Image Enhancement and Reduction of Computational Complexity Using Various Interpolation Techniques

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Abstract:
This paper contains Image interpolation techniques which are frequently requisite from clinical image to satellite image. Since various ideal interpolation functions are defined in spatial domain with unconstrained functions, interpolation kernels of finite size should introduce necessarily. B - Spline Interpolation Methods and Quadratic B - spline functions are such kernels and truncate ideal interpolator and produces phase distortion in the output and re-sampled images. This decreases the computing time and increases the quality of re-sampled images. Further these re-sampled images are enhanced by cubic, sinc, linear, B-spline, ideal, adaptive and quadratic interpolation methods. Cubic Spline interpolations with minimal distortion of the sub-sampled images are enhanced and reduced the computational complexity. The quadratic spline interpolator resulting from a quadratic B-spline weighted function is decomposed by manipulating the weighted sum of quadratic B-spline weight functions. Subsequently by using spatial analysis, the images are assessed its chromatic eminence value, error identification and reduction of error, reduction of computational complexity, and run time measurement.

Key words: Image re-sampling, B – Spline, Quadratic B - Spline, Interpolation.

I INTRODUCTION

In this paper we proposed novel recursive image interpolation concepts that support the B-spline quadratic algorithm with limited frequencies [11,12,13,14,15]. In image processing and enhancing techniques, general recursive interpolation preserves the primary edges of the image and this interpolation technique does not create any additional noise and does not destroy the smoothness of the image [7, 9]. To explore this in further, various adaptive interpolations techniques like recursive interpolation algorithm are used in image processing and image enhancement which leads to consistent enhanced image with the formation of directional pattern of the edges of the input image, and this greatly increase the interpolation performance [2]. Our investigational result reveals that it produces higher quality in enhancing and classifying the image in various fields [14].

This paper is organized as follows: In section II, the ideal interpolator and B-spline interpolator derived the exact value of interpolator changes from positive value to negative value at various points in a particular domain. Section III provides adaptive and quadratic spline interpolation in a recursive manner. Convolution of infinite impulse and direction of rotation of the image and its reconstructed image from its original image was arrived. Section IV focuses the sinc and cubic interpolation operation which provides reconstruction of the images. There are two common approaches are mentioned to proceed further by truncating and making windows with a windowing operation $w(x)$. Finally section V concludes the paper.

II Ideal and B - Spline interpolation

The ideal interpolator deals with the hypothetical and theoretical exploration to explicate the interpolator in the image enhancement. Basic properties of any interpolator are derived from this ideal interpolation with convolution operation [4]. The exact value of interpolator changes from positive value to negative value at various points in a particular domain and these nodes referred as zero crossing points. If the interpolators meet the subsequent condition along with this zero crossing points, and it is anticipate smoothing and maintain the high frequency even in the edges of the image. Initially, the ideal interpolator function assumes the following values:

$h(0) = 1,\ h(x) = 0,\ |x| = 1,2,3, \ldots$ (1)

In general, the ideal interpolator is spatially restricted and the common approaches to proceed through ideal interpolators are windowing and truncation method. Truncation method produces interpolator ringing (Gibbs) effects when it is recursively repeated and either it may be a signal noise or image noise and interpolate as a result of Gibbs growth or decay within the frequency domain when the discontinuities cause. Windowing method is the way to multiply the best interpolator with less rigid and less redundancy, hence windowing method is most popular over the truncation technique [1].

B - Spline interpolations measures are typically employed in the family of spline functions. B - splines of order n measures piecewise unitary polynomial functions of degree n. These functions are produced $n - 1$ differentiable units to ensure its convergence. Any continuous polynomial of degree n is linearly combined into piecewise and in addition differentiable to $n - 1$ times, these results a schematic exploitation of B-spline functions of the same order [5]. In the case of regular separation between the nodes, then the convolution operation is schematic at various periodic intervals [3]. Now define:

$$\phi^n(x) = \sum_{k=-\infty}^{\infty} c_n(k) * \beta^n(x-k) \quad (2)$$

where $\beta^n(x)$ is the n-th order B-spline coefficient function; n be a degree of piecewise polynomials connected by edges of the images. $\phi^n(x)$ is function exclusively determined by its B-spline constant $c_n(k)$.

The B-spline property is derived from several self-convolutions and its associate degrees describes its polynomial of order n. Now define $\beta^0(x)$ with the following conditions:

$$\beta^0(x) = \begin{cases} 1, & \frac{-1}{2} \le x \le \frac{1}{2} \\ 0, & otherwise \end{cases} \quad (3)$$

Therefore, B-spline coefficient functions of any order satisfy the convolution property:

$$\beta^n(x) = \beta^{n-1}(x) * \beta^0(x) \quad (4)$$

when $n = 2$, the quadratic B-spline operations are defined by the subsequent equation:

$$\beta^2(x) = \begin{cases} \frac{3}{4} - |x|^2, & |x| \le \frac{1}{2} \\ \frac{1}{2!}\left(|x| - \frac{3}{2}\right)^2, & \frac{1}{2} \le |x| \le \frac{3}{2} \\ 0, & otherwise \end{cases} \quad (5)$$

In this stage, Quadratic B-spline functions create the distortion in the output image. To overcome this, we plug in as $n = 3$, in (5) and we get the following isometric B-spline function:

$$\beta^3(x) = \begin{cases} \frac{1}{2}|x|^3 - |x|^2 + \frac{2}{3}, & |x| \le 1 \\ \frac{1}{3!}(2 - |x|)^3, & 1 \le |x| \le 2 \\ 0, & otherwise \end{cases} \quad (6)$$

This $\beta^3(x)$ shows the operation of transferring B-spline weight functions from order zero to order 3 of magnitude response of the B-spline coefficient functions [10]. When the transfer functions decrease periodically, it leads to convergence by indicating that the B-spline interpolation value averaging an excessive redundant value [8]. This increases the interpolations quality and smoothing the image as well as the edge of the image. Within the edge estimation, quadratic interpolation of the spline is performed by manipulating the interpolation coefficients adaptively which support the classification of the edges [12]. The way to classify the edges (e_1 to e_2) should be consistent and the direction of rotation of the image should be noted while classifying the edges [6]. Exploding the four edges at periodical intervals the interpolation values are noted in d_1 to d_4 and is shown as follows:

$$d_1 = |e_1 - e_2|,\ \ d_2 = |e_2 - e_3|,\ \ d_3 = |e_3 - e_4|,\ \ d_4 = |e_4 - e_1| \qquad (7)$$

Now find d_{min} and d_{max} based on the edges and these values are treated as the primary and secondary values in an individual basis.

III Adaptive and quadratic spline interpolation

In this section we considered a quadratic spline interpolator derived from a quadratic B-spline weighted function and its characteristics focused within its frequency domain. Initially the interpolation considered the convolution of infinite impulse and direction of rotation of the image and its reconstructed image from its original image is classified as follows:

$$f(x) = \sum_{k=-\infty}^{\infty} c(k) * q(x-k) \qquad (8)$$

where $c(k)$ histogram analyzer value and $q(x)$ be a nonstop impulse response from the spline quadratic interpolator. Now we allow the contemplated image in the $k-m$ level by changing the function from $f(x)$ to $f(k)$ and is defined as,

$$f(k) = \sum_{m=-\infty}^{\infty} c(m) * q(k-m) \qquad (9)$$

and we obtain

$$f(k) = \frac{1}{8}\{c(k-1) + 6c(k) + c(k+1)\} \qquad (10)$$

This can be delineated within the type of convolution

$$f(k) = c(k) * q(k) \qquad (11)$$

From equations (10) and (11), we derive the subsequent equation

$$f(k) = c(k) * \frac{1}{8}\{\delta(k-1) + 6 * \delta(k) + \delta(k+1)\} \qquad (12)$$

To calculate the weight function in the frequency domain, the Fourier series convolution is written like

$$F(\omega) = C(\omega) * \frac{1}{4}\{3 + cos(\omega)\} \qquad (13)$$

Then the inverse Fourier remodel of $C(\omega)$ is calculated through

$$c(x) = \sqrt{2} \sum_{k=-\infty}^{\infty} (2\sqrt{2}-3)^{|x-k|} * f(k) \qquad (14)$$

Therefore, the continual function $f(x)$ is coordinated along with quadratic spline interpolation algorithm as,

$$f(x) = \sum_{i=-\infty}^{\infty} f(i) h_{quad}(x - i) \quad (15)$$

The spline quadratic interpolator $h_{quad}(x\ )$ is defined as

$$h_{quad}(x\ ) = \sqrt{2} \sum_{k=-\infty}^{\infty} (2\sqrt{2} - 3)^{|k|} \beta^2(x - k) \quad (16)$$

Before applying the transformation, identify the $\beta^2(x)$ and $\beta^3(x)$ is listed below.

x	$\beta^2(x)$	$\beta^3(x)$	$\lvert\Delta\rvert$
0	0.75	0.667	0.083
|0.1|	0.74	0.6575	0.0825
|0.2|	0.71	0.631	0.079
|0.3|	0.66	0.5905	0.0695
|0.4|	0.59	0.539	0.051
|0.5|	0.5	0.4795	0.0205
|0.6|	0.405	0.415	0.01
|0.7|	0.32	0.3485	0.0285
|0.8|	0.245	0.283	0.038
|0.9|	0.18	0.2215	0.0415
|1|	0.125	0.1667	0.0417
|1.1|	0.08	0.1215	0.0415

Table 1: B-spline weight functions of order 2 and 3

Equation (16) illustrates the quadratic interpolator of the spline, its frequency, magnitude and the zero crossing conditions related to equilibrium exponent. Table 1 shows the B – Spline weight functions of order 2 and 3. At identical time, the image and its pixel size will not reduce its eminence. To enhance the image further, we worked with sinc and cubical interpolation, and the corresponding weighted value which produces a parameter that controls the interpolation and this leads to the enhancement of the image.

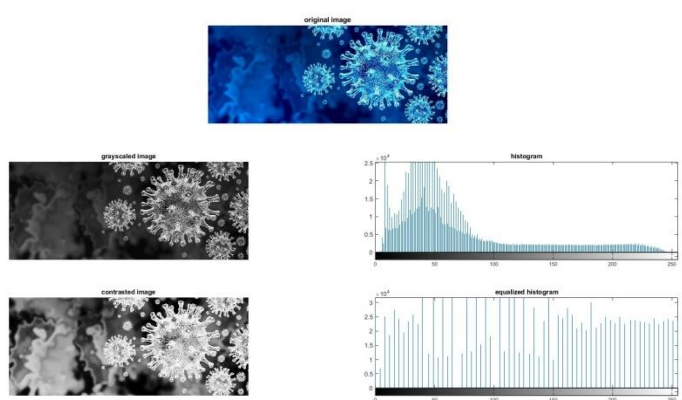


Figure 1: Original (sampled) image and histogram analyzer of original image

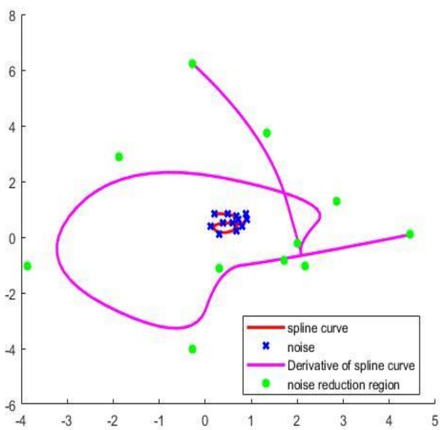


Figure 2: Original Image's - Spline curve, noise, noise region and its first derivative

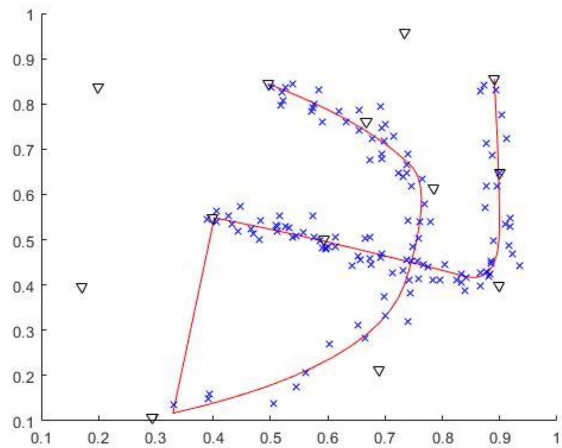
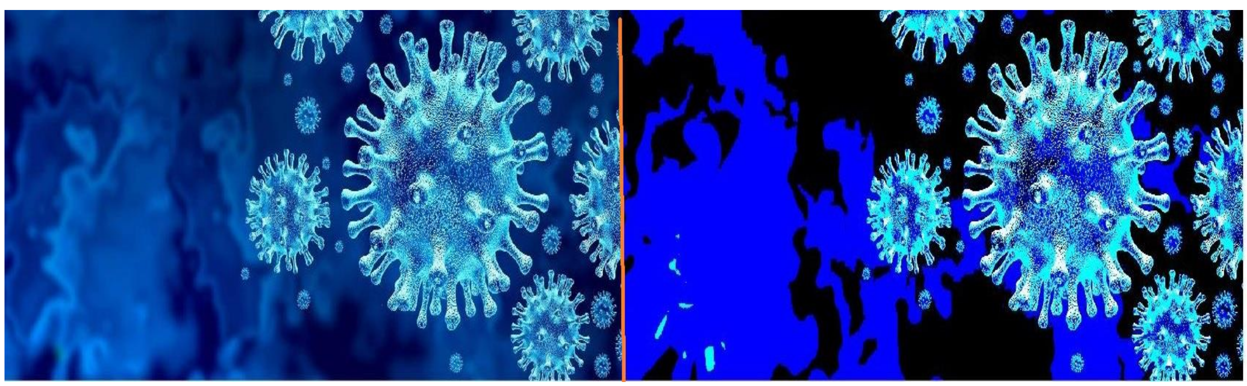


Figure 3: Reconstructed shape of the original sampled image



4a **4b**

Figure 4a: Original Image **4b : Distorted Image**

Figure 1shows the original (sampled) image and histogram analyzer of original image. Figure 2shows the original Image's - Spline curve, noise, noise region and its first derivative. Figure 3 explains the reconstructed shape of the original sampled image. Figure 4a and 4b classifies the original image and distorted image.

IV Sinc and Cubic Interpolation

The sinc operation provides reconstruction of $s(x,y)$, and is spatially unlimited. There are two common approaches to proceed further by truncating and making windows with a windowing operation $w(x)$. This Sinc interpolation is defined along with the ideal interpolation is as follows:

$$Sinc_{h_{\mathcal{N}}(x)} = \begin{cases} Ideal_{h(x)*w(x)}, & 0 \leq |x| < N/2 \\ 0 & elsewhere \end{cases} \quad (17)$$

where $\mathcal{N}$ indicates the number of pixels and as per sinc interpolation and these values are converged as well as locally uniformed in its kernel. In other words, all windowed or truncated synchronized kernels are basically sinc interpolators. Truncation is like multiplying $Ideal_{h(x)}$ by a rectangular operation among the domain of abstraction, which is sort of a convolution with a sinc operation among the frequency domain. Therefore, truncating the proper interpolator produces ringing effects among the frequency domain as results of an oversized quantity of energy being discarded functioned by a truncated synchronization operation with various values of $\mathcal{N}$. This kernel value differs to a great extent between the dimensions of the even-numbered and odd-numbered kernel. Significantly by increasing the pixel size within the intervals are used to measure various properties of a truncated kernel and abstraction convolution of Sinc interpolation.

Now we compared various window functions for interpolation with sinc kernels through exploitation analysis and Fourier transformation $F(x)$ [13] with the weighted windows w_0, w_1 and w_2 .

$$F(x) = w_0 + w_1 \cos\left(2\pi \frac{2x}{\mathcal{N}}\right) + w_2 \cos\left(2\pi \frac{4x}{\mathcal{N}}\right) \tag{18}$$

Cubic polynomials are often used due to its ability to adapt forward and backward interpolation. Furthermore, the B-spline approximate Cubic h_2, as defined in (13), as well as the Lagrange interpolator $Spline_{h(x)}$ fragmentarily from cubical polynomials. Of course, cubical polynomials also can be accustomed approximate the following types of interpolation.

1) *Two-point interpolation*: Cubical interpolation with 2 points leads to a symmetrical kernel and this is outlined through $A, B, C, D \in \mathbb{R}$

$$Cubic_{h_2(x)} = \begin{cases} A|x|^3 + B|x|^2 + C|x| + D & , \quad 0 \le |x| < 1 \\ 0 & , \quad elsewhere \end{cases} \tag{13}$$

Parameters A to D are determined according to the following boundary conditions:

- $h(k^-) = h(k^+)$, continuity;
- $h'(k^-) = h'(k^+)$, continuity;
- $h(k) = 1$ for $k = 0$,
- $h(k) = 0$ for $k \neq 0$

For $\mathcal{N} = 2$, these boundary conditions produce the subsequent parameters say;

$$Cubic_{h_2(x)} = \begin{cases} 2|x|^3 - 3|x|^2 + 1 & , \quad 0 \le |x| < 1 \\ 0 & , \quad elsewhere \end{cases} \tag{14}$$

It should be noted that by definition, $Cubic_{h_2(x)}$ is a constant interpolator and this resultant curves looks like linear interpolation, within the spatial domain.

2) *Four-point interpolation:* Interpolation with $\mathcal{N} = 4$ determined its initial value with the negative kernel value is ranging from 0 to 1.

V Conclusion

This paper contains various image interpolation techniques which are defined in spatial domain. These interpolation techniques provide the output and re-sampled images. This decreases the computing time and increases the quality of re-sampled images.

Further these re-sampled images are enhanced by cubic, sinc, linear, B-spline, ideal, adaptive and quadratic interpolation methods. Cubic Spline interpolations with minimal distortion of the sub-sampled images are enhanced and reduced the computational complexity. The quadratic spline interpolator resulting from a quadratic B-spline weighted function is decomposed by manipulating the weighted sum of quadratic B-spline weight functions. The ideal interpolator and B-spline interpolator derives its exact value of interpolator changes from positive value to negative value at various points in a particular domain. Convolution of infinite impulse and direction of rotation of the image and its reconstructed image (distorted) from its original image was arrived along with the noise.

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