Research Article

Products Of $\Gamma-$ Reset Fuzzy Automata

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Abstract

we study the Characterization of $\Gamma-$ reset FA through their products. We prove that the direct product and restricted direct product of $\Gamma-$ reset FA is $\Gamma-$ reset FA.

Key words: $\Gamma-$ reset FA, direct product, restricted directed product. AMS Mathematics subject classification: 03D05,20M35,18 B20,68Q45, 68Q70, 94A45.

1 Introduction

Fuzzy sets was introduced by Zadeh in 1965[6]. The fuzzy automaton was ntroduced by Wee [5]. Directable automata is also known as synchronizable, reset automata. $\mu-$ necks of FA, generalized products of directable FA were discussed in [2,3]. $\Gamma-$ reset FA was introduced in [1]. We prove that the direct product, restricted direct product of $\Gamma-$ reset FA is $\Gamma-$ reset FA.

2. Related Work

Automata theory is a fundamental subject known and used by everyone, this theory has formed a part of the intellectual landscape for so long that it is no longer noticed. It is also called as the study of abstract computing devices. In the 1930's, Turing designed a prototype of a machine which is widely known as computer, today. Turing's goal was to describe precisely the boundary between what a computing machines could do and what it could not do. The conclusions apply not only to his abstract Turing machines but to today's real machines.

Theories of fuzzy sets introduced by [1] and rough sets introduced by [2] are generalizations of classical set theory for modeling vagueness and uncertainty. [3] has shown that both the theories model different types of uncertainty. The fuzzy set theory deals with the ill-definition of the boundary of a class through a generalization of set characteristic functions, while the rough set theory takes into consideration the indiscernibility between objects, which is typically characterized by an equivalence relation. The indiscernibility between objects is not used in fuzzy set theory.

During the recent years, the researchers began to work on fuzzy automata with membership values in complete residuated lattices, lattice-ordered monoids and some other kind of lattices [4]. It is therefore quite natural to wonder if a similar program can be carried out for such fuzzy automata. Somewhat surprisingly, our investigations show that several results related to the algebraic properties of fuzzy automata introduced in [5] may not hold well in the case of fuzzy automata with membership values in lattice ordered monoids.

Making decisions about processes that contain uncertainity in natural language has been shown to be less than perfect. The idea proposed by [6] suggested that set membership was the key to decision making when faced with uncertainity. The sets on the universe X that can accommodate degrees of membership were termed as fuzzy sets.

[7] studied about group semi automata. A special feature of the radical theory of group semi automata is the existence of complimentary radical and semisimple classes. The concept of T-fuzzy semiautomata, T-fuzzy subsemiautomata and T-fuzzy kernel were introduced by [8] using the notion of group semiautomata. [9] uses a T-norm, introduced the notion of idempotent T-fuzzy subquasi groups of quasi groups, and investigated some of their properties. They also constructed T-fuzzy subquasi groups in the finite direct product of quasi groups.

[10] gave the definition of a fuzzy subgroup with threshold of a group. By using implication operators of fuzzy logic, they also defined an implication based fuzzy subgroup and discussed the relation between two fuzzy subgroups. [11] investigated fuzzy automata, which gave a great diversity of growth patterns that had sensitivity to background conditions which maintained the symmetry of snowflakes. The automata use simple local arithmetic combinations, depending upon the neighbourhood configuration, so that the 6-fold symmetry persists. They were

interested in exploring the diversity of growth patterns that 13 these automata can generate. In 2004, a new class of Mealy type of fuzzy finite automata was described and its state wise equivalence relations were defined by [12]. From the equivalence relations its state wise minimization algorithm was obtained. They have classified fuzzy finite automata into two kinds of basic models one has initial states and no outputs; the other has outputs and no initial states.

Fuzzy set theory which is a generalization of conventional set theory was proposed by Lofti A. Zadeh in 1965 with his seminal paper 'Fuzzy Sets' . Fuzzy set provides a simple mathematical tool to represent vagueness, uncertainty and imprecision inherently present in day to day life. Fuzzy Logic provides a simple way to arrive at a definite conclusion based on vague, ambiguous, imprecise, noisy or missing input information. Since 1965, fuzzy set theory has witnessed enormous development by several researchers. Fuzzy logic based applications range from consumer products and industrial systems to biomedicine, decision analysis, information sciences and control engineering. In short, there is no field in which fuzzy logic has not made its impact. All concepts of Mathematics based on set theory have been extended to fuzzy sets. There exist several text books which deal with the basic concepts of fuzzy sets and their applications.

[13] introduced fuzzy mealy machine with output associated with each transition. They also introduced the concept of interval partition and semigroups of fuzzy mealy machines and have derived more general results.

In a very interesting paper, [14] has shown that a fuzzy automaton can be expressed as Advances in Fuzzy Automata Literature Survey. The NFAs in the chain are determined by different α-cut values of the fuzzy language accepted by the given fuzzy automaton. This paper deals with two categories namely category of fuzzy automata with fuzzy morphisms and category of chains of non-deterministic automata with morphisms. The principal result of this paper is that these two categories are isomorphic. The conclusion of this paper is that instead of fuzzy automaton, one can deal equivalently with a chain of non-deterministic automata.

[15] studied regular fuzzy expressions, fuzzy automata and their applications to decision processes. [16] established that a fuzzy language is recursive if and only if it is generated by a fuzzy grammar.

[17] has introduced a max-min fuzzy language and has found a fuzzy automaton that generates this language. [18] introduced the concept of lattice valued regular expressions and proved the equivalence between lattice-valued regular expressions and lattice-valued finite automata (LVA).

Algebraic properties such as union, intersection, Kleene-closure and complement of LVA-languages have been studied [19].

In [20], applications of fuzzy languages and fuzzy automata in the areas of pattern recognition, intelligent information retrieval, intelligent selective packet discarding, syntactic pattern recognition, fuzzy signal processing, fault diagnosis, intelligent target tracking and fuzzy lexical analysis have been discussed with illustrative examples by different authors. In [21], concepts related to minimization of fuzzy automaton such as reduction, behaviour, state equivalence, quotient machines and minimization of fuzzy automata have all been dealt by different authors.

2 Preliminaries

2.1 Fuzzy automata [4]

A FA is $F = (S, A, \eta)$ where S - set of states $\{s_1, s_2, \ldots\ldots s_n\}$

$A -$ alphabets $\eta -$ function from $S \times A \times S \rightarrow [0,1]$

$\eta(s_i, a, s_j) = \nu \; 0 \leq \nu \leq 1$

2. 2 $\Gamma -$ reset FA[1]

Let $F = (S, A, \eta)$ be a FA. FA is $\Gamma -$ -reset FA at s_l if there exist a real number Γ with $0 \leq \Gamma \leq 1$ and a string $g \in A^*$, for all s_i of S into s_l such that $\eta(s_i, g, s_l) \geq \Gamma$.

3 Products of $\Gamma -$ Reset Fuzzy Automata

3.1 Definition

Let $F_k = (S_k, A_k, \eta_k), k = 1,2$ be FA. The D.P of F_1 and F_2 is denoted by $F_1 \times F_2$ and is defined by $F_1 \times F_2 = (S_1 \times S_2, A_1 \times A_2, \eta_1 \times \eta_2)$

Where $(\eta_1 \times \eta_2)\left((s_i, s_j), (a_1, a_2), (s_k, s_l)\right) = \eta_1(s_i, a_1, s_k) \wedge \eta_2(s_j, a_2, s_l)$

$\forall (s_i, s_j), (s_k, s_l) \in S_1 \times S_2, (a_1, a_2) \in A_1 \times A_2$

3. 2 Definition

Let $F_k = (S_k, A, \eta_k), k = 1,2$ be FA. The R.D.P of F_1 and F_2 is denoted by $F_1 \wedge F_2$, defined by $F_1 \wedge F_2 = (S_1 \times S_2, A, \eta_1 \times \eta_2)$ where

$$(\eta_1 \wedge \eta_2)\left((s_i, s_j), b, (s_m, s_l)\right) = \eta_1(s_i, b, s_m) \wedge \eta_2(s_j, b, s_l)$$

$$\forall (s_i, s_j), (s_m, s_l) \in S_1 \times S_2, \forall b \in A$$

Theorem 3.1 Let $F_k = (S_k, A, \eta_k), k = 1,2$ be FA. Let $F_1 \times F_2$ is the D.P of FA F_1 and F_2. Then $\forall (s_i, s_j), (s_m, s_l) \in S_1 \times S_2$, and $\forall (y_1, y_2) \in (A_1 \times A_2)^* (\eta_{1^*} \times \eta_{2^*})\left((s_i, s_j), (y_1, y_2), (s_m, q_l)\right) = \eta_{1^*}(s_i, y_1, s_m) \wedge \eta_{2^*}(s_j, y_2, s_l)$

Proof.

Let $y_1 \in A_1^*, y_2 \in A_2^*$ and $|y_1| = |y_2| = c$. Then the result is true for $c = 1$. Suppose the result is true for all $u_1 \in A^*, u_2 \in A^*$ and $|u_1| = c - 1 = |u_2|, c > 1$. Let $y_1 = a_1 u_1; y_2 = a_2 u_2$, where $a_1 \in A_1, a_2 \in A_2$ and $u_1 \in A_1^*, u_2 \in A_2^*$. Then

$$(\eta_1 \times \eta_2)^* \left((s_i, s_j), (a_1 u_1, a_2 u_2), (s_m, s_l)\right)$$

$$(\eta_1 \times \eta_2)^* \left((s_i, s_j), (a_1 u_1), (s_m, s_l)\right) \wedge (\eta_1 \times \eta_2)^* \left((s_i, s_j), (a_2 u_2), (s_m, s_l)\right)$$

$$= \left\{\vee \left\{(\eta_1 \times \eta_2)\left((s_i, s_j), a_1, (s_s, s_t)\right) \wedge (\eta_1 \times \eta_2)^*\left((s_s, s_t), u_1, (s_m, s_l)\right) \mid (s_s, s_t) \in S_1 \times S_2\right\}\right\} \wedge \left\{\vee \left\{(\eta_1 \times \eta_2)\left((s_i, s_j), a_2, (s_u, s_v)\right) \wedge (\eta_1 \times \eta_2)^*\left((s_u, s_v), u_2, (s_k, s_l)\right) \mid (s_u, s_v) \in S_1 \times S_2\right\}\right\}$$

$$= \left\{\vee \{\eta_1((s_i, a_1, s_s) \wedge \eta_1^*(s_s, u_1 s_k) | s_s \in S_1\}\right\} \wedge \left\{\vee \left\{\eta_2\left((s_j, a_2, s_u\right) \wedge \eta_2^*(s_u, u_2 s_l) \mid s_u \in\right.\right.$$

$$= \eta_1^*(s_i, a_1 u_1, s_k) \wedge \eta_2^*(s_j, a_2 u_2, s_l)$$

$$= \eta_1^*(s_i, y_1, s_k) \wedge \eta_2^*(s_j, y_2, s_l)$$

Theorem 3.2

Let $F_k = (S_k, A, \eta_k), k = 1,2$ be FA. Let $F_1 \wedge F_2$ is the R.D.P of F_1 and F_2. Then $\forall (s_i, s_j), (s_m, s_l) \in S_1 \times S_2$, and $\forall y \in A^*$

$$(\eta_1 \wedge \eta_2)^* \left((s_i, s_j), y, (s_m, s_l)\right) = \eta_{1^*}(s_i, y, s_k) \wedge \eta_{2^*}(s_j, y, s_l)$$

Proof.

The proof of the result is follows by induction on $|y| = c$. If $c = 1$ then the result is obivious. Suppose the result is true for all $y \in A^*$. Let $y = bv$, where $b \in A, v \in A^*$ and $|v| = c - 1, c > 1$. Then $(\eta_1 \times \eta_2)^* \left((s_i, s_j), y, (s_k, s_l)\right) = (\eta_1 \times \eta_2)^* \left((s_i, s_j), bv, (s_m, s_l)\right)$

$$= \vee \left\{(\eta_1 \times \eta_2)\left((s_i, s_j), b, (s_s, s_t)\right) \wedge (\eta_1 \times \eta_2)^*\left((s_s, s_t), v, (s_k, s_l) \mid (s_s, s_t) \in S_1 \times S_2\right)\right\}$$

$$= \vee \left\{\eta_1(s_i, b, s_s) \wedge \eta_2(s_j, b, s_t) \wedge \eta_1^*(s_s, v, s_k) \wedge \eta_2^*(s_t, v, s_l) \mid (s_s, s_t) \in S_1 \times S_2\right\}$$

$$= \eta_1^*(s_i, bv, s_k) \wedge \eta_2^*(s_j, bv, s_l)$$

$$= \eta_1^*(s_i, y, s_k) \wedge \eta_2^*(s_j, y, s_l)$$

Theorem 3. 3

The D.P of $\Gamma -$ reset FA is $\Gamma -$ reset FA.

Proof.

Let $F_k = (S_k, A, \eta_k), k = 1,2$ be $\Gamma -$ reset FA. Then the D.P of F_i is given by $F_1 \times F_2 = (S_1 \times S_2, A_1 \times A_2, \eta_1 \times \eta_2)$.

Define $\eta_1 \times \eta_2 : (S_1 \times S_2) \times (A_1 \times A_2) \times (S_1 \times S_2) \to [0,1]$.

Since F_1 and F_2 are $\Gamma -$ reset FA then there exists $\Gamma -$ reset strings $w_i \in A^*, i = 1{,}2$ and the states $s_m \in S_1$ and $s_l \in S_2$ such that

$\eta_1^*(s_i, w_1, s_m) > 0$ and $\eta_2^*(s_j, w_2, s_l) > 0$

Now,
$$\left(\eta_1^* \times \eta_2^*\right)\left(\left(s_i, s_j\right), w, \left(s_m, s_l\right)\right) > 0 \Leftrightarrow \left(\eta_1^* \times \eta_2^*\right)\left(\left(s_i, s_j\right), w_1 w_2, \left(s_m, s_l\right)\right) > 0$$
$$\Leftrightarrow \eta_1^*(s_i, w_1, s_m) \wedge \eta_2^*\left(s_j, w_2, s_l\right) > 0$$

Hence $F_1 \times F_2$ is $\Gamma -$ reset FA.

Theorem 3.4

The R.D.P of $\Gamma -$ reset FA is $\Gamma -$ reset FA.

Proof.

Let $F_k = \left(S_k, A, \eta_k\right), k = 1{,}2$ be $\Gamma -$ reset FA. Then the R.D.P of F_i is given by

$F_1 \wedge F_2 = \left(S_1 \times S_2, A, \eta_1 \times \eta_2\right)$

Define $\eta_1 \times \eta_2 \colon (S_1 \times S_2) \times A \times (S_1 \times S_2) \to [0,1]$. Since F_1 and F_2 are $\gamma -$ reset FA then there exists $\Gamma -$ reset strings $w \in A^*$, and the states $s_m \in S_1$ and $s_l \in S_2$ such that

$$\eta_1^*(s_i, w, s_m) > 0 \text{ and } \eta_2^*\left(s_j, w, s_l\right) > 0$$

Now,
$$\left(\eta_1^* \times \eta_2^*\right) \quad \left(\left(s_i, s_j\right), w, \left(s_m, s_l\right)\right) > 0 \Leftrightarrow \left(\eta_1^* \times \eta_2^*\right)\left(\left(s_i, s_j\right), w, \left(s_m, s_l\right)\right) > 0$$
$$= \eta_1^*(s_i, w, s_k) \wedge \eta_2^*\left(s_j, w, s_l\right) > 0$$

Hence $F_1 \wedge F_2$ is $\Gamma -$ reset FA.

4 Conclusion

In this paper, we study the properties of $\Gamma -$ reset FA. We prove that direct product and restricted direct product of $\Gamma -$ reset FA is $\Gamma -$ reset FA.

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