Research Article

A Historical overview of Four Color Problem

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Abstract: Graph Theory is one of the trending domain of mathematics which has a great importance in the modern technology. The first concept about the Four Color problem was given by A.F.Mobius in his innovative speech in the year 1840. After that there began war of thoughts to solve the Four Colour problem. It was in the year 1852 when a genius student namely Francis Gutherie from university of London came with a positive solution but failed while applying the same in the four color problem. His brother Fradric himself convinces that Francis was not satisfactory with the solution. Later many mathematicians like Kempe, Heawood, and Tait etc. tried their best to solve the problem but were unsuccessful. Finally, it was the year 1976 when Kenneth Appel and Wolfgang Haken proved conclusively the problem by means of computer.

Key Words: - Graph Theory, Four Colour, Innovative, Conclusively, Satisfactory.

Introduction

The historical perspective of graph theory is believed to be originated from an innovative speech delivered by A.F. Mobius by presenting the four-color problem in 1840. During antiquity the concept of map drawing and coloring was considered as an art but after this graphical representation there emerged a connection between map coloring and mathematics. This inventive concept was developed in 1852 by a genius student namely Francis Guthrie enrolled in the University of London who declared this pioneering concept to his mathematics mentor Augustus De Morgan that he was busy in coloring many maps of English counties and detected that each and every map he had attempted; no matter how intricate and complicated, could be colored with four colors. Moreover, the destinations and regions sharing a common border must be colored with different colors. Augustus De Morgan got impressed and examined this factual novel concept, with his contemporaries and intellectual giants of mathematics of that time, including his bosom friend and famous mathematician William Rowan Hamilton. The notion and the new factual idea that any map could be colored with just four colors, seems rational and so simple that everyone assumed it could be easily proved mathematically.

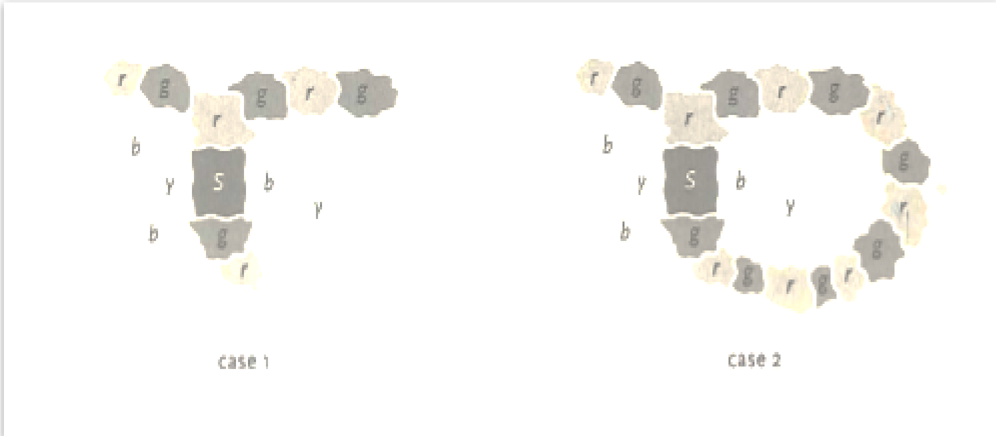
This new idea of four-color conjecture which clarifies that every map can be colored with four colors seemed not much interesting and innovative to the mathematician Hamilton. This domain of mathematical wisdom was almost forgotten till the 1860’s when a mathematician C.S. Peirce presented with an evidence in a seminar held at Harvard. The conjecture achieved fame and prominence in 1878 when Arthur Cayley, in a meeting of London Mathematical Society, asked whether the theory seems logical and settled the same. Shortly after that, Cayley with his originality and subjectivity published a concise and short description elaborating the four color conjecture problem, duly pointing out the excellencies and faults. After that, there was another innovation and emerged in the newly published American Journal of Mathematics that gave an authentic proof of the four- color conjecture. The proof and innovative concept came from the mind of A. B. Kempe’s which survived and revolutionized the mathematical world for more than ten years but later there was found an error in the novel concept of A.B. Kempe . In 1880 P.G. Tait, a well acclaimed and known Professor of Natural philosophy in Edinburgh, came out with his authentic proofs of the four color conjecture, and figured out that, “the edges of every cubic map can be colored with only three colors in a way that the three meeting point of the edges at each vertex are assigned with different colors” (729)

The year 1890 has also a worth as another mathematician P.J. Haewood published a vital and important treatise by rebuffing and refuting Kempe’s proof regarding four color conjecture and he with his new innovation and idea proved that, “the map of every country can be properly colored with not only four colors but by five colors” (205).

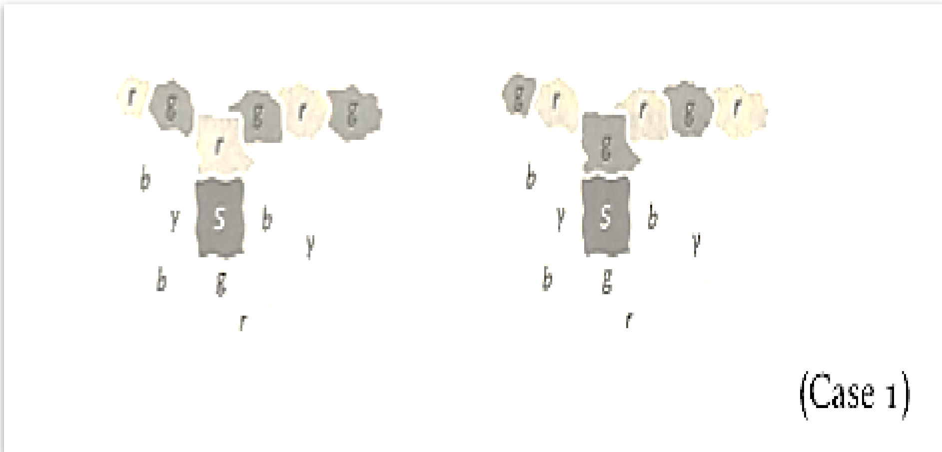
Kempe’s proof of the four color theorem.

Proof. As it is evident that there exists a minimal graph which is not four colourable, thus we can say that every smaller graph can be four colored. We will use the colors: red, green, blue and yellow in order to color the graphs. Is it mandatory that we have to use a fifth color? Thus the fifth color we use will be orange color. We will use to cover upon the face partition of the graph, that will be equivalent and will shrink at a point. Since the graph contains less faces as compared to our minimal criminal, which might be four colored. Thus after using the Euler’s formula, we can use only five neighbours theorem, to guarantee one square (having face with four neighbours) or pentagon (having face with five neighbours) exists at least in the graph. Is it possible that there be a face with lesser than four neighbours, for example like a triangle (having face with three neighbours) or we can use an example of a digon (which contains face with two neighbours)? Thus to apply this procedure we need to shrink the graph, then color the graph and then replace the face. Although it is evident that there exists only three or less neighbours to this face, so we will use the remaining colors in order to color the face.

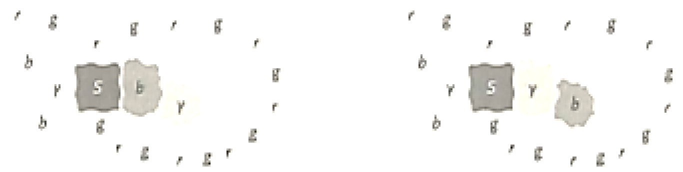
While applying the procedure on the Square. We clearly shrink it down If our minimal criminal contains a Square and thus the rest of the graph is colored, and then we will apply to replace the face. Let us take a Supposition that the colors green and red are not adjacent. By adjacent we mean, “two regions share a common boundary curve segment, not merely a corner where three or more regions meet” (Gonthier, 1388). There arise two cases, either the Kempe chain is connected by green and red face, or they are not. A Kempe chain can be defined as a chain of faces having only two colors, which is illustrated in a figure of Red-Greeen Kempe Chain. When we tend to start with a red face, we can clearly check that the chain contains that face, every green face adjacent to red face, every red face adjacent to any of the green faces and so on the chain continues to progress.



If the procedure given by the Kempe is applied on red and green combination which is not connected by a Kempe chain (case 1) then after applying the red-green Kempe chain which is connected to only one of the faces (but not connected to the both) and reverse the process of the colors upon the chain, we may see that every red face becomes green and conversely, then there remains only three colors which are neighboring to the square and thus the square can be colored with the remaining fourth color.



Now let us suppose they are linked through a (case 1) Kempe chain in which we yield the blue yellow Kempe chain which is connected to the blue or yellow face that is neighbouring to the square (but here the case is not both) and thus replace the colors of that mentioned chain. This clearly defines that we now possess only three colors neighbouring to that square and so we can utilize the fourth color in order to color the square. Since all possible cases containing a square, a minimal criminal cannot contain a square have been dealt to find out the link of the Kempe Chain.



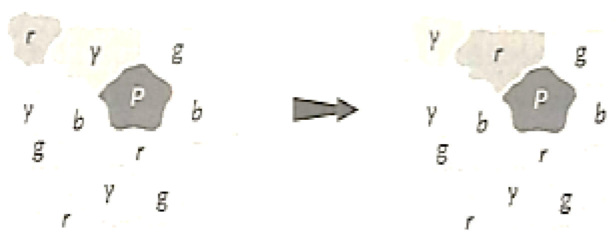
(Case 2)

The Pentagon. If we apply the procedure of kempe chain on a pentagon, at that instance we shrink the pentagon down, and then color the remaining graph, and then try to replace the face. Thus there is an occurance to two cases, now here we are applying the procedure on a pentagon and we have to use only four colors in order to color the faces neighboring to it. Here we have to do that, two of the faces must have the same color after the coloring procedure is applied. Although it is evident that two neighbouring faces must not have the same color and there should be a face between the two faces with the same color, here we assume that two faces which are adjacent to the pentagon can be colored blue, as shown in the below figure, alike as the face in between the blue faces should be colored red.

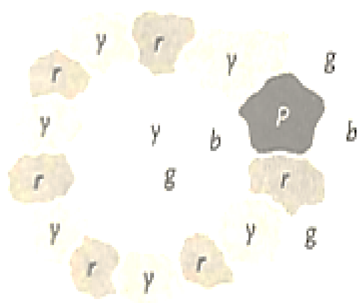
Case1: At most one of the two following statements are true: “The red and green faces are connected by a Kempe chain.” and “The red and yellow faces are connected by a Kempe chain.”

If this is the case, then first consider the yellow and red faces adjacent to the pentagon. If the yellow-red part connected to the red face adjacent to the pentagon does not link up with the

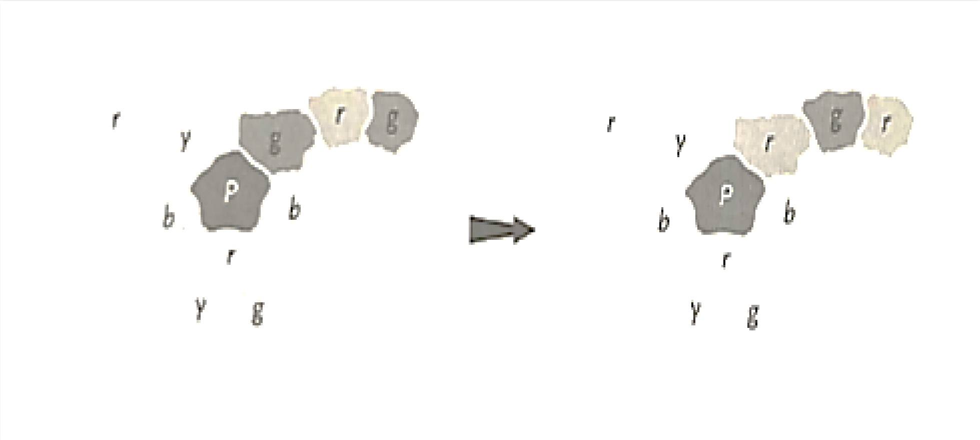
yellow-red part connected to the yellow face adjacent to the pentagon, then we can simply interchange the colors of one of these chains. This ensures that one color is now available to use on one of the faces adjacent to the pentagon, and thus gives us a four coloring



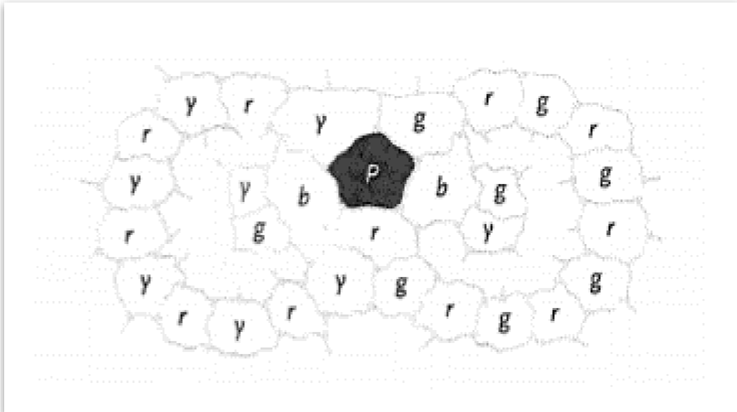
If the yellow and red faces are connected by a yellow-red chain, as shown here



We can be sure here that the green and the red faces are not connected. We simply interchange the colors of one of green red chain. This guarantees that there exists one color such that no face adjacent having the same color. In this way we can color the pentagon with that one. This method gives us a four coloring of the graph.

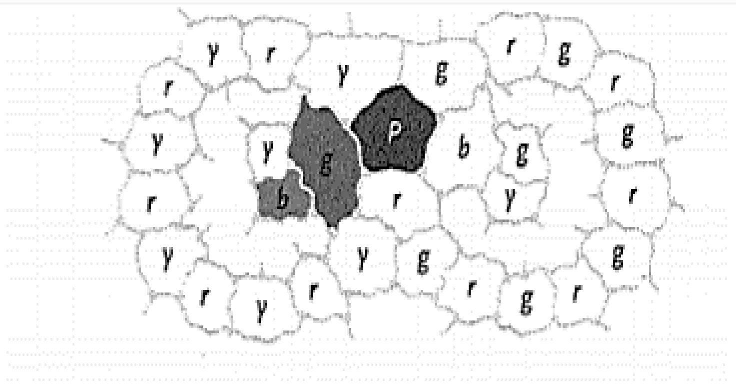


Case 2: Above the both statements “The red and green faces are connected by a Kempe chain.” and “The red and yellow faces are connected by a Kempe chain.” Which is true.



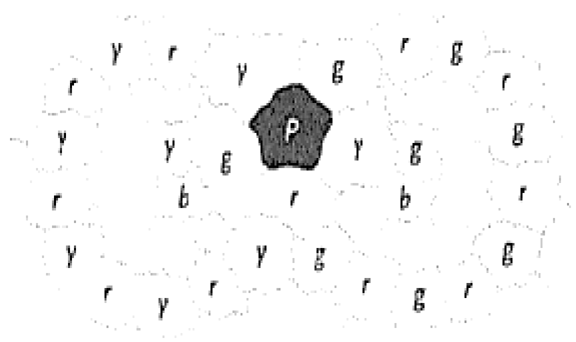
Now we will try to apply the interchanging of Kempe Chain, then it is obvious that we take the green-blue chain which should be connected to the blue face which is not neighbouring to the face and that is green and the blue-yellow chain that is connected to the blue face which is not adjacent to the yellow face. Thus this interchanging of Kempe Chain is also illustrated in the below figure.

(Interchanging the blue-yellow Kempe chain)



(Interchanging the blue-green Kempe chain)

Thus it is obvious that the adjacent edges of the Pantagon does not contain a blue color. Now in order to solve the four color problem we can use the blue colour to justify our statement.



All the possible assumptions have been properly analysed and verified and thus four color problem has been solved by Kemp.

The proof seems to be false was stated earlier in 1889 by Percy John Heawood, “who had succeeded to find a counter example who clearly managed to find out that Kempe’s method was faulty and full of errors” (335). This problem which makes the Kempe proof as false arises when dealing with the pentagon. Heawood through his intellectuality found that there exists the problem when are replacing the main colors of two Kempe chains simultaneously, thus when we replace and invert the colors of one chain we are certain that it will never interfere with the graphical coloring, but when at the same time inverting two chains, at that time we do not have a guarantee that this will not affect and interfere with the coloring of the graph. This method gave rise to a new theorem called as five color problem in the realm of Math’s.

After 100 years the concept of four color problems revolutionized the mathematical world as a difficult notion and riddle as whosoever attempted to find out the proof failed to attempts a genuine and logical proof. But the mathematical brains were always busy in thinking about this problem and find a proper solution. It was the year 1976 when Kenneth Appel and Wolfgang Haken came with a solution Four-Color Conjecture by hailing from the University of Illinois. Yes, indeed, Guthrie was right: Every map can be colored with four colors or less. Though the problem seems very short and easy but the solution to this simple question came out with unending valour and hard work and it was solved on 500 pages and about one thousand hours of computer time. Thus the four color problem which was solved opened different avenues in the field of mathematics and more in the field of graph theory.

Conclusion

Thus the four color problem was at last solved by using the computer technology. The computer proof given by Appel and Haken is too long and contains hundreds of textual pages and diagrams. It is still the case that that mathematicians are familiar with and most comfortable with a paper proof that consists of sequence of logical steps recorded on a piece of paper. I am sure that the mathematicians in future will solve the riddle with the paper proof that should be less hectic and should not use the terminology of computer science.

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