Sadik Transform, The Generalization Of All The Transform Who's Kernal Is Of Exponential Form With The Application In Differential Equation With Variable Coefficients

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Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 4 June 2021

Abstract:
In This Paper We Have Tried To Produce General Form Of All The Integral Transforms Whose Kernel Is Of Exponential Form By Changing Different Values Of Alfa And Beta In Sadik Transform, We Have Produces General Form Of Sadik Transform Over Laplace Transform, Sumudu Transform, Elzaki Transform, Kamal Transform, Abood Transform, Tarig Transform, Mohand Transform Etc. Also We Have Solved Differential Equation Of Order One With Variable Coefficients By Sadik Transform And Analysis Which Transform Can Solve This Differential Equation For Particular Value Of Coefficients A And B.
Keywords: The Sadik Transform, Laplace Transform, Sumudu Transform, Elzaki Transform, Kamal Transform, Abood Transform, Tarig Transform, Mohand Transform Etc, Differential Equation With Variable Coefficients.

1. Introduction

In Mathematics, An Integral Transform Maps An Equation From Its Domain Into Another Domain Of New Variable Where It Might Be Converted And Solved Algebraically And Easily Than In The Original Domain. Different Integral Transform Have Been Successfully Used For Two Centuries In Solving Many Problems In Applied Mathematics, Mathematical Physics, Physical Chemistry And Engineering Science. The Solution Is Back To The Original Domain Using The Inverse Integral Transform. There Are So Many Integral Transform To Solve Differential Equations, Partial Differential Equations, Integral Transform Etc. Who Claim Their Superiority Over Each Other. Recently A New Integral Transform Named The Sadik Transform Has Been Introduced By Sadikali Latif Shaikh In 2018 See [2]. The Sadik Transform Is Nothing But Generalization Of The Laplace Transform, Sumudu Transform, Elzaki Transform And All Those Integral Transforms Whose Kernels Are Of Exponential Type Or Similar To The Kernel Of The Laplace Transform. In This Chapter We Have Tried To Generalized Many Integral Transform By New Integral Transform Sadik Transform, We Tried To Prove Generality Of Sadik Transform Over Laplace Transform, Sumudu Transform, Elzaki Transform, Kamal Transform, Abood Transform, Tarig Transform, Mohand Transform Etc. In Solving Linear Differential Equation Of Homogenous And Non Homogenous Form, Sadik Transform Is Now Very Powerful Tool Because After Applying The Sadik Transform We Can Choice Whether We Proceed By Sadik Transform Or Any Other Existing, Non-Existed Integral Transforms Just By Fixing Values Of Alpha And Beta According To A Convenience And Situation Of The Problem.

2. Definitions And Theorems Of Sadik Transform:

Definition: If, (1) F(t) Piecewise Continuous On The Interval $0\le t \le A$ For Any $A>0$.

(2) $|F(t)|\le K$. When $t\ge M$, For Any Real Constant A. And Some Positive Constant K And M.

Then Sadik Transform Of F (t) Is Defined By

$$S(\upsilon^{\alpha},\beta)=S[f(t)]=\frac{1}{\upsilon^{\beta}}\int_{0}^{\infty}e^{-\upsilon^{\alpha}t}\,f(t)dt,\quad (1)\qquad for\ \mathrm{Re}(\upsilon^{\alpha})>w^{\alpha}$$

Where, υ Is Complex Variable, α Is Any Non-Zero Real Numbers, And β Is Any Real Number.

Theorem 1. If $F\left(\upsilon^{\alpha},\beta\right), k\left(\upsilon^{\alpha},\beta\right)$ Are The Sadik Transform Of Functions F (T) And K (T) Respectively And F (T) *K (T) Is A Convolution Product Then

$S\left[f(t)*k(t)\right]=\upsilon^{\beta}F\left(\upsilon^{\alpha},\beta\right).k\left(\upsilon^{\alpha},\beta\right)$; [3]

Theorem 2. If, $S_1\left(\upsilon^{\alpha},\beta\right),S_2\left(\upsilon^{\alpha},\beta\right).....S_n\left(\upsilon^{\alpha},\beta\right)$, Are Sadik Transforms Of $f_1(t),f_2(t).....,f_n(t)$ Respectively Then

$S\left[f_1(t)*f_2(t)*.....*f_n(t)\right]=\upsilon^{(n-1)\beta}S_1\left(\upsilon^{\alpha},\beta\right)S_2\left(\upsilon^{\alpha},\beta\right)......S_n\left(\upsilon^{\alpha},\beta\right)$

Theorem 3. Laplace - Sadik Transform Duality Theorem

If $L\{f(t)\}=F(s)$ Is Laplace Transform Of $f(t)$ And $S\left\{f(t)\right\}=G\left(\upsilon^{\alpha},\beta\right)$ Is A Sadik Transform Of $f(t)$ Then $G\left(\upsilon^{\alpha},\beta\right)=\upsilon^{-\beta}F(\upsilon^{\alpha})$

Theorem 4. First Shifting Theorem Of Sadik Transform,

If $S\left\{f(t)\right\}=G\left(\upsilon^{\alpha},\beta\right)$ then $S\left\{e^{at}f(t)\right\}=G\left(\upsilon^{\alpha}-a,\beta\right)$ [2]

Theorem 5.:-If $S\left(e^{at}\right)=\dfrac{\upsilon^{-\beta}}{\upsilon^{\alpha}+a}$ Then $S\left\{e^{at}t^{n}\right\}=\dfrac{n!\upsilon^{-\beta}}{\left(\upsilon^{\alpha}-a\right)^{n+1}}$

3. Generalization Of Different Transform By Sadik Transform

F(T)	Sadik Transform $S\{f(t)\}=G(\upsilon^{\alpha},\beta)$ & $S\{f(t)\}=\dfrac{1}{\upsilon^{\beta}}\int_0^{\infty}e^{-\upsilon^{\alpha}t}f(t)dt$	Laplace Transform Put $\alpha=1,\beta=0, L\{f(t)\}=F(\upsilon)$ $L\{f(t)\}=\int_0^{\infty}e^{-\upsilon t}f(t)dt$	Sumudu Trasform Put $\alpha=-1,\beta=1,\ S\{f(t)\}=G(\upsilon)$ $S\{f(t)\}=\dfrac{1}{\upsilon}\int_0^{\infty}e^{-\frac{t}{\upsilon}}f(t)dt$
1	$\frac{1}{\upsilon^{\alpha+\beta}}$	$\frac{1}{\upsilon}$	1
$[t^n]$	$\frac{\upsilon^{-\beta}n!}{\upsilon^{(n+1)\alpha}}$, N Is Positive Integer	$\frac{n!}{\upsilon^{(n+1)}}$	$n!\upsilon^{n}$
[sin(at)]	$\frac{a\upsilon^{-\beta}}{\upsilon^{2\alpha}+a^2}$	$\frac{a}{\upsilon^2+a^2}$	$\frac{a\upsilon}{1+a^2\upsilon^2}$
[cos(at)]	$\frac{\upsilon^{\alpha-\beta}}{\upsilon^{2\alpha}+a^2}$	$\frac{\upsilon}{\upsilon^2+a^2}$	$\frac{1}{1+a^2\upsilon^2}$
$[e^{(at)}]$	$\frac{\upsilon^{-\beta}}{\upsilon^{\alpha}-a}$	$\frac{\upsilon}{\upsilon-a}$	$\frac{1}{1-a\upsilon}$
[sinh(at)]	$\frac{a\upsilon^{-\beta}}{\upsilon^{2\alpha}-a^2}$	$\frac{a\upsilon}{\upsilon^2-a^2}$	$\frac{a\upsilon}{1-a^2\upsilon^2}$
[cos(at)]	$\frac{\upsilon^{\alpha-\beta}}{\upsilon^{2\alpha}-a^2}$	$\frac{\upsilon}{\upsilon^2-a^2}$	$\frac{1}{1-a^2\upsilon^2}$
$\left[J_0(at)\right]$	$\frac{1}{\upsilon^{\beta}\sqrt{a^2+\upsilon^{2\alpha}}}$	$\frac{1}{\sqrt{a^2+\upsilon^2}}$	$\frac{1}{\sqrt{1+a^2\upsilon^2}}$
$[f^{'}(t)]$	$\upsilon^{\alpha}S\left(\upsilon^{\alpha},\beta\right)-\upsilon^{-\beta}f(0)$	$\upsilon F(\upsilon)-f(0)$	$\frac{G(\upsilon)}{\upsilon}-\frac{f(0)}{\upsilon}$

$[f''(t)]$	$\upsilon^{2\alpha}S(\upsilon^{\alpha},\beta)-\upsilon^{\alpha-\beta}f(0)-\upsilon^{-\beta}f'(0)$	$\upsilon^{2}F(\upsilon)-\upsilon f(0)-f'(0)$	$\frac{G(\upsilon)}{\upsilon^2}-\frac{f(0)}{\upsilon^2}-\frac{f'(0)}{\upsilon}$
$[f^{n}(t)]$	$\upsilon^{n\alpha}S(\upsilon^{\alpha},\beta)-\sum_{k=0}^{n-1}\upsilon^{k\alpha-\beta}f^{n-k-1}(0)$	$\upsilon^{n}F(\upsilon)-\sum_{k=0}^{n-1}\upsilon^{k}f^{n-k-1}(0)$	$\frac{G(\upsilon)}{\upsilon^n}-\sum_{k=0}^{n-1}\frac{f^{n-k-1}(0)}{\upsilon^{k+1}}$
	First Shifting Theorem $S\{f(t)\}=S(\upsilon^{\alpha},\beta)$ then $S\{e^{at}f(t)\}=S(\upsilon^{\alpha}-a,\beta)$ $S\{e^{at}\sin(bt)\}=\frac{b\upsilon^{-\beta}}{(\upsilon^{\alpha}-a)^2+b^2}$	First Shifting Theorem $L\{f(t)\}=F(\upsilon)$ then $L\{e^{at}f(t)\}=F(\upsilon-a)$ $L\{e^{at}\sin(bt)\}=\frac{b}{(\upsilon-a)^2+b^2}$	First Shifting $S\{e^{at}\sin(bt)\}=\frac{b\frac{1}{\upsilon}}{\left(\frac{1}{\upsilon}-a\right)^2+b^2}$ $=\frac{b\upsilon}{(1-a\upsilon)^2+b^2\upsilon^2}$

F(T)	Elzaki Transform Put $\alpha=-1,\beta=-1, E\{f(t)\}=T(s)$ $E\{f(t)\}=\upsilon\int_0^\infty e^{-\frac{t}{\upsilon}}f(t)dt$	Abood Transform Put $\alpha=1,\beta=1\ \&\ A\{f(t)\}=K(\upsilon)$ $A\{f(t)\}=\frac{1}{\upsilon}\int_0^\infty e^{-\upsilon t}f(t)dt$	Tarig Trasform Put $\alpha=-2,\beta=1\ \&\ T\{f(t)\}=F(\upsilon)$ $T\{f(t)\}=\frac{1}{\upsilon}\int_0^\infty e^{-\upsilon^2 t}f(t)dt$
1	υ^2	$\frac{1}{\upsilon^2}$	υ
$[t^n]$	$n!\upsilon^{n+2}$	$\frac{n!}{\upsilon^{(n+2)}}$	$n!\upsilon^{2n+1}$
[sin(at)]	$\frac{a\upsilon^3}{1+a^2\upsilon^2}$	$\frac{a}{\upsilon(\upsilon^2+a^2)}$	$\frac{a\upsilon^3}{1+a^2\upsilon^4}$
[cos(at)]	$\frac{\upsilon^2}{1+a^2\upsilon^2}$	$\frac{1}{\upsilon^2+a^2}$	$\frac{\upsilon}{1+a^2\upsilon^4}$
$[e^{(at)}]$	$\frac{\upsilon^2}{1-a\upsilon}$	$\frac{1}{\upsilon^2-a\upsilon}$	$\frac{\upsilon}{1-a\upsilon^2}$
[sinh(at)]	$\frac{a\upsilon^3}{1-a^2\upsilon^2}$	$\frac{a}{\upsilon(\upsilon^2-a^2)}$	$\frac{a\upsilon^3}{1-a^2\upsilon^4}$
[cos(at)]	$\frac{\upsilon^2}{1-a^2\upsilon^2}$	$\frac{1}{\upsilon^2-a^2}$	$\frac{\upsilon}{1-a^2\upsilon^4}$
$[J_0(at)]$	$\frac{\upsilon^2}{\sqrt{1+a^2\upsilon^2}}$	$\frac{1}{\upsilon\sqrt{a^2+\upsilon^2}}$	$\frac{\upsilon}{\sqrt{a^2\upsilon^4+1}}$
$[f'(t)]$	$\frac{T(\upsilon)}{\upsilon}-\upsilon f(0)$	$\upsilon.K(\upsilon)-\frac{f(0)}{\upsilon}$	$\frac{F(\upsilon)}{\upsilon^2}-\frac{f(0)}{\upsilon}$
$[f''(t)]$	$\frac{T(\upsilon)}{\upsilon^2}-f(0)-\upsilon f'(0)$	$\upsilon^2K(\upsilon)-f(0)-\frac{f'(0)}{\upsilon}$	$\frac{F(\upsilon)}{\upsilon^4}-\frac{f(0)}{\upsilon^3}-\frac{f'(0)}{\upsilon}$

$[f^{n}(t)]$	$\frac{T(\upsilon)}{\upsilon^{n}}-\sum_{k=0}^{n-1}\frac{f^{n-k-1}(0)}{\upsilon^{k-1}}$	$\upsilon^{n}K(\upsilon)-\sum_{k=0}^{n-1}\upsilon^{k-1}f^{n-k-1}(0)$	$\frac{F(\upsilon)}{\upsilon^{2n}}-\sum_{k=0}^{n-1}\upsilon^{-2k-1}f^{n-k-1}(0)$
	First Shifting $T\{e^{at}\sin(bt)\}=\frac{b\upsilon}{\left(\frac{1}{\upsilon}-a\right)^{2}+b^{2}}$ $=\frac{b\upsilon^{3}}{(1-a\upsilon)^{2}+b^{2}\upsilon^{2}}$	First Shifting $S\{e^{at}\sin(bt)\}=\frac{b\frac{1}{\upsilon}}{(\upsilon-a)^{2}+b^{2}}$ $=\frac{b}{\upsilon\left[(\upsilon-a)^{2}+b^{2}\right]}$	First Shifting $T\{e^{at}\sin(bt)\}=\frac{b\frac{1}{\upsilon}}{\left(\frac{1}{\upsilon^{2}}-a\right)^{2}+b^{2}}$ $=\frac{b\upsilon^{3}}{(1-a\upsilon^{2})^{2}+b^{2}\upsilon^{4}}$

F(T)	Mohand Transform Put $\alpha=1,\beta=-2, M\{f(t)\}=R(\upsilon)$ $M\{f(t)\}=\upsilon^{2}\int_{0}^{\infty}e^{-\upsilon t}f(t)dt$	Kamal Transform Put $\alpha=-1,\beta=0$ & $K\{f(t)\}=G(\upsilon)$ $K\{f(t)\}=\int_{0}^{\infty}e^{-\frac{t}{\upsilon}}f(t)dt$
1	υ	υ
$[t^{n}]$	$\frac{n!}{\upsilon^{n-1}}$	$n!\upsilon^{(n+1)}$
[sin(at)]	$\frac{a\upsilon^{2}}{\upsilon^{2}+a^{2}}$	$\frac{a\upsilon^{2}}{1+a^{2}\upsilon^{2}}$
[cos(at)]	$\frac{\upsilon^{3}}{\upsilon^{2}+a^{2}}$	$\frac{\upsilon}{1+a^{2}\upsilon^{2}}$
$[e^{(at)}]$	$\frac{\upsilon^{2}}{\upsilon-a}$	$\frac{\upsilon}{1-a\upsilon}$
[sinh(at)]	$\frac{a\upsilon^{2}}{\upsilon^{2}-a^{2}}$	$\frac{a\upsilon^{2}}{1-a^{2}\upsilon^{2}}$
[cosh(at)]	$\frac{\upsilon^{3}}{\upsilon^{2}-a^{2}}$	$\frac{\upsilon}{1-a^{2}\upsilon^{2}}$
$[J_{0}(at)]$	$\frac{\upsilon^{2}}{\sqrt{a^{2}+\upsilon^{2}}}$	$\frac{\upsilon}{\sqrt{a^{2}\upsilon^{2}+1}}$
$[f'(t)]$	$\upsilon R(\upsilon)-\upsilon^{2}f(0)$	$\frac{G(\upsilon)}{\upsilon}-f(0)$
$[f''(t)]$	$\upsilon^{2}R(\upsilon)-\upsilon^{3}f(0)-\upsilon^{2}f'(0)$	$\frac{G(\upsilon)}{\upsilon^{2}}-\frac{f(0)}{\upsilon^{2}}-f'(0)$
$[f^{n}(t)]$	$\upsilon^{n}R(\upsilon)-\sum_{k=0}^{n-1}\upsilon^{k+2}f^{n-k-1}(0)$	$\frac{G(\upsilon)}{\upsilon^{n}}-\sum_{k=0}^{n-1}\frac{f^{n-k-1}(0)}{\upsilon^{k}}$
	First Shifting $M\{e^{at}\sin(bt)\}=\frac{b\upsilon^{2}}{(\upsilon-a)^{2}+b^{2}}$	First Shifting

		$S\left\{e^{at}\sin(bt)\right\}=\dfrac{b}{\left(\frac{1}{\upsilon}-a\right)^2+b^2}$ $=\dfrac{b\upsilon^2}{(1-a\upsilon)^2+b^2\upsilon^2}$

4. Main Result

Solution Of Ordinary Different Equation With Variable Coefficient By Sadik Transform And Its Generalization Over Above Integral Transform.

Theorem 6: If F(T) Is A Function Which Is Piecewise Continuous On Every Finite Interval I.E. $0 \le t \le A$ For Any A > 0 And $S\left[f\left(t\right)\right]=S\left(\upsilon^{\alpha},\beta\right)$

Then $S\left\{t.f\left(t\right)\right\}=(-1)\upsilon^{-\beta}\dfrac{d}{d\upsilon^{\alpha}}\left[\upsilon^{\beta}\ S\left(\upsilon^{\alpha},\beta\right)\right],$ (2)

Theorem 7: If F(T) Is A Function The Prove

$$S\left[t^n f\left(t\right)\right]=\left(-1\right)^n\upsilon^{-\beta}\frac{d^n}{(d\upsilon^{\alpha})^n}\left[\upsilon^{\beta}S\left(\upsilon^{\alpha},\beta\right)\right] \qquad (3)$$

Corollary 1: If $F(\upsilon)$ Be Laplace Transform Of F (T), Then By Using Sadik Transform Shows That

$$\therefore\ L(tf(t))=(-1)\frac{d}{d\upsilon}F(\upsilon)$$

Proof: Put $\alpha=1,\beta=0,L\left\{f\left(t\right)\right\}=F(\upsilon)$ In (2) We Get Expression For Laplace Transform

Corollary 2: If $F(\upsilon)$ Be Laplace Transform Of F(T), Then By Using Sadik Transform Show That

$$\therefore\ L(t^n f(t))=(-1)^n\frac{d^n}{d\upsilon^n}F(\upsilon)$$

Proof: Put $\alpha=1,\beta=0,L\left\{f\left(t\right)\right\}=F(\upsilon)$ In (3) We Get Expression For Laplace Transform

Corollary 3: If $G(\upsilon)$ Be Sumudu Transform Of F(T), Then By Using Sadik Transform Show That

$$\therefore\ S(tf(t))=\upsilon\frac{d}{d\upsilon}(\upsilon G(\upsilon))$$

Proof: Put $\alpha=-1,\beta=1,S\left\{f\left(t\right)\right\}=G(\upsilon)$ In (2) We Get Expression For Sumudu Transform,

Corollary 5: If $T(\upsilon)$ Be Elzaki Transform Of F (T), Then By Using Sadik Transform Then Show That,

$$\therefore\ E(tf(t))=\upsilon^2\frac{d}{d\upsilon}T(\upsilon)-\upsilon T(\upsilon)$$

Proof: Put $\alpha=-1,\beta=-1,E\left\{f\left(t\right)\right\}=T(\upsilon)$ In (2) We Get Expression For Elzaki Transform,

Corollary 6: If $G(\upsilon)$ Be Kamal Transform Of F (T), Then By Using Sadik Transform Then Show That,

$$\therefore\ K(t.f(t))=\upsilon^2\frac{d}{d\upsilon}G(\upsilon)$$

Proof: Put $\alpha=-1,\beta=0,K\left\{f\left(t\right)\right\}=G(\upsilon)$ In (2) We Get Expression For Kamal Transform,

Corollary 7: If $G(\upsilon)$ Be Abood Transform Of F (T), Then By Using Sadik Transform Then Show That, $\therefore\ A(tf(t))=-\frac{d}{d\upsilon}K(\upsilon)-\frac{1}{\upsilon}K(\upsilon)$

Proof: Put $\alpha=1,\beta=1,A\left\{f\left(t\right)\right\}=K(\upsilon)$ In (2) We Get Expression For Abood Transform,

Corollary 8: If $F(\upsilon)$ Tarig Transform Of F (T), Then By Using Sadik Transform Then Show That, $\therefore\ T(t.f(t))=\frac{\upsilon^3}{2}\frac{d}{d\upsilon}F(\upsilon)+\frac{\upsilon^2}{2}F(\upsilon)$

Proof: Put $\alpha=-2,\beta=1,T\left\{f\left(t\right)\right\}=F(\upsilon)$ In (2) We Get Expression For Tarig Transform,

Corollary 9: If $R(\upsilon)$ Mohand Transform Of F (T), Then By Using Sadik Transform Then Show That, $\therefore\ M(tf(t))=-\frac{d}{d\upsilon}R(\upsilon)+\frac{2}{\upsilon}R(\upsilon)$

Proof: Put $\alpha=1,\beta=-2,M\left\{f\left(t\right)\right\}=R(\upsilon)$ In (2) We Get Expression For Tarig Transform,

Bessel Equation And Sadik Transform:

Theorem 8: Prove That Solution Of Bessel Equation For Order N=0

$$t\frac{d^2x(t)}{dt^2}+\frac{dx(t)}{dt}+a^2t.x(t)=0,\quad x(0)=1\ \&\ x^{'}(0)=0$$

By Sadik Transform Is A General Transform Method Of Laplace, Sumudu, Elzaki, Kamal, Abood Transform, Tarig, Mohand Transform Etc.

Proof: - Applying Sadik Transform To $t\frac{d^2x(t)}{dt^2}+\frac{dx(t)}{dt}+a^2.t.x(t)=0,\qquad (4)$

$$S\left[t\frac{d^2x(t)}{dt^2}\right]+S\left[\frac{dx(t)}{dt}\right]+a^2S\left[t.x(t)\right]=0,$$

Applying (2) $S\left\{t.f\left(t\right)\right\}=(-1)\upsilon^{-\beta}\frac{d}{d\upsilon^{\alpha}}\left[\upsilon^{\beta}\ S\left(\upsilon^{\alpha},\beta\right)\right]$

$$\left(-1\right)\upsilon^{-\beta}\frac{d}{d\upsilon^{\alpha}}\left[\upsilon^{\beta}S\left(\frac{d^2x(t)}{dt^2}\right)\right]+S\left[\frac{dx(t)}{dt}\right]+a^2\left(-1\right)\upsilon^{-\beta}\frac{d}{d\upsilon^{\alpha}}\left[\upsilon^{\beta}S\left(\upsilon^{\alpha},\beta\right)\right]=0$$

$$\frac{d}{d\upsilon^{\alpha}}\left[\upsilon^{2\alpha}\upsilon^{\beta}S\left(\upsilon^{\alpha},\beta\right)-\upsilon^{\alpha}\right]-\upsilon^{\alpha}\upsilon^{\beta}S\left(\upsilon^{\alpha},\beta\right)+1+a^2\frac{d}{d\upsilon^{\alpha}}\left[\upsilon^{\beta}S\left(\upsilon^{\alpha},\beta\right)\right]=0\qquad (5)$$

$$\frac{d\left[\upsilon^{\beta}S\left(\upsilon^{\alpha},\beta\right)\right]}{\upsilon^{\beta}S\left(\upsilon^{\alpha},\beta\right)}=\frac{-\upsilon^{\alpha}d\upsilon^{\alpha}}{\left(\upsilon^{2\alpha}+a^2\right)}$$

Integrating Both Sides, $\log\left(\upsilon^{\beta}S\left(\upsilon^{\alpha},\beta\right)\right)=\frac{-1}{2}\log\left(\upsilon^{2\alpha}+a^{2}\right)+\log c$

$$S\left(\upsilon^{\alpha},\beta\right)=\frac{\upsilon^{-\beta}c}{\left(\upsilon^{2\alpha}+a^{2}\right)^{\frac{1}{2}}}$$

By Sadik Inverse Transform

$$x(t)=cJ_{0}\left(at\right)$$

By Initial Condition C=1 $\therefore$ Solution is $x(t)=J_{0}\left(at\right)$, (6)

Note: By Opting Different Values Of α and β In **(5)** And Above Corollaries For Different Transform We Get Equations In Terms Of Different Transform And Taking Inverse Related Transform For Bessel's Function Mention In The Table We Get Same **(6).**

For Example Choose $\alpha=1$ and $\beta=1$ In **(5)** Which Becomes Abood Transform

$$\frac{d}{d\upsilon}\left[\upsilon^{2}\upsilon K\left(\upsilon\right)-\upsilon\right]-\upsilon\upsilon K\left(\upsilon\right)+1+a^{2}\frac{d}{d\upsilon}\left[\upsilon K\left(\upsilon\right)\right]=0$$

$$\frac{d}{d\upsilon}\left[\upsilon^{3}K\left(\upsilon\right)-\upsilon\right]-\upsilon^{2}K\left(\upsilon\right)+1+a^{2}\frac{d}{d\upsilon}\left[\upsilon K\left(\upsilon\right)\right]=0$$

$$K\left(\upsilon\right)=\frac{c}{\upsilon\left(\upsilon^{2}+a^{2}\right)^{\frac{1}{2}}}$$

Using Inverse Abood Transform From Table We Get, $x(t)=cJ_{0}\left(at\right)$

Hence Solution Is True. That Shows That Sadik Transform Is General Form Of All These Transforms.

Theorem 9: Show That Sadik Transform Is General Form Of Laplace, Sumudu, Elzaki, Kamal, Abood, Tarig, Mohand Transform, And All The Transform Whose Kernel Is Of Exponential Form In Solving First Order Differential Equation Particularly With Variable Coefficient

$$a\left(t\frac{dx(t)}{dt}\right)+b\left(x(t)\right)=t,\ \text{with special condition } x(0)=0, \quad (7),$$

Proof: By Applying Sadik Transform To (7) We Get,

$$aS\left(t\frac{dx(t)}{dt}\right)+bS\left(x(t)\right)=S\left(t\right)$$

Applying (2)

$$-a\upsilon^{-\beta}\frac{d}{d\upsilon^{\alpha}}\left[\upsilon^{\beta}S\left(\frac{dx(t)}{dt}\right)\right]+bS\left(\upsilon^{\alpha},\beta\right)=\frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}}$$

$$a\upsilon^{-\beta}\upsilon^{\alpha+\beta}\frac{d}{d\upsilon^{\alpha}}S\left(\upsilon^{\alpha},\beta\right)+a\upsilon^{-\beta}\left(1+\frac{\beta}{\alpha}\right)\upsilon^{\beta}S\left(\upsilon^{\alpha},\beta\right)-bS\left(\upsilon^{\alpha},\beta\right)=-\frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}}$$

$$a\upsilon^{\alpha}\frac{d}{d\upsilon^{\alpha}}\left[S\left(\upsilon^{\alpha},\beta\right)\right]+\left[a\left(1+\frac{\beta}{\alpha}\right)-b\right]S\left(\upsilon^{\alpha},\beta\right)=-\frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}}$$

Solution Of (7) Exist Only If, $b=a\left(1+\frac{\beta}{\alpha}\right)$ (8)

$$\therefore\frac{d}{d\upsilon^{\alpha}}\left[S\left(\upsilon^{\alpha},\beta\right)\right]=-\frac{\upsilon^{-\beta}}{a\upsilon^{3\alpha}}$$

Integrating With Respect To $d\upsilon^{\alpha}$ We Get, $S\left(\upsilon^{\alpha},\beta\right)=\left(\frac{\alpha}{a\left(2\alpha+\beta\right)}\right)\frac{\upsilon^{-\beta}}{\upsilon^{2\alpha}}$

By Inverse Sadik Transform We Get Solution Of (7), $x(t)=\left(\frac{\alpha}{a\left(2\alpha+\beta\right)}\right)t$ (9)

By Choosing Values Of α and β In (8) And (9) According To The Table For Different Transforms Above We Get Condition For Solution And Solution For (7). There For Sadik Transform Is General Form Of All Mentioned Transform.

Corollary 10: Show That Solution Of (7) Exist By Laplace Transform Only If A=B And Solution Is $x(t)=\frac{t}{2a}$

Proof: Put $\alpha=1,\beta=0$ and $L\left\{f\left(t\right)\right\}=F(\upsilon)$ In (8) & (9) We Get B=A Solution As $x(t)=\frac{t}{2a}$

Corollary 11: Show That Solution Of (7) Exist By Sumudu Transform Only If B=0 And Solution Is $x(t)=\frac{t}{a}$

Proof: Put $\alpha=-1,\beta=1$ & $S\left\{f\left(t\right)\right\}=G(\upsilon)$ In (8) & (9) We Get B=A Solution As $x(t)=\frac{t}{a}$

Corollary 12: Show That Solution Of (7) Exist By Elzaki Transform Only If B=2a And Solution Is $x(t)=\frac{t}{3a}$

Proof: Put $\alpha=-1,\beta=-1,E\left\{f\left(t\right)\right\}=T(\upsilon)$ In (8) & (9) We Get B=2a Solution As $x(t)=\frac{t}{3a}$

Corollary 13: Show That Solution Of (7) Exist By Kamal Transform Only If B=A And Solution Is $x(t)=\frac{t}{2a}$

Proof: Put $\alpha=-1,\beta=0,K\left\{f\left(t\right)\right\}=G(\upsilon)$ In (8) & (9) We Get B=A Solution As $x(t)=\frac{t}{2a}$

Corollary 14: Show That Solution Of (7) Exist By Abood Transform Only If B=2a And Solution Is $x(t)=\frac{t}{3a}$

Proof: Put $\alpha=1,\beta=1,A\{f(t)\}=K(\upsilon)$ In (8) & (9) We Get B=2a Solution As $x(t)=\dfrac{t}{3a}$

Corollary 15: Show That Solution Of (7) Exist By Tarig Transform Only If $b=\dfrac{a}{2}$ And Solution Is $x(t)=\dfrac{2t}{3a}$

Proof: Put $\alpha=-2,\beta=1,T\{f(t)\}=F(\upsilon)$ In (8) & (9) We Get $b=\dfrac{a}{2}$ Solution As $x(t)=\dfrac{2t}{3a}$

Corollary 16: Show That (7) Cannot Be Solved In Any Condition By Mohand Transform.

Proof: Put $\alpha=1,\beta=-2,M\{f(t)\}=R(\upsilon)$ In (8) & (9) We Get B= -A But Solution Does Not Exist. Because Applying Integration Before Inverse Mohand Transform We Get Term In Logarithmic Form For Which Inverse Mohand Transform Does Not Exit.

Note: We Can Cross Check All These Condition By Respective Transform Formulae.

5 Conclusions:

In This Chapter We Have Tried To Prove Generality Of Sadik Transform Over Laplace Transform, Sumudu Transform, Elzaki Transform, Kamal Transform, Abood Transform, Tarig Transform, Mohand Transform Etc. We Have Formulate All Transform In One Table By Putting Different Valued Of α & β. Also We Have Derived Equation For Solution Of Homogeneous And Non Homogeneous Higher Order Differential Equation With Constant Coefficient And Showed Generality Of Sadik Transform Over Laplace, Sumudu, Elzaki, Kamal, Abood, Tarig, Mohand Transform By Demonstrating Different Values Of N.

Also We Have Successfully Proved Different Result Related To $t^n f(t)$ Of Mentioned Transform By Using Sadik Transform Of Multiplication $t^n f(t)$ We Are Unable To Solve (7) By Most Renowned Methods That Are Sumudu, Elzaki, Abood, Mohand, Tarig If A=B. Simultaneously Sadik, Laplace And Kamal Transform Are Able To Solve (7) If A=B . Also We Conclude A General Idea That (7) Is Solvable For A=B Only By Those Transform In Which $\beta=0$

By Theorem (9) We Conclude That Sadik Transform Is General Transform Of All Those And Whose Kernel Is Exponential Form Which Are Not Defined In Research Yet.

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