Research Article

q-Continuous on (q-open, q-closed, q-interior, q-closure) and separation axiom in quad Topological Space

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ABSTRACT

The purpose of this paper is to the q-continuousspace from a quad topological space to a triple topological space,we introduce a conditions to make us able to change the separation axioms and regular space, normal space in q-topologicalspace to the separation axioms and regular space, normal space in tri-topological space and study some of their properties

Keywords: Quad topological spaces, q-T_0 space, q-T_1 space, q-T_2 space, q-T_3space ,regular space, q-T_4 space, normalspace, q-continuous, q-homeomorphism.

1- Introduction

J .C. Kelly [2] introduced bi-topological spaces in 1963 and Luma [4], . The study of tri-topological spaces was firstinitiated by Martin M. Kovar [4] in 2000,where a non empty set X with three topologies is calledtri-topological spaces. N.F. Hameed & Mohammed Yahya Abid [1] studied separation axioms in tri-topologicalspaces. D.V. Mukundan [5] introduced the concept on topological structures with fourtopologies, quad topology and defined new types of open (closed)sets. In thispaper, we use q-open and q-closed sets defined by D.V. Mukundan [5] to explain the concept of separation axioms in quad topological spaces

Definition 2.1: A quad topological space X is called $(T_{o\_}q)$ space iff to each pair of distinct point x, y in X, there exist a (q_open) set containing one of the points but not other.

Definition 2.11: A quad topological space X is called $(T_{1\_}q)$ space iff to each pair of distinct point x, y in X, there exist a pair (q_open) set containing X but not y and the other containing y but not X.

Definition 2.22: A quad topological space X is called $(T_{2\_}q)$ space iff to each pair of distinct point x, y of X, there exist a pair of distinct (q_open) sets one containing x and the other containing y and called $(Hausd\ orff\_q)$.

Definition 2.33: A quad topological space X is said to be $(regular\_q)$ space iff for each q_{closed} set F and each point $x \notin F$. there exist disjoint $\left(q_{open}\right)$ sets G, H such that $x \in G, F \subseteq H$.

A $regular\_q$ with $T_{1\_}q$ space is called $T_{3\_}q$ space.

A $regular\_q + T_{1\_}q$ space $= T_{3\_}q$ space

Definition 2.43: A quad topological space X is called to be $normal\_q$ space iff for each two disjoint q_colsed set $F_1, F_2 \subseteq X$, there exist two disjoint $\left(q_{open}\right)$ sets G_1, G_2 such that $F_1 \subseteq G_1, F_2 \subseteq G_2$.

A $Normal\_q$ with $T_{1\_}q$ space is called $T_{4\_}q$ space.

A $Normal\_q + T_{1\_}q$ space $= T_{4\_}q$ space

2- Preliminaries

Definition 2.1 [5][6] :Let X be a nonempty set and $(\tau)_{i=1,2,3,4}$are general topologies on X.
Then a subset A of space X is said to be quad-open(q-open) set if$A \subset \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$ and itscomplement is said to be q-closed and set X with four topologies called $(X, \tau_i)_{i=1,2,3,4}$. q-opensets satisfy all the axioms of topology.

Definition2.2 [5][6]: A subset of a q-topological space $(X, \tau_i)_{i=1,2,3,4}$is called q- Neighbourhood of a point if and only if there exist q-open sets such that$x \subset X \subset A$.

Note 2.3[5][6] : We will denote the q-interior (resp. q-closure) of any subset ,say of by q-intA(qclA),is the intersection of all q-closed sets containing A.where q-intA is the union of all q-open sets contained in A, and q-clA is the intersection of allq-closed sets containing A.

3- q-Continuous in quad Topological Space

Definition3.1: Let $(X, \tau_i)_{i=1,2,3,4}$, be quad-topological space and $(\psi, \tau_{i\psi})_{i=1,2,3}$ be a tri-topological space, a function of $f: (X, \tau_i)_{i=1,2,3,4} \to (\psi, \tau_{i\psi})_{i=1,2,3}$ is said to be $q-$ccontinuous at $x \in X$ if f for every tri-open set V in ψ containing $f(x)$ there exists q-open set U in X containing x such that $f(U) = V$, we say that f is q-continuous on X if f is $q-$continuous at each $x \in X$.

Definition3.2 : Let $(X, \tau_i)_{i=1,2,3,4}$ be quad topological space and $(\psi, \tau_{i\psi})_{i=1,2,3}$ be a tri topological space and $f: (X, \tau_i)_{i=1,2,3,4} \to (\psi, \tau_{i\psi})_{i=1,2,3}$be a function, then:

1. $f$is said to be q-open function if and only if $f(G)$ is tri-open in ψ for every q-open set G in X.
2. f is q-closed function if and only if $f(F)$ is tri-closed in ψ for every q-closed set F in X.
3. $f$is q-homeomorphism if and only if:
   i. $f$is bijective (1-1, onto)
   ii. f and f^{-1} are q-continuous.

Example 3.3: Let $X = \{a, b, c\}$, $\tau_i = \{x, \varphi, \{a\}\}$, τ_i =$\{x, \varphi, \{b\}\}$, τ_i =$\{x, \varphi, \{a, c\}\}$,
τ_i =$\{x, \varphi, \{b\}, \{c\}, \{b, c\}\}$, (X,T_1) , (X,T_2) , (X,T_3) , (X,T_4) are quad topological space, such that:
$T_{\cup X}$= $\{x, \varphi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$
Let $\psi = \{1,2,3\}\tau_{1\psi} = \{\psi, \varphi, \{1,2\}\}, \tau_{2\psi}$=$\{\psi, \varphi, \{2,3\}\}$, $\tau_{3\psi}$=$\{\psi, \varphi, \{1\}, \{3\}, \{1,3\}\}$where$(X,\mathrm{T}_1)$, (X,T_2) , (X,T_3) aretri-topological space, such that$T_{\cup Y}$ = $\{\psi, \varphi, \{1\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}\}$
Define $f: (X, \tau_i)_{i=1,2,3,4} \to (\psi, T_i)_{i=1,2,3}$by $f(a) = 1, f(b) = 2, f(c) = 3$
Then f is not q-open and not q-continuous because$f^{-1}(\psi) = \{a, b, c\} = X$ is q-open in X,and $f^{-1}(\varphi) = \varphi$ is q-open in X, $f^{-1}(\{1\}) = \{a\}$ is q-open in X, $f^{-1}(\{3\}) = \{c\}$ is q-open in X
but $f^{-1}(\{1,2\}) = \{a, b\}$is not q-open in X.
Hence $f$is not q-continuous. And since $f(X) = \{1,2,3\} = \varphi$ is tri-open in ψ and $f(\varphi) = \varphi$ is tri-open in ψ, also $f(\{a\}) = \{1\}$ is tri-open in ψ and $f(\{b\}) = \{2\}$ is not tri-open in ψ, Hence f is not q-open.
Early $T^{C}{}_{\cup X} = \{x, \varphi, \{b, c\}, \{a, c\}, \{a, b\}, \{a\}, \{b\}\}$ and $T^{C}{}_{\cup Y}$=$\{ \psi, \varphi, \{2,3\}, \{1,2\}, \{3\}, \{1\}, \{2\}\}$
Then $f(\varphi) = \varphi$ is tri-open in ψ,$f(x) = \psi$ is tri-open in ψ and $f(\{b, c\}) = \{2,3\}$ is tri-open in ψ
And $f(\{a, c\}) = \{1,3\}$is not tri-open in ψ. Hence f is not q-closed.
Clearly f is bijective, but its not q-continuous. Thane is notq-homeomorphism.

Propositions 3.4: Let $(X, \tau_i)_{i=1,2,3,4}$ be a quad topological space and $(\psi, T_i)_{i=1,2,3}$be a tri topological space the function $f: X \to \psi$ is q-continuous if and only if the inverse image under f of every t=open set V of ψ is a q-open set of X.
Proof: Let f a q-continuous, and V is tri-open in ψ, to prove that $f^{-1}(V)$is q-open in X. if $f^{-1}(V) = \varphi$ so, $f^{-1}(V)$ is q-open inX. if $f^{-1}(V) \neq \varphi$, Let $x \in f^{-1}(V)$then $f(x) \in V$,Bydefinition ofq-continuous there exist q-open set Gx in X containing x such that $f(Gx) \in V$.

$x \in Gx \in f^{-1}(V)$this shows that $f^{-1}(V)$is a q-nbd of each is points. Hence $f^{-1}(V)$is q-open in X.
Conversely, let $f^{-1}(V)$is q-open set in X, for each V is a tri-open set in ψ to prove f is q-continuous.
Let $x \in X$ andV is a tri-open set in ψ containing $f(x)$so$f^{-1}(V)$is q-open in X-containing $f(x)$ so $f^{-1}(V)$ is q-open in X-containing x and $(f^{-1}(V)) \subset V$
Then f is q-continuous on X.

Propositions 3.5: Let $(X,\tau_i)_{i=1,2,3,4}$ be a quad topological space and $(\psi,\tau_{i\psi})_{i=1,2,3}$ be a tri topological space. A function $f: X \to \psi$ is q-continuous iff if the inverse image under f of every tri-closed set in ψ is q-closed set in X.
Proof: Assume that f is q-continuous and let F be any tri-closed set in ψ. To show that f^{-1}(F) is q-closed in X, since f is q-continuous and $\psi - F$ is t-open in ψ, it follows from proposition (3.3.4), $f^{-1}(\psi$ -F) = X- $f^{-1}(F)$ is q-open in X, that is $f^{-1}(F)$ is q-closed in X.
Conversely, let $f^{-1}(F)$ be q-closed in X for every t-closed set F in ψ, we want to show that f is q-continuous function. Let G be an t-open set in ψ. Then ψ -G is t-closed in ψ and so by hypothesis,
$f^{-1}(\psi - G) = X - f^{-1}(G)$ is q-closed in X, that is $f^{-1}(G)$ is q-open in X, hence f is q-continuous by proposition (3.1.4).

Proposition 3.6: Let $(X,\tau_i)_{i=1,2,3,4}$be a quad topological space, and $(\psi,\tau_{i\psi})_{i=1,2,3}$be a tri topological space, a function $f: X \to \psi$ is q-continuous iff. $f(q - cl(A) \subset tri - cl(f(A)$ for every $A \in X$
Proof: let f be q-continuous, since $tri - cl(f(A))$ is a tri-closed set in Ψ. Then by proposition (3.3.5)
$f^{-1}(tri - cl(f(A))$ isq-closed in X,
$q - cl(f^{-1}(tri - cl(f(A)) = f^{-1}(t - cl(f(A))$ ………………… (3.3.4)
Now $f(A) \subset t - cl(f(A), A \subset f^{-1}(f(A)) \subset f^{-1}(t - cl(f(A))$
Then $q - cl(A) \subset q - cl(f^{-1}(tri - cl(f(A)) = f^{-1}(tri - cl(f(A))$ by (3.3.4)
Then $f(q - cl(A)) \subset tri - cl(f(A))$
Conversely, let $f(q - cl(A)) \subset tri - cl(f(A))$ for every $A \subset X$.
Let be any tri-closed set in ψ, so that $tri - cl(F) = F$,
Now $f^{-1}(F) \subset X$ by hypothesis, $f(q - cl(f^{-1}(F)))C\ tri - cl(f(f^{-1}(F))) \subset tri - cl(F) = F$
Therefore, $q - cl(f^{-1}(F)) \subset f^{-1}(F)\ but f^{-1}(F) \subset q - cl(f^{-1}(F))$ always
Hence $q - cl(f^{-1}(F)) = f^{-1}(F)$ and so $f^{-1}(F)$ is q-closed in X, hence by proposition (3.3.5), f is q-continuous.

Proposition 3.7: : Let $(X,\tau_i)_{i=1,2,3,4}$be a quad topological space, and $(\psi,\tau_{i\psi})_{i=1,2,3}$be a tri-topological space, a function $f: X \to \psi$is q-continuous iff:
$q - cl(f^{-1}(B)) \subset f^{-1}(t - cl(B))$ for every B$\subset \psi$
Proof: Let f be q-continuous. Since $tri - cl(B)$ is t-closed in Ψ, then by proposition (3.3.5) $f^{-1}(tri - cl(B))$ is q-closed in X and therefore,
$q - cl(f^{-1}(tri - cl(B)) = f^{-1}(tri - cl(B))$ …………. $(3-3-5)$
Now $B \subset tri - cl(B)$, then $f^{-1}(B) \subset f^{-1}(tri - cl(B))$, then $q - cl(f^{-1}(B)) \subset q - cl(f^{-1}(t - cl(B)) = f^{-1}(tri - cl(B))$ by (3-3-5)
Conversely, let the condition hold and let Fbe any tri-closed set in ψ so that $tri - cl(F) = F$ by hypothesis
$q - cl\left(f^{-1}(F)\right) \subset f^{-1}\left(q - cl(F)\right) = f^{-1}(F)butf^{-1}(F) \subset q - cl(f^{-1}(F))$ always.
Hence $q - cl(f^{-1}(F)) = f^{-1}(F)$ and so$f^{-1}(F)$ is q-closed in X. it follows from proposition (3.3.5) that f is q-continuous.

Proposition 3.8: Let $(X,\tau_i)_{i=1,2,3,4}$be a quad topological space, and $(\psi,\tau_{i\psi})_{i=1,2,3}$be a tri topological space, a function $f: X \to \psi$is q-continuous iff$f^{-1}(tri - int(B) \subset q - int(f^{-1}(B))$ for every $B \subset \psi$.
Proof: Let f be q-continuous, since $tri - int(B)$is tri-open in ψ, then by proposition (3.1.4) $f^{-1}(tri - int(B)$isq-openin Xand therefor$q - int(f^{-1}\left(tri - int(B)\right) = f^{-1}\left(tri - int(B)\right)$ ....……….(3.3.6)

Now $tri - int(B) \subset B$ then $f^{-1}(tri - int(B)) \subset f^{-1}(B)$, then $q - int(f^{-1}(tri - int(B) \subset q - int(f^{-1}(B))$ Hence $f^{-1}(tri - int(B))\ C\ q - int(f^{-1}(B))\ by\ (3.3.6)$.

Conversely, let the condition hold and let G by any tri-open set in ψ. So that $tri - int(G) = G$ by hypothesis, $f^{-1}(tri - int(G)) \subset q - int(f^{-1}(G))$,since $f^{-1}(tri - int(G)) = f^{-1}(G)$ then $f^{-1}(G) \subset q - int(f^{-1}(G))$, but $q - int(f^{-1}((G)) \subset f^{-1}(G)$ always and so $q - int(f^{-1}((G)) = f^{-1}((G)$therefore$f^{-1}(G)$ is a q-open in X and consequently by proposition (3.3.4)f is q-continuous.

4- **Separation axiom in quad** T**opological Space**

Proposition 4.1 : Let $(\psi, T_i)_{i=1,2,3}$be a T_0-tri space if $f: (X, T_i)_{i=1,2,3,4} \rightarrow (\psi, \tau_{i\psi})_{i=1,2,3}$ q-continuous and 1-1 function, then $(X, \tau_i)_{i=1,2,3,4}$ is a T_0-q space.

Proof: Let $x_1, x_2 \in$ X, $x_1 \neq x_2$. Since f is 1-1 function then $f(x_1) \neq f(x_2)$, $f(x_1)$ and, $f(x_1) \in G$ and ψ us T_0-tri-space, then there exist q-open G in ψ such that $f(x_1) \in G$ and $f(x_2) \notin G$.

So $x_1 \in f^{-1}(G)$, $x_2 \notin f^{-1}(G)$. therefore $f^{-1}(G)$ is q-open set in X containing x_1 but not x_2 , hence $(X, T_i)_{i=1,2,3,4}$is T_0-q space.

Proposition4.2:let$f: (X, \tau_i)_{i=1,2,3,4} \rightarrow (\psi, \tau_{i\psi})_{i=1,2,3}$ be an q-open bijective function if$(X, \tau_i)_{i=1,2,3,4}$is a T_0-q space then $(\psi, \tau_{i\psi})_{i=1,2,3}$is T_0-tri space.

Proof: suppose $y_1, y_2 \in \psi, y_1 \neq y_2$ since f is onto, there exist $x_1, x_2 \in$ X such that $y_1 = f(x_1), y_2 = f(x_2)$ and since f is bijective, then $(x_1) \neq (x_2)$, since X is T_0-q space, then there exist q-open set G such that $x_1 \in G$, $x_2 \notin G$.

Hence $y_1 = f(x_1) \in f(G), x_2 = f(x_2) \notin f(G)$, since f is q-open function, then $f(G)$ is tri-open set in ψ. Therefore $(\psi, T_i)_{i=1,2,3}$is T_0-t space.

Proposition 4.3: Let $(\psi, \tau_{i\psi})_{i=1,2,3}$be T_1-tri space, If $f: (X, \tau_i)_{i=1,2,3,4} \rightarrow (\psi, \tau_{i\psi})_{i=1,2,3}$ is q-continuous and 1-1 function, then X is a T_1-q space.

Proof: Let$x_1, x_2 \in X$, $x_1 \neq x_2$ since f is 1-1 function then $f(x_1), f(x_2) \in \psi$, ψ is T_1-tri space, then there exist U_1, U_2, tri-open set in ψ such that:$f(x_1) \in U_1, f(x_2) \in U_2, f(x_1) \notin cU_2, f(x_2) \notin U_1$ then $x_1 \in f^{-1}(U_1)$ but $x_2 \notin f^{-1}(U_1)$, and $X_2 \in f^{-1}(U_2)$, but $x_1 \notin f^{-1}(U_2)$ and$f^{-1}(U_1)$, $f^{-1}(U_2,)$ are q-open set inX, Hence$(X, T_i)_{i=1,2,3,4}$ is a T_1-q space.

Proposition 4.4: Let $f: (X, \tau_i)_{i=1,2,3,4} \rightarrow (\psi, \tau_{i\psi})_{i=1,2,3}$ be bijective function and q-open function,

If $(X, \tau_i)_{i=1,2,3,4}$is a T_1-q space then $(\psi, \tau_{i\psi})_{i=1,2,3}$is a T_1-tri space

Proof: Suppose $y_1, y_2 \in \psi$, $y_1 \neq y_2$since f is onto, there exist x_1, $x_2 \in$ X, such that

$y_1 = f(x_1)$, $y_2 = f(x_2)$ and since f is bijective, then $x_1 \neq x_2 \in X$, $f(x_1) \neq f(x_2)$ and since X is T_1-q space, then there exist q-open sets G,H such that $x_1 \in G$ but $x_2 \notin G$ and $x_2 \in H$ but $x_1 \notin$ H, hence $f(x_1) \in f(G)$ and $f(x_2) \in f(H)$. since f is q-open function hence $f(G), f(H)$ are tri-open sets of ψ, such that $y_1 \in f(G)$ but $y_2 \notin f(G)$ and $y_2 \in f(H)$ but $y_1 \notin f(H)$, then $(\psi, \tau_{i\psi})_{i=1,2,3}$is a T_1-tri space.

Proposition4.5: Let $(\psi, \tau_{i\psi})_{i=1,2,3}$be T_2-tri-space, if $f: (X, \tau_i)_{i=1,2,3,4} \rightarrow (\psi, \tau_{i\psi})_{i=1,2,3}$ is q-continuous and 1-1 function, then $(X, \tau_i)_{i=1,2,3,4}$is a T_2-q space.

Proof: Let $x_1, x_2 \in X$, $x_1 \neq x_2$since$f$ is1-1 function then

$f(x_1) \neq f(x_2)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$, $y_1 \neq y_2$.

Since ψ is T_2-tri-space, there exist tow tri-open sets G,H in ψ such that$y_1 \in G, y2 \in H, G \cap H = \varphi$, hence $x_1 \in f^{-1}(G), x_2 \in f^{-1}((H)$ since f is q-continuous and $f^{-1}(G)$ and $f^{-1}((H)$ are q-open sets in X.Also$f^{-1}(G) \cap f^{-1}((H) = 0$ and $f^{-1}(G \cap H) = f^{-1}(\varphi) = \varphi$Thus $f: (X, T_i)_{i=1,2,3,4}$ isa T_2-q space.

Proposition4.6: let $f:(X,\tau_i)_{i=1,2,3,4} \to (\psi,\tau_{i\psi})_{i=1,2,3}$ be bijective and q-open function, if $(X,T_i)_{i=1,2,3,4}$ is a T_2-q space then $(\psi,\tau_{i\psi})_{i=1,2,3}$is a T_2-t space.

Proof: Let $y_1 \neq y_2$ since f is bijective function and onto, then there exist $x_1 \neq x_2 \in$ X such that$y_1 = f(x_1)$ and $y_2 = f(x_2)$ since X is T_2-q space, then there exist q-open sets G, H in X such that $x_1 \in G, x_2 \in H, G \cap H = \varphi$. Since f is q-open function then $f(G)$ and $f(H)$ are tow t-open sets in ψ and $f(G \cap H) = f(G) \cap f(H) = \varphi$, Also $y_1 = f(x_1) \in f(G), y_2 = f(x_2) \in f(H)$ hence $(\psi,\tau_{i\psi})_{i=1,2,3}$is a T_2-tri space.

Proposition4.7: Let $(X,\tau_i)_{i=1,2,3,4}$be a regular-qspace and the function $f:(X,\tau_i)_{i=1,2,3,4} \to (\psi,\tau_{i\psi})_{i=1,2,3}$ be q-homeomorphism then $(\psi,\tau_{i\psi})_{i=1,2,3}$is a regular-t space.

Proof: let F be a t-closed in ψ, $q \in \psi$, $q \notin F$. Since f is bijective and onto function, then there exist. P $\in$ X such that $f(p) = q$, $p = f^{-1}(q)$ since f is q-continuous so $f^{-1}(F)$ is q-closed in X, $q \notin Fp = f^{-1}(q) \notin f^{-1}(F)$. since $(X,T_i)_{i=1,2,3,4}$is regular-q-space, there exist q-open sets G,H in X. such that $p \in G, f^{-1}(F) \in H$ and $G \cap H = \varphi \ so \ q = f(p) \in f(G), F \in f(f^{-1}(F) \in H)$, since f is a q-open function, hence$f(G), f(H)$are tri-open sets in ψ and $f(G \cap H) = f(G) \cap f(H) = F(\varphi) = \varphi$, therefore $(\psi,T_i)_{i=1,2,3}$is a regular-tri-space.

Proposition4.8: Let $(X,\tau_i)_{i=1,2,3,4}$ be T_3-q space and the function $f:(X,\tau_i)_{i=1,2,3,4} \to (\psi,\tau_{i\psi})_{i=1,2,3}$ be q-homeomorphism, then $(\psi,T_i)_{i=1,2,3}$is T_3-t space.

Proof: Easy by using Propositions (3.4.4) and (3.4.7)

proposition4.9: Let $(X,\tau_i)_{i=1,2,3,4}$be a normal-q space and the function $f:(X,T_i)_{i=1,2,3,4} \to (\psi,\tau_{i\psi})_{i=1,2,3}$ be q-homeomorphism, then $(\psi,\tau_{i\psi})_{i=1,2,3}$is a normal-t.

Proof: Let $(X,\tau_i)_{i=1,2,3,4}$ be a normal-q space and let $(\psi,\tau_{i\psi})_{i=1,2,3}$ be a q-homeomorphism image of $(\psi,\tau_{i\psi})_{i=1,2,3}$under a q-homeomorphism to show that $(\psi,\tau_{i\psi})_{i=1,2,3}$also normal-t space.

Let S,B be a pair of disjoints t-closed subsets of ψ, since f is q-continuous function then $f^{-1}(S)$ and $f^{-1}(B)$ are q-closed subsets of X, also$f^{-1}(S) \cap f^{-1}(B) = f^{-1}(S \cap B) = f^{-1}(\varphi) = \varphi$

then $f^{-1}(S), f^{-1}(B)$ are disjoint pair of q-closed subset of X. since the space $(X,T_i)_{i=1,2,3,4}$is normal-q, then there exist q-open sets G, H in X such that $f^{-1}(B) \in G, f^{-1}(S) \in H$ and $G \cap H = \varphi$ but $f^{-1}(B) \in G$, then $f(f^{-1}(B)) \in f(G), B \in f(G)$ similarly, $S \in f(H)$. also since f is a q-open function $f(G)$ and $f(H)$ are tri-open sets of ψ, such that $f(G) \cap f(H) = f(G \cap H) = f(\varphi) = \varphi$

thus, there exist tri-open subsets in ψ, $G_1 = f(G) \ and \ H_1 = f(H)$ such that $B \cap G_1$, $S \cap H_1$,

and $G_1 \cap H_1 = \varphi$, it follows that $(\psi,\tau_{i\psi})_{i=1,2,3}$also normal-tri-space.

Proposition4.10: Let $(X,\tau_i)_{i=1,2,3,4}$be a T_4-q space and the function $f:(X,\tau_i)_{i=1,2,3,4} \to (\psi,\tau_{i\psi})_{i=1,2,3}$ be q-homeomorphism, then $(\psi,\tau_{i\psi})_{i=1,2,3}$is T_4-trspace.

Proof: Easy by using proposition (3.4.4) and (3.4.9)

Proposition4.11: The q-continuous image of a q-continuous space is a tri-compact.

Proof: let$f:(X,\tau_i)_{i=1,2,3,4} \to (\psi,\tau_{i\psi})_{i=1,2,3}$ be a q-continuous, let X be a q-compact, let C be a tri-open covering of the set $f(X)$ by sets of tri-open in ψ. The collection $\{f^{-1}(A): A \in C\}$ is a collection of q-open covering of X, these sets are q-open in X because f is q-continuous hence finitely many of them, say $f^{-1}(A_1), \dots, f^{-1}(A_n)$ cover X, then the sets $A_1, \dots, A_n$ are cover of X.

Conclusions

In this paper, the concept of continuity from a quad topological space to a triple topological space is presented, and the separation axioms are studiedbetween these spaces. between

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