(\delta_{12}^{L^p}, \delta_{12}^{M^p}, \delta_{12}^{U^p} \right) \right\} > \dots$$

$$y_{1n} =< x_{1n}, \left\{ \left(\delta_{1n}^{L^1}, \delta_{1n}^{M^1}, \delta_{1n}^{U^1} \right) \left(\delta_{1n}^{L^2}, \delta_{1n}^{M^2}, \delta_{1n}^{U^2} \right) .... \left(\delta_{1n}^{L^p}, \delta_{1n}^{M^p}, \delta_{1n}^{U^p} \right) \right\} >$$

$$y_{m1} =< x_{m1}, \left\{ \left(\delta_{m1}^{L^1}, \delta_{m1}^{M^1}, \delta_{m1}^{U^1} \right) \left(\delta_{m1}^{L^2}, \delta_{m1}^{M^2}, \delta_{m1}^{U^2} \right) .... \left(\delta_{m1}^{L^p}, \delta_{m1}^{M^p}, \delta_{m1}^{U^p} \right) \right\} >$$

$$y_{m2} =< x_{m2}, \left\{ \left(\delta_{m2}^{L^1}, \delta_{m2}^{M^1}, \delta_{m2}^{U^1} \right) \left(\delta_{m2}^{L^2}, \delta_{m2}^{M^2}, \delta_{m2}^{U^2} \right) .... \left(\delta_{m2}^{L^p}, \delta_{m2}^{M^p}, \delta_{m2}^{U^p} \right) \right\} >$$

$$\dots y_{mn} =< x_{mn}, \left\{ \left(\delta_{mn}^{L^1}, \delta_{mn}^{M^1}, \delta_{mn}^{U^1} \right) \left(\delta_{mn}^{L^2}, \delta_{mn}^{M^2}, \delta_{mn}^{U^2} \right) .... \left(\delta_{mn}^{L^p}, \delta_{mn}^{M^p}, \delta_{mn}^{U^p} \right) \right\}$$

$$y_{ij} = \left(\min\left(\delta_{ij}^{L^1}, \delta_{ij}^{L^2} ... \delta_{ij}^{L^p} \right) = \varepsilon_{ij}^{L}, \varepsilon_{ij}^{M} = \text{mean}\left(\delta_{i2}^{M^1}, \delta_{i2}^{M^2}, ..., \delta_{i2}^{M^p} \right), \max\left(\delta_{ij}^{U^1}, \delta_{ij}^{U^2}, ..., \delta_{ij}^{U^p} \right) = \varepsilon_{ij}^{U} \right)$$

$$y_{ij} = \left\{ \left(\varepsilon_{ij}^{L}, \varepsilon_{ij}^{M}, \varepsilon_{ij}^{U} \right) \right\} \quad i = 1, 2..., m, \ j = 1, 2..., n \qquad \dots \quad (3)$$

Step-2

It is clear that the alternatives are rated for dissimilarAttributesof dissimilar dimensions. To keep the rating y_{ij} ofeach alternative on a normalized scale, the procedure of

normalization is applied. The normalized matrix *N* can be obtainedas follows

$$N = \left[N_{ij} \right]_{m\times n} \dots \quad (4)$$

where $N_{ij} = \left(\frac{\varepsilon_{ij}^{L}}{\rho}, \frac{\varepsilon_{ij}^{M}}{\rho}, \frac{\varepsilon_{ij}^{U}}{\rho} \right)$ where $\rho = \max\left\{ \varepsilon_{11}^{U}, \varepsilon_{12}^{U}, ..., \varepsilon_{mn}^{U} \right\}, i, j = 1, 2, ..., n.$ The process of normalization which is presented above normalized THFS be supposed to

variety in the period [0, 1] is well preserved.:

Step-3

It has been observed that calculation withTHFS for a MCDM problem having large number of alternatives and attributes may result in computational monotony. To avoidthis computational complexity, in this process of decision making,first a weighted normalized matrix its entries will be obtained, by using the following equation

$$V = N \otimes w$$

*The Weighted Normalized*Decision matrix using THFS is as follows.

$$w=\left\{\begin{array}{l}
<x_{11},\left\{\left(w_{11}^{L^1},w_{11}^{M^1},w_{11}^{U^1}\right)\left(w_{11}^{L^2},w_{11}^{M^2},w_{11}^{U^2}\right).............\left(w_{11}^{L^p},w_{11}^{M^p},w_{11}^{U^p}\right)\right\}>,\\
<x_{12},\left\{\left(w_{12}^{L^1},w_{12}^{M^1},w_{12}^{U^1}\right)\left(w_{12}^{L^2},w_{12}^{M^2},w_{12}^{U^2}\right).............\left(w_{12}^{L^p},w_{12}^{M^p},w_{12}^{U^p}\right)\right\},...,\\
<x_{1n},\left\{\left(w_{1n}^{L^1},w_{1n}^{M^1}w_{1n}^{U^1}\right)\left(w_{1n}^{L^2},w_{1n}^{M^2},w_{1n}^{U^2}\right).............\left(w_{1n}^{L^p},w_{1n}^{M^p},w_{1n}^{U^p}\right)\right\}\\
\\
<x_{21},\left\{\left(w_{21}^{L^1},w_{21}^{M^1},w_{21}^{U^1}\right)\left(w_{21}^{L^2},w_{21}^{M^2},w_{21}^{U^2}\right).............\left(w_{21}^{L^p},w_{21}^{M^p},w_{21}^{U^p}\right)\right\},\\
<x_{22},\left\{\left(w_{22}^{L^1},w_{22}^{M^1},w_{22}^{U^1}\right)\left(w_{22}^{L^2},w_{22}^{M^2},w_{22}^{U^2}\right).............\left(w_{22}^{L^p},w_{22}^{M^p},w_{21}^{U^p}\right)\right\},...,\\
<x_{2n},\left\{\left(w_{2n}^{L^1},w_{2n}^{M^1},w_{2n}^{U^1}\right)\left(w_{2n}^{L^2},w_{2n}^{M^2},w_{2n}^{U^2}\right).............\left(w_{2n}^{L^p},w_{2n}^{M^p}w_{2n}^{U^p}\right)\right\}\\
...\\
<x_{m1},\left\{\left(w_{m1}^{L^1},w_{m1}^{M^1},w_{m1}^{U^1}\right)\left(w_{m1}^{L^2},w_{m1}^{M^2},w_{m1}^{U^2}\right).............\left(w_{m1}^{L^p},w_{m1}^{M^p},w_{m1}^{U^p}\right)\right\},\\
<x_{m2},\left\{\left(w_{m2}^{L^1},w_{m2}^{M^1},w_{m2}^{U^1}\right)\left(w_{m2}^{L^2},w_{m2}^{M^2},w_{m2}^{U^2}\right).............\left(w_{m2}^{L^p},w_{m2}^{M^p},w_{m2}^{U^p}\right)\right\},...,\\
<x_{mn},\left\{\left(w_{mn}^{L^1},w_{mn}^{M^1},w_{mn}^{U^1}\right)\left(w_{mn}^{L^2},w_{mn}^{M^2},w_{mn}^{U^2}\right).............\left(w_{mn}^{L^p},w_{mn}^{M^p},w_{mn}^{U^p}\right)\right\}
\end{array}\right\}$$

$$w=\begin{bmatrix} w_{11} & w_{12} & ... & w_{1n} \\ w_{21} & w_{22} & ... & w_{2n} \\ ... & ... & ... & ... \\ w_{m1} & w_{m2} & ... & w_{mn} \end{bmatrix}$$

where

$$w_{ij}=\left\{\left(\min\left(w_{ij}^{L^1},w_{ij}^{L^2}...w_{ij}^{L^p}\right)=w_{ij}^{L},w_{ij}^{M}=\text{mean}\left(w_{i2}^{M^1},w_{i2}^{M^2},...,w_{i2}^{M^p}\right),\max\left(w_{ij}^{U^1},w_{ij}^{U^2},...,w_{ij}^{U^p}\right)=w_{ij}^{U}\right)\right\}$$

$$w_{ij}=\left(w_{ij}^{L},w_{ij}^{M},w_{ij}^{U}\right) \quad ... \quad (5)$$

Step-4: Determine Fuzzy Positive and Negative Ideal solutions

According to the weighted normalized fuzzy decision matrix, normalized positive trapezoidal fuzzy number can be approximate by the elements $\tilde{v}_{ij}$. Then, the FPIS (Fuzzy Positive Ideal Solution) A^{+} FNIS (Fuzzy Negative Ideal Solution) A^{-} can be *defined as;*

$$A^{+}=\left\{\left(v_{ij1}^{+},v_{ij2}^{+},v_{ij3}^{+}\right)\right\}\ A^{-}=\left\{\left(v_{ij1}^{-},v_{ij2}^{-},v_{ij3}^{-}\right)\right\} \quad ... \quad (6)$$

Where $v_3^{+}=\max_i v_{ij3}$ and $v_1^{-}=\min_i v_{ij1}$ $i=1,2,....m,\ j=1,2,....n$. In the index, v_{ij1} and v_{ij3} are the first and third numbers in theTriangular Hesitant Fuzzy set, respectively.

Step-5: Distance of each alternative from Fuzzy positive Ideal solution and Fuzzy negative Ideal solutions using Triangular Hesitant Fuzzy set

The Distance measure using Triangular Hesitant Fuzzy set is

$$d\left(\tilde{g}_{\tilde{F}}\left(x_i\right),\tilde{g}_{\tilde{F}}\left(x_j\right)\right)=\max\left\{\left|\in_{\tilde{A}}-\Delta_1^L\right|,\left|\Delta_3^U-\in_{\tilde{A}}\right|\right\} \dots \quad (7)$$

Step-6

The closeness coefficient of each alternative is denoted by cc_i and isdefined as;

$$cc_i=\frac{d_i^-}{d_i^++d_i^-} \dots \quad (8)$$

Step-7: Best Alternative

*According to the*coefficient cc_i value, the larger closeness coefficient has a higher level ranking order. So the best alternative has the maximum closeness coefficient cc_i .

5. Proposed method for the critical path selection

A very methodical approach is put forth in this segment to deal with the critical path selection problem under a fuzzy environment. The importance weights of various criteria and the ratings of qualitative criteria are contemplated as Hesitant fuzzy sets in this paper. The times of each activity can be expressed in a Triangular Fuzzy Hesitant Fuzzy Set. The detailing of the proposed method is given hereunder.

A large project can be broken into several fragment which are called activities, determining the durations and precedence of these activities. The relationship of precedence among these activities is visualized in a network representation of the project so as to draw the precedence project network the arcs of which express the activities alongside determining all benchmarks which are important to choose a critical path under these criteria. A path intersecting a project network is one of the routes from the starting node to the ending node. Therefore, in this step, it is important to indicate all paths that start with the starting event and end with the ending event. These are our alternatives from which we need to choose one as a critical path. Also pick the most suitable Triangular Fuzzy Hesitant Fuzzy Set for quantitative criteria. The length of a path is the sum of spans of activities on the path. Hence add up the Triangular Fuzzy Hesitant Fuzzy Set to ascertain the value of the final evaluation of each criterion for paths. The duration of the project equals the length of the longest path in the network and a project is considered complete if the work along all paths is complete. Thus build up the fuzzy decision matrix in which all the alternatives are the paths which start with the starting event and end with the ending event. It is important to determine the critical path in the project network under different criterion in order to calculate the completion time of the project. Applying fuzzy TOPSIS method can be very handy with the ratings of both qualitative as well as quantitative criteria to designate a suitable alternative critical path under various criteria and the same is explained in section -5. The algorithm put forth to deal with the selection of critical path is summarized the following steps – draw the network of the precedence project (activity on arrows) – choose the befitting Triangular Fuzzy Hesitant Fuzzy set for the importance weight of the criteria – make a mention of all paths which start with the starting event and end with the ending event. Employ Triangular Fuzzy Hesitant Fuzzy Set to identify the fuzzy time under each criterion. Then sum up Triangular Fuzzy Hesitant Fuzzy Set to ascertain the value of the final evaluation of each criterion for paths – construct the fuzzy decision matrix in which all its alternatives are the parts that start with the starting event and end with the ending event. Then apply the fuzzy TOPSIS method that is proposed in Section 4. A numerical example is solved to exemplify the efficiency of the proposed method.

Numerical Example:

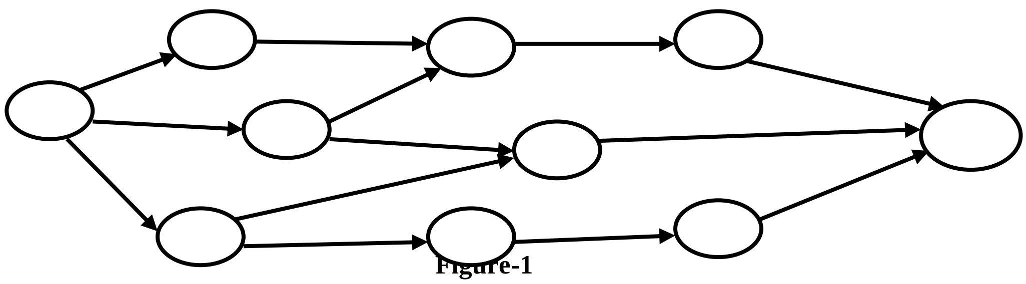

Figure-1

The proposed method is presently applied to solve the example the computational procedure of which is summarized hereunder;Draw the precedence project network as shown in the figure-1. The decision makers estimate the time criteria of each activity of the project network in THFS which shows in table-1 as a decision matrix . Add up all activitie Times in each the path that start with the starting event and end with the ending event. The ratings of the activities by the decision makers under various criteria are described. So the normalized fuzzy decision matrix is built up as in table-3.Next the Weighed normalized fuzzy decision matrix is constructed as in table-4. Now collect the Triangle Hesitant Fuzzy set positive and negative ideal solution ideal solution . Then determine and FNIS so as to calculate the distance of each from Fuzzy positive ideal solution and Fuzzy negative ideal solution d- with respect to each criterion respectively as shown in tables 5 and 6. Calculate di+ and di – of the five possible paths and then calculate the closeness coefficient of each path as shown in table-6. As per the closeness coefficients of five paths, we get to know that the second path (1 – 3 – 6 – 10) is the critical path under the criteria of time. The problem is solved only with the time criterion.Further we can apply on cost ,risk and quality.

Decision Matrix

As per the times of each activity in the project network the following decision matrix is constructed and presented in the Table -1

(i,j)	***Activity times* (Triangular Hesitant Fuzzy set)**
$x_1=(1,2)$	$< x_1,\{(0.1,0.3,0.4),(0.1,0.5,0.6)\} >$
$x_2=(1,3)$	$< x_2,\{(0.1,0.4,0.5),(0.2,0.6,0.7),(0.8,0.9,0.9)\} >$
$x_3=(1,4)$	$< x_3,\{(0.1,0.2,0.6),(0.2,0.6,0.7)\} >$
$x_4=(2,5)$	$< x_4,\{(0.2,0.4,0.6)\} >$
$x_5=(3,5)$	$< x_5,\{(0.1,0.7,0.9)\} >$
$x_6=(4,6)$	$< x_6,\{(0.1,0.2,0.3),(0.4,0.5,0.6),(0.4,0.6,0.7)\} >$
$x_7=(8,9)$	$< x_7,\{(0.2,0.3,0.3),(0.1,0.5,0.6)\} >$
$x_8=(3,6)$	$< x_8,\{(0.2,0.2,0.4),(0.3,0.4,0.6)\} >$
$x_9=(5,7)$	$< x_9,\{(0.1,0.3,0.4),(0.2,0.3,0.4)(0.5,0.6,0.9)\} >$
$x_{10}=(4,8)$	$< x_{10},\{(0.6,0.7,0.75),(0.5,0.5,0.6),(0.3,0.5,0.7)\} >$
$x_{11}=(6,10)$	$< x_{11},\{(0.7,0.8,0.8)\} >$
$x_{12}=(7,10)$	$< x_{12},\{(0.4,0.5,0.6),(0.1,0.3,0.5)\} >$
$x_{13}=(9,10)$	$< x_{13},\{(0.3,0.5,0.6),(0.3,0.4,0.6)\} >$

Table-1

Now, Add the times (which are in Triangular Hesitant Fuzzy set) *of each paths in the project network presented in Table -2*

Paths of the Network	***Activity times* (Triangular Hesitant Fuzzy set)**
$1-2-5-7-10$	$\{(0.1, 0.4, 0.9)\}$
$1-3-6-10$	$\{(0.1, 0.55, 0.9)\}$
$1-3-5-7-10$	$\{(0.1, 0.4625, 0.9)\}$
$1-4-6-10$	$\{(0.1, 0.51667, 0.9)\}$
$1-4-8-9-10$	$\{(0.1, 0.44375, 0.9)\}$

Table-2

Normalized decision Matrix

Applying step-2, to construct Normalized decision Matrix is presented in Table-3

Paths of the Network	***Normalized decision Matrix* (Triangular Hesitant Fuzzy set)**
$1-2-5-7-10$	$\{(0.111, 0.444, 1)\}$
$1-3-6-10$	$\{(0.111, 0.611, 1)\}$
$1-3-5-7-10$	$\{(0.111, 0.513, 1)\}$
$1-4-6-10$	$\{(0.111, 0.574, 0.88)\}$
$1-4-8-9-10$	$\{(0.1, 0.493, 0.833)\}$

Table-3

Weighted Normalized decision Matrix

Applying the procedure presented in step-3 to find the weighted normalized decision matrix

Paths of the Network	**(Triangular Hesitant Fuzzy set)**
$1-2-5-7-10$	$\{(0.111, 0.444, 0.99)\}$
$1-3-6-10$	$\{(0.11, 0.605, 1)\}$
$1-3-5-7-10$	$\{(0.111, 0.508, 0.99)\}$

$1-4-6-10$	$\{(0.111, 0.568, 0.88)\}$
$1-4-8-9-10$	$\{(0.1, 0.488, 0.825)\}$

Table-4

Fuzzy Positive Negative Ideal Solutions

Using Step-4,to identify Triangular Hesitant Fuzzy Set *Positive Negative Ideal Solutions presented in step-4* $A^- = \{(0.1, 0.1, 0.1)\}, A^+ = \{(1,1,1)\}$

Distance between Fuzzy Positive Ideal Solutions and

By means of step-5 to identify Distance between paths and A^+ with respect to each criterion presented in following table-5

Criteria Activity	**Time**
$d^+(A_1, A^+)$	0.61
$d^+(A_2, A^+)$	0.69
$d^+(A_3, A^+)$	0.64
$d^+(A_4, A^+)$	0.67
$d^+(A_5, A^+)$	0.63

Table-5

Distance between Fuzzy Negative Ideal Solutions and

By means of step-5 to identify distance between paths and A^- with respect to each criterion

Criteria Activity	**Time**
$d^-(A_1, A^-)$	0.17
$d^-(A_2, A^-)$	0.25
$d^-(A_3, A^-)$	0.20
$d^-(A_4, A^-)$	0.23
$d^-(A_5, A^-)$	0.19

Table-6

Closeness coefficient:

Using step-6 to caliculate the closeness cofficent of each alternative presented in the following .

Criteria Activity	d_i^+	d_i^-	$d_i^+ + d_i^-$	$cc_i = \frac{d_i^-}{d_i^- + d_i^+}$
1-2-5-7-10	0.61	0.17	0.78	0.217949
1-3-6-10	0.69	0.25	0.95	0.267196
1-3-5-7-10	0.64	0.20	0.85	0.240795
1-4-6-10	0.67	0.23	0.91	0.257798
1-4-8-9-10	0.63	0.19	0.83	0.23434

Table-7

Observe that the largest value of the closeness cofficent of each alternative gives the best path of ghe project network 1-3-6-10.

5.Conclusion:

In this paper TOPSIS method has been applied to fuzzy project network to determine the critical path using several criteria. Triangular Hesitant Fuzzy Set have been used as fuzzy activity times, to find critical path of the project network . A new Triangular Hesitant Fuzzy Set‘s distances measure has been proposed to select critical path in new TOPSIS method using Triangular Hesitant Fuzzy Set as activity times. A numerical example related to this problem has provided to explain the procedure of proposed TOPSIS method in determining critical path with different criteria.

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