New Wavelet Method For Solving Partial Differential Equations

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Abstract: In this paper a new wavelet formula is derived based on the definition of the convolution between the Haar and CAS wavelets, while finding the integrals of the proposed formula analytically. An outline of the proposed method is written with the collocation points for solving partial differential equations. From the comparison of the numerical results of the proposed methods with the exact solution to solve three problems, we concluded that the suggested method is more accurate, better, and nearer to an exact solution.

Keywords: Haar wavelets, Cosine and Sine (CAS) wavelets, partial differential equations, Operational Matrix.

INTRODUCTION

Wavelet analysis is a new numerical concept that allows one to represent a function in terms of basic functions, called wavelets, that are identified in space. Wavelets were introduced relatively recently in the early 1980s. It has attracted great interest from the mathematical community and from members of many disciplines in which wavelets have had promising applications. Among the results of this interest is the emergence of many books on this topic and a large amount of research articles [11].

Wavelet decomposition analysis is most often used in the processing of the wavelet signal. It is utilized in sign compression in addition to in sign identification. The transform of wavelet to a function, such as the Fourier transform, is an effective tool for studying additives of stationary phenomena. However, the wavelet transform has the advantage of being able to analyze unstable phenomena where the Fourier transform fails [14, 15, 3].

When the expansion parameter a and the translation parameter b change continuously, we have the following set of continuous wavelets:

$$\Psi_{a,b}(t) = |a|^{\frac{-1}{2}}\Psi\left(\frac{t-b}{a}\right). a,b \in R, a \neq 0 \quad (1)$$

If we restrict the parameters a and b to discrete values which are

$$a = a_0^{-k}, b = nb_0\, a_0^{-k}, a_0 > 1, b_0 > 0$$

Where n and k are positive integers, Then we have the next family of separate wavelets

$$\Psi_{k,n}(t) = |a_0|^{\frac{k}{2}}\,\Psi\,(a_0^k\, t - n\, b_0) \quad (2)$$

Where $\Psi_{k,n}(t)$ is a basis of wavelet for $L^2(R)$. In particular, when $a_0 = 2, b_0 = 1, the\ form\ \Psi_{k,n}(t)$ is an orthonormal basis [2]

In recent years, the wavelet method has become more and more popular in the field of numerical methods. Various wavelet types and approximate functions were used for this.

Siddu C. S. and Lata are Solved the Stochastic integral equations by using Haar wavelet and CAS wavelet schemes and generated the operational matrix of integration of these wavelets [19,17]. where In [18] Siddu C. S. and R. A. M. are introduced wavelet full-approximation scheme to solve nonlinear Voltera-Frodholm integral equations and obtained good accuracy numerical results. CAS wavelet function method are solve nonlinear fractional order Volterra integral equation in [6], general two-dimensional PDEs of higher order in [8], Haar wavelet method are solved three dimensional and time depending PDEs in [10], nonlinear two – dimensional BBM-BBM system are solved and obtained the accuracy of numerical solutions is very high even if the number of calculated points is small in [7], and an operational matrix of integrations based on the Haar wavelet method is applied for finding the numerical solutions of non-linear third –order boussinesq system in [9].

The paper is organized in the following structure. In section 2, Haar wavelets and their integrals are reported. CAS wavelets with their integrals are introduced in section 3. In section 4, new convolution wavelet are derived and their integrals. The steps of proposed method for solving PDEs is performed in section 5. Then in section 6 some numerical examples are given and solved. Finally, conclusion of numerical results is presented and future research are offered.

2. Haar wavelets and the integrals

The Haar wavelets family for $x \in [0,1)$ is defined as follows [4,5,12]

$$h_i(x) = \begin{cases} 1 & for\ x \in [\xi_1(i), \xi_2(i)\ ) \\ -1 & for\ x \in [\xi_2(i), \xi_3(i)\ ) \\ 0 & elsewhere \end{cases} \quad (3)$$

where

$$\xi_1(i) = \frac{k}{2^j}, \xi_2(i) = \frac{k+0.5}{2^j}, \xi_3(i) = \frac{k+1}{2^j} \quad (4)$$

The interval [0,1) is participated into 2M subintervals of equal length, the length of each subinterval is $\Delta x = 1/(2M)$. Integer j = 0,1,2,…., J, indicates the wavelet plane ;
k= 0,1,2,…., 2^j -1 is the translation Parameter. The maximal accuracy level is J. The formula of index i is calculated from i= 2^j +k+1. Within the case of smallest values 2^j =1, k=0, we've got i=2, and i= 2M = 2^{J+1} is the greatest value of i. It is assumed that for i=1 the scaling function is $h_1(x)$=1 in [0,1).
The collocation points are:

$$x_l = \left(l - \frac{1}{2}\right) \Delta x, l = 1,2, \dots, 2M \quad (5)$$

It is convenient to introduce the Haar matrices $H(i,l\ ) = h_i(x_l)$ which has the dimension 2M*2M.

We find the integrals for the Haar wavelets defined in equation (3) analytically, and these integrals in turn can be used in the numerical solution of higher order differential equations. We'll use these integrals to compute the numerical solution of one- dimensional linear system. If we integrate equation (3) from (0) to (x), we obtain the operational matrix of integration p,

$$p_{i,1}(x) = \int_0^x h_i\,(x')dx'$$

$$p_{i,1}(x) = \begin{cases} x - \xi_1(i) & ,x \in [\xi_1(i), \xi_2(i)\ ), \\ 2\,\xi_2(i) - x - \xi_1(i) & ,x \in [\xi_2(i), \xi_3(i)\ ) \\ 0 & elsewhere \end{cases} \quad (6)$$

$$p_{i,2}(x) = \begin{cases} \frac{(x - \xi_1(i))^2}{2} & ,x \in [\xi_1(i), \xi_2(i)\ ) \\ \frac{1}{4m^2} - \frac{(\xi_3(i) - x)^2}{2} & ,x \in [\xi_2(i), \xi_3(i)\ ) \\ \frac{1}{4m^2} & ,x \in [\xi_3(i), 1\ ) \\ 0 & elsewhere \end{cases} \quad (7)$$

In general

$$p_{i,v+1}(x) = \int_0^x p_{i,v}\,(x')dx', v = 1, 2, \dots. \quad (8)$$

The general from of v- times of integrals [13]

$$p_{i,v}(x) = \begin{cases} 0 & ,for\ x < \xi_1(i) \\ \frac{1}{v!}\,[\,x - \xi_1(i)]^{\,v} & ,for\ x \in [\xi_1(i), \xi_2(i)\ ) \\ \frac{1}{v!}\,\{[\,x - \xi_1(i)]^{\,v} - 2[x - \xi_2(i)]^{\,v}\,\} & ,for\ x \in [\xi_2(i), \xi_3(i)) \\ \frac{1}{v!}\,\{[\,x - \xi_1(i)]^{\,v} - 2[x - \xi_2(i)]^{\,v} + [x - \xi_3(i)\,]^{\,v}\,\} & ,for\ x > \xi_3(i) \end{cases} \quad (9)$$

and In the case $i = 1$, we have $\xi_1 = 0, \xi_2 = \xi_3 = 1$ These formulas hold for $i\ > 1$;

$$p_{1,v}(x) = \frac{1}{v!}\,(x)^{\,v}, \forall\, x \in\ [0,1] \quad (10)$$

3. CAS Wavelets and the integrals

In this section, we give some essential definitions and mathematical preliminaries of Cosine and Sine (CAS) wavelets, and we introduce function approximation via CAS wavelets and block pulse function.

CAS wavelet $\Psi_{n,m}(x) = \Psi(k, ň, m, x\ )$ have four arguments ; $n = 0,1,2, \dots\dots, 2^k - 1$, k can assume any positive integer, m is any integer, and x is the normalized time.

The orthonormal CAS wavelets are defined on the interval [0, 1) by [16]:

$$\Psi_{n,m}(x) = \begin{cases} 2^{\frac{k}{2}}\,CAS_m\,(\,2^k\,x - n\,) & ,for\ \frac{n}{2^k} \le x < \frac{n+1}{2^k} \\ 0 & otherwise \end{cases} \quad (11)$$

Where

$$CAS_m(x) = \cos(2m\pi x) + \sin(2m\pi x), \quad (12)$$

and $m \in \{\,-M, -M+1 \dots, M\,\}$. The CAS wavelets are orthonormal with appreciate to the weight function w(x)=1.

Now, We integrate the CAS wavelets in equation (11) analytically. The CAS Wavelets described in phrases of trigonometric functions whose integration is periodical. A general form for n of the integrals to these wavelets can be computed.

If we integrate equation (11) from (0) to (x), we obtain

$$P_{2^k(2M+1),1}(x) = \int_0^x \Psi_{n,m}^{CAS}(x')dx'$$

$$P_{2^k(2M+1),1}(x) = \begin{cases} 0 & ,0 \le x < \frac{n}{2^k} \\ \begin{cases} 2^{\frac{k}{2}} \frac{1}{2^{k+1}\pi m}[\sin(2\pi m(2^k x - n)) - \cos(2\pi m(2^k x - n))] \\ -2^{\frac{k}{2}} \frac{-1}{2^{k+1}\pi m} \end{cases} & ,\frac{n}{2^k} \le x < \frac{n+1}{2^k} \\ 2^{\frac{k}{2}} \frac{1}{2^{k+1}\pi m}[\sin(2\pi m) - \cos(2\pi m)] - 2^{\frac{k}{2}} \frac{-1}{2^{k+1}\pi m} & ,\frac{n+1}{2^k} \le x < 1 \end{cases} \quad (13)$$

$$P_{2^k(2M+1),2}(x) = \begin{cases} 0 & ,0 \le x < \frac{n}{2^k} \\ \begin{cases} 2^{\frac{k}{2}} \frac{(-1)}{(2^{k+1}\pi m)^2}[\cos(2\pi m(2^k x - n)) + \sin(2\pi m(2^k x - n))] \\ -[2^{\frac{k}{2}} \frac{(-1)}{(2^{k+1}\pi m)^2} + 2^{\frac{k}{2}} \frac{(-1)}{(2^{k+1}\pi m)}(x - \frac{n}{2^k})] \end{cases} & ,\frac{n}{2^k} \le x < \frac{n+1}{2^k} \\ \begin{cases} \left[1 + \left(x - \frac{n+1}{2^k}\right)\right][[2^{\frac{k}{2}} \frac{(-1)}{(2^{k+1}\pi m)^2}(\cos(2\pi m) + \sin(2\pi m))] \\ -(2^{\frac{k}{2}} \frac{(-1)}{(2^{k+1}\pi m)^2} + 2^{\frac{k}{2}} \frac{(-1)}{(2^{k+1}\pi m)}\left(\frac{1}{2^k}\right))] \end{cases} & ,\frac{n+1}{2^k} \le x < 1 \end{cases} \quad (14)$$

Repeating the integration v times, we find [8]

$$P_{i,v}(x) = \begin{cases} 0 & ,0 \le x < \frac{n}{2^k} \\ \begin{cases} 2^{\frac{k}{2}} \frac{(-1)^{c_v}}{(2^{k+1}\pi m)^v} \cos(2\pi m(2^k x - n)) \\ +2^{\frac{k}{2}} \frac{(-1)^{d_v}}{(2^{k+1}\pi m)^v} \sin(2\pi m(2^k x - n)) \\ -\sum_{jj=0}^{v-1} 2^{\frac{k}{2}} \frac{1}{jj!} \frac{(-1)^{c_v}}{(2^{k+1}\pi m)^{v-jj}} \left(x - \frac{n}{2^k}\right)^{jj} \end{cases} & ,\frac{n}{2^k} \le x < \frac{n+1}{2^k} \\ \begin{cases} \sum_{jj=0}^{v-1} \frac{1}{j!}(x - \frac{n+1}{2^k})^j. \\ .((2^{\frac{k}{2}} \frac{(-1)^{c_v}}{(2^{k+1}\pi m)^v} \cos(2\pi m) + 2^{\frac{k}{2}} \frac{(-1)^{d_v}}{(2^{k+1}\pi m)^v} \sin(2\pi m) - \sum_{jj=0}^{v-1} 2^{\frac{k}{2}} \frac{1}{jj!} \frac{(-1)^{c_v}}{(2^{k+1}\pi m)^{v-jj}} (\frac{1}{2^k})^{jj}), \end{cases} & \frac{n+1}{2^k} \le x < 1 \end{cases} \quad (15)$$

where

$$c_v = \begin{cases} 0 \; if \; v = 3,4,7,8,11,12, \ldots\ldots. \\ 1 \; if \; v = 1,2,5,6,9,10, \ldots\ldots.. \end{cases}$$

and

$$d_v = \begin{cases} 0 \; if \; v = 1,4,5,8,9,12, \ldots\ldots. \\ 1 \; if \; v = 2,3,6,7,10,11, \ldots\ldots.. \end{cases}$$

4. New wavelet with the integrals

Definition:[1]

Let f and g be two functions defined on R. Then the convolution of f and g is defined by the symbol $h = f * g$ by $f * g\,(x) = \int_R f(t)\, g(x-t)dt$ whenever the integration is logical.

The following theorem presents a technique for establishing a new wavelet from a given one.

Theorem:-[1]

If Ψ is a wavelet and φ is bounded integrate function, then the convolution function $\Psi * \varphi$ is also a wavelet .

Now, we derive a new wavelet formula which obtain from the convolution between the two wavelets, Haar and CAS wavelets where CAS wavelets are defined in terms of trigonometric functions whose integration is periodical and bounded.

Let $\Psi_{n,m} = CAS\, wavelet, and\ H_i = Haar\ wavelet$ since the convolution is Commutative we have

$$W_{n,m}^{New}(x) = (\Psi_{n,m} * H_i)(x) = \int_0^x \Psi_{n,m}(t).H_i(x-t)dt \quad (16)$$

$$W_{n,m}^{New}(x) = \begin{cases} \begin{cases} \frac{2^{\frac{-k}{2}-1}}{\pi m}[\cos(2m\pi(2^k x - n)) - \sin(2m\pi(2^k x-n)) + 2\sin(m\pi) \\ -2\cos(m\pi) + 1] & , \frac{n}{2^k} \le x < \frac{n+1}{2^k} \end{cases} \\ 0 & , otherwise \end{cases} \quad (17)$$

Where $m \in \{-M, -M+1, \dots, M\}$

Any function $f(x) \in L^2[0,1)$ may be expanded using $W_{n,m}^{New}$ wavelets as :

$$f(x) = \sum_{n=1}^{\infty}\sum_{m\in z} C_{n,m} W_{n,m}^{New}(x), \quad (18)$$

$C_{n,m} = < f(t), W_{n,m}^{New} >$. Where

If the infinite series in equation (18) is truncated, then equation (18) can be written as

$$f(x) = \sum_{n=0}^{2^k-1}\sum_{m=-M}^{M} C_{n,m} W_{n,m}^{New}(x) = C^T W_{n,m}^{New}(x), \quad (19)$$

where C and $W_{n,m}^{New}$ are $2^k\ (2M+1)\times 1$ matrices given by

$$C = \left[c_{0,(-M)}, c_{0,(-M+1)}, \dots, c_{0,M}, c_{1,(-M)}, \dots, c_{1,(M)}, c_{2^K-1,(-M)}, \dots \dots, c_{2^K-1,(M)}\right]^T \quad (20)$$

$$W_{n,m}^{New}(x) = \left[W_{0,(-M)}(x), W_{0,(-M+1)}(x), \dots, W_{0,M}(x), W_{1,(-M)}(x), \dots, W_{2^K-1,(-M)}(x), \dots, W_{2^K-1,M}(x)\right]^T \quad (21)$$

For convenience, in numerical solution, we rewrite equation (19) as follows:

Let $i = n(2M+1) - M + m$, then

$$f(x) = \sum_{i=1}^{2^k(2M+1)} C_i\ W_{2^k(2M+1),i}^{New}(x), \quad (22)$$

Now, If we want to solve a second order PDE we need the two integrals, If we integrate equation (17) from (0) to (x), we obtain New operational matrix (NP).

$$NP_{2^k(2M+1),1}(x) = \int_0^x (\Psi_{n,m} * H_i)(x')dx'$$

Then

$$NP_{2^k(2M+1),1}(x) =$$

$$= \begin{cases} 0 & , 0 \le x < \frac{n}{2^k} \\ \begin{cases} \frac{2^{\frac{-3k}{2}-2}}{\pi^2 m^2}[\sin(2m\pi(2^k x-n)) - 4\pi m(2^k x - n)\cos(\pi m) \\ + \cos(2m\pi(2^k x-n)) + 2m\pi(2^k x - n) + 4\pi m(2^k x-n)\sin(\pi m) - 1] \end{cases} & , \frac{n}{2^k} \le x < \frac{n+1}{2^k} \\ \begin{cases} \frac{2^{\frac{-3k}{2}-2}}{\pi^2 m^2}[\sin(2\pi m) - 4\pi m\cos(\pi m) \\ + \cos(2\pi m) + 4\pi m\sin(\pi m) + 2\pi m - 1] \end{cases} & , \frac{n+1}{2^k} \le x < 1 \end{cases} \quad (23)$$

$$NP_{2^k(2M+1),2}(x) = \begin{cases} 0 & ,0 \le x < \frac{n}{2^k} \\ \begin{cases} \frac{2^{\frac{-5k}{2}-3}}{\pi^3 m^3}[\sin(2\pi m(2^k x - n)) - \cos(2\pi m(2^k x - n)) + 2^{-k}(2^k x - 2n)\, x\sin(m\pi) \\ -2^{-k}(2^k x - 2n)x\cos(m\pi) + \frac{x^2}{2} - \frac{nx}{2^k} - \frac{x}{m\pi 2^{k+1}}] - \frac{2^{\frac{-5k}{2}-3}}{\pi^3 m^3}[-1 - (\frac{n}{2^k})^2 \sin(m\pi) \\ +(\frac{n}{2^k})^2\cos(m\pi) - \frac{n^2}{2^{2k+1}} - \frac{n}{m\pi 2^{k+1}}] \end{cases} & ,\frac{n}{2^k} \le x < \frac{n+1}{2^k} \\ \begin{cases} \frac{2^{\frac{-5k}{2}-2}}{\pi^2 m^2}[\sin(2\pi m) + \cos(2\pi m) + 4\pi m\sin(\pi m) - 4\pi m\sin(\pi m) + 2\pi m - 1] \\ + \frac{2^{\frac{-5k}{2}-3}}{\pi^3 m^3}[\sin(2\pi m) - \cos(2\pi m) + \frac{1-2n^2}{2^{2k}}\cos(\pi m) + \frac{1}{2^{2k+1}} - \frac{1}{m\pi 2^{2k+1}}] \\ - \frac{2^{\frac{-5k}{2}-3}}{\pi^3 m^3}[-1 - (\frac{n}{2^k})^2 \sin(m\pi)] \end{cases} & ,\frac{n+1}{2^k} \le x < 1 \end{cases} \tag{24}$$

5. The suggestion algorithm

We solve partial differential equation using new wavelet method.

The general form for PDE is

$$F(x, t, u, Du, D^2u, \dots, D^{\mu+\gamma}u) = f(x, t),$$
$$D^{\mu+\gamma}u = \frac{\partial^{(\mu+\gamma)}u(x,t)}{\partial t^\mu \partial x^\gamma}, \tag{25}$$

Where $f(x,t)$ is known function

We intend to do J levels of resolutions, hence we let $2M = 2^{J+1}$. The interval $[a, b]$ will be divided into $2M$ subintervals as a result $\Delta x = \frac{b-a}{2M}$ and the matrices are of dimensions $2M \times 2M$.

This new procedure is given in the following six steps.

Step(1): In the differential equation(25),Expand the derivative in its wavelet series.

$$\frac{\partial^{(\mu+\gamma)}u(x_*, t)}{\partial t^\mu \partial x_*{}^\gamma} = \sum_{i=0}^{m-1} a_i\, w_i^{new}(x) \tag{26}$$

a_i are the wavelet coefficients .

Step (2): Integrate the expansion in step (1) repeatedly to t from $(t_s) to\ (t)$, and x from $(0) to\ (x)$, we obtain

$$u(x,t) = \frac{(t-t_s)^\mu}{(\mu)!}\sum_{i=0}^{m-1} a_i\, p_{\gamma,i}^{\,new}(x) + \vartheta(x,t) \quad , \tag{27}$$

$\vartheta(x,t)$ is calculated from the initial and boundary conditions

Step (3): Substitute the expansion of the solution and its derivatives obtained in step(2) in to the equation (25) we get

$$\sum_{i=0}^{m-1} a_i\, [w_i^{new}(x_l) + \delta_1(x_l)p_{1,i}^{new}(x_l) + \delta_2(x_l)p_{2,i}^{\,new}(x_l) = \mathrm{R(X)} \tag{28}$$

where $\mathrm{R(X)} = f(x_l) - \delta_1(x_l)\gamma - \delta_2(x_l)[x_l\gamma - \vartheta]$ (29)

Step(4): replace $u(x,t)$ and all its derivatives in relation to t and x into the problem .
to the collocation points $x_l = \frac{l-0.5}{m}, l = 1,2, \dots, 2m$. And also to the collocation points $t_s = \frac{(s-1)}{N}$, $\mathrm{s} = 1,2, \dots, \mathrm{N}$ for a given resolution M,where $\mathrm{M} = \frac{m}{2}$,and get
a system of linear equation.

$$\sum_{i=0}^{m-1} a_i\, [w_i(x_l) + \delta_1(x_l)p_{1,i}(x_l) + \delta_2(x_l)p_{2,i}(x_l) = f(x_l) - \delta_1(x_l)\gamma - \delta_2(x_l)[x_l\gamma - \vartheta(x,t)] \tag{30}$$

Step(5): Solve the system of algebraic equations obtained in step (4), for the coefficients of

wavelet a_i.
Step(6): Evaluate the numerical solution for $u(x,t)$ by using the coefficients a_i in the wavelet series expansion of the solution .

6. Numerical Experiments

To show the efficiency of the proposed method, we will apply Haar and CAS wavelet method with the new wavelet method to obtain the approximate solution of the following examples. All of the computations have been performed using MATLAB .

Example(1): Consider the one-dimensional diffusion equation .

$$\boldsymbol{u_t = u_{xx}} \quad , x \in [0,1) \quad , t > 0$$

With the initial condition $u(x,0) = \sin(\pi x)$,

and the boundary conditions $u(0,t) = u(1,t) = 0 \, , t > 0$

The exact solution is $\boldsymbol{u(x,t)} = \mathbf{e}^{-\pi^2 t} \boldsymbol{sin}\,(\boldsymbol{\pi x})$

Results obtained using Haar, CAS wavelets and new wavelet methods are compared in table (1) for the amplitude of the matrix is m=16.

Table (1): Shows the approximate solution with the exact solution using a Haar, CAS wavelets and new wavelet for m = 16 , at t = 0.001 of example (1)

(x/32)	Haar wavelet	CAS wavelet	New wavelet	Exact solution
1	0.09780735	0.09770275	0.09771317	0.09705451
3	0.28859259	0.28830811	0.28837030	0.28743377
5	0.46797801	0.46772453	0.46781865	0.46676712
7	0.62938789	0.62915494	0.62924357	0.62816287
9	0.76665295	0.76642851	0.76650463	0.76541867
11	0.87450334	0.87428093	0.87437159	0.87325986
13	0.94881016	0.94857124	0.94869376	0.94754217
15	0.98686352	0.98644582	0.98658395	0.98541096
17	0.98754979	0.98644965	0.98657215	0.98541096
19	0.94986189	0.94858281	0.94867097	0.94754217
21	0.87559688	0.87430060	0.87436777	0.87325986
23	0.76775438	0.76645776	0.76653906	0.76541867
25	0.63049408	0.62919868	0.62930775	0.62816287
27	0.46909272	0.46779825	0.46791909	0.46676712
29	0.28975516	0.28845907	0.28857339	0.28743377
31	0.09939298	0.09807644	0.09818162	0.09705451

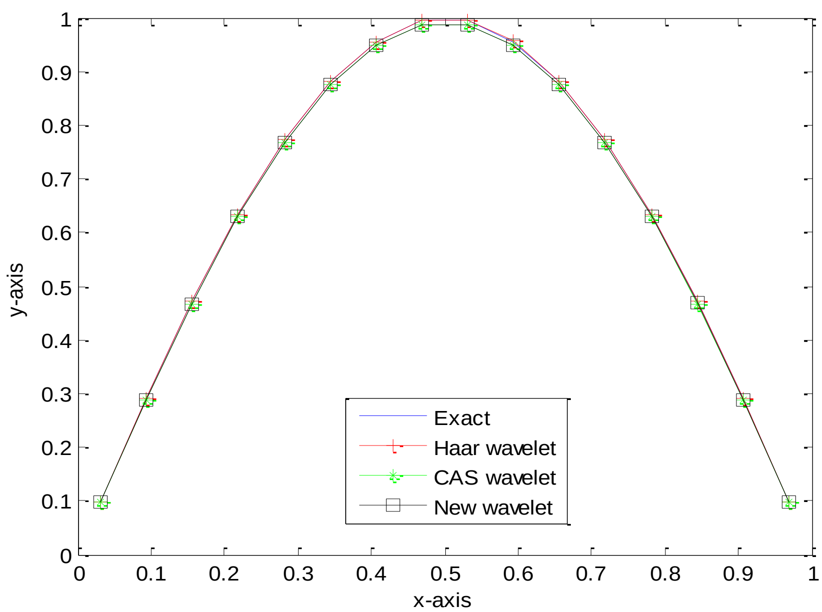

Figure (1): Compared the numerical solutions with the exact solution of example (1) at t = 0.001

From table (1) and Figure (1), we see that the solution of suggestion wavelet is better and nearer to the exact solution.

Example (2): Consider the wave equation .

$$u_{tt} = a^2 u_{xx} \quad , \; 0 < x < 1 \;\; , t > 0$$

With the initial conditions $u(x,0) = u_0 \sin(\pi x)$, $u_t(x,0) = 0$,$0 < x < 1$,

and the boundary conditions $u(0,t) = 0$, $u(1,t) = 0$,$t > 0$

The exact solution is

$$u(x,t) = u_0 \cos(\pi a t) . \sin(\pi x)$$

Also we use Haar, CAS and new wavelets method to obtain the results which are in table (2) and we plotted Fig (2) to illustrate the numerical and exact solutions for the amplitude of the matrix m=16.

Table (2): shows the approximate solution with the exact solution using a Haar, CAS wavelets and new wavelet for m = 16 , at t = 0.02 of example (2)

(x/32)	Haar wavelet	CAS wavelet	New wavelet	Exact solution
1	0.07808598	0.07811496	0.07809324	0.09628099
3	0.26620014	0.26617697	0.26616909	0.28514294
5	0.44330237	0.44329475	0.44329388	0.46304700
7	0.60269110	0.60268779	0.60268485	0.62315643
9	0.73824006	0.73823816	0.73823213	0.75931831
11	0.84474020	0.84473892	0.84473577	0.86630001
13	0.91809828	0.91809791	0.91810193	0.93999029
15	0.95554444	0.95549616	0.95550414	0.97755727
17	0.95544883	0.95549654	0.95550115	0.97755727
19	0.91809924	0.91809904	0.91809586	0.93999029
21	0.84474019	0.84474084	0.84473294	0.86630001
23	0.73824006	0.73824104	0.73823606	0.75931831
25	0.60269110	0.60269237	0.60269365	0.62315643
27	0.44330238	0.44330392	0.44330800	0.46304700
29	0.26619906	0.26620077	0.26620317	0.28514294
31	0.07819367	0.07818992	0.07819003	0.09628099

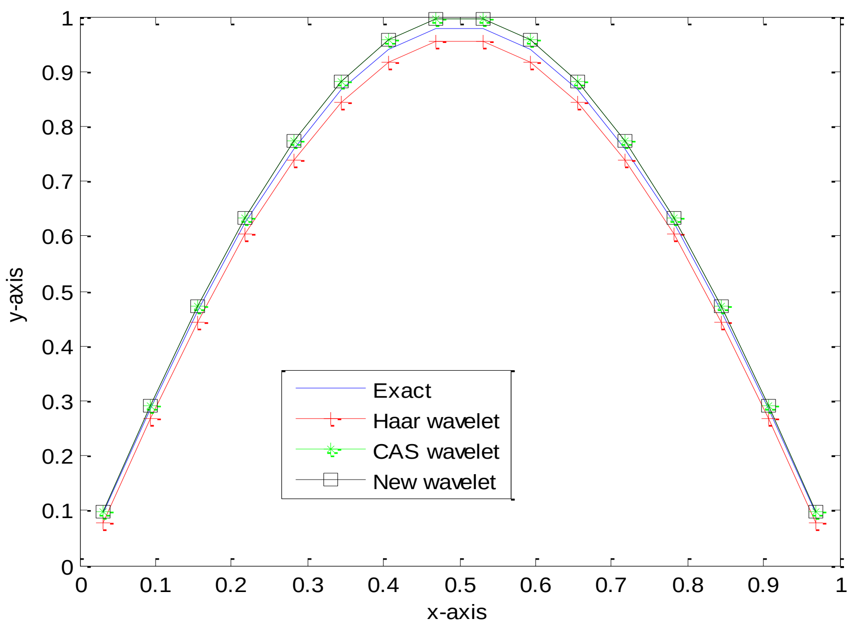

Figure (2): Compared the numerical solutions with the exact solution of example (2) at t = 0.02

Example(3): Consider the one-dimensional wave –like equation .

$$u_{tt} - \frac{x^2}{2} u_{xx} = 0 \quad , \; 0 < x < 1 \quad , t > 0$$

with the initial conditions $u(x,0) = x$, $\dot{u}\,(x,0) = x^2$,

and the boundary conditions $u(0,t) = u(1,t) = 1 + \sinh(t)$ $, t > 0$

The exact solution is

$$u(x,t) = x + x^2 sinh\,(t)$$

Results obtained using Haar, CAS and new wavelets method are compared in table (3), and Figure(3) shows the numerical solutions plot this example by using presented methods with m=16.

Table (3) shows the approximate solution with the exact solution using a Haar, CAS wavelets and new wavelet for m = 16, at t = 0.02 of example (3)

(x/32)	Haar wavelet	CAS wavelet	New wavelet	Exact solution
1	0.03126950	0.03127924	0.03127924	0.03126953
3	0.09392570	0.09401353	0.09401353	0.09392579
5	0.15673817	0.15698223	0.15698223	0.15673831
7	0.21970689	0.22018534	0.22018534	0.21970710
9	0.28283188	0.28362287	0.28362287	0.28283214
11	0.34611313	0.34729481	0.34729481	0.34611344
13	0.40955065	0.41120116	0.41120116	0.40955100
15	0.47314442	0.47534192	0.47534192	0.47314482
17	0.53689447	0.53971710	0.53971710	0.53689491
19	0.60080077	0.60432669	0.60432669	0.60080125
21	0.66486334	0.66917069	0.66917069	0.66486386
23	0.72908217	0.73424911	0.73424911	0.72908272
25	0.79345727	0.79956193	0.79956193	0.79345785
27	0.85798862	0.86510918	0.86510918	0.85798923
29	0.92267625	0.93089083	0.93089083	0.92267688
31	0.98752012	0.99690689	0.99690689	0.98752078

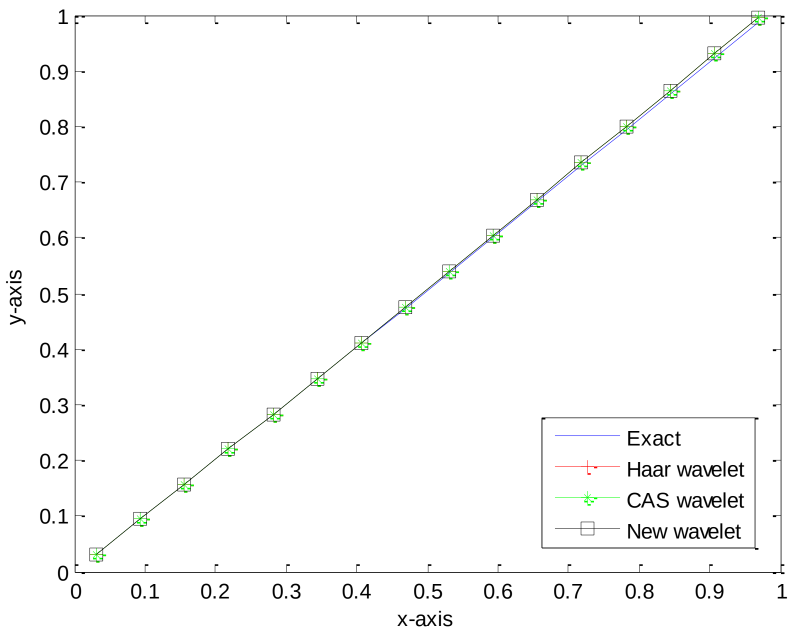

Figure (3): Compared the numerical solutions with the exact solution of example (3) at t = 0.02

CONCLUSIONS

In this paper, we drive a new wavelet from the convolution between Haar and CAS wavelets. the suggestion method is applied to solve the PDEs with collocation points. we compared between the new convolution, Haar and CAS wavelets methods with the exact solution from three examples. Figure 1-3 show the numerical solution of new method, Haar, CAS wavelets methods and the exact solution of the PDEs proposed in examples 1-3 respectively. The obtained results shows that the new technique is better and nearer to the exact solution. In this paper only linear problems were solved, but the suggestion method is applicable for nonlinear PDEs.

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