Research Article

Diagonally implicit and explicit Euler's methods for RLC circuit

M.Malarvizhi[1] , Dr.S.Karunanithi[2]
[1,2] P G & Research Department of Mathematics,
Government Thirumagal Mills College, Tamil Nadu , India.
E-Mail: malar2308@gmail.com , kap232008@gmail.com

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Abstract:
This paper integrates the intuitive look at the natural responses of Euler's components with inductors, capacitors and resistors (RLC circuits). Within these components, the current-voltage relationship is modeled by differential equations (according to Euler's implicit and explicit), and we analyzed computational methods for simulating RLC circuits. This computational method leads the application of higher-order diagonally implicit and explicit Euler methods to an RLC circuit (static as well as dynamic circuit solving strategies) in the presence and absence of equilibrium. Typically, existing numerical methods for RLC circuits have accuracy in the first or second order. Our high order method allows larger time steps, which are critical and damped when large circuits are simulated over a significant period of time and phase angles.

Key Words : Euler's implicit and explicit identity, RLC Circuits, complex circuits, Phasor current, Phasor voltage.

I Introduction

The RLC circuit is symbolic of real life circuits, and there is some finite resistance in every real circuit. In many areas of electrical engineering, this circuit has a rich and dynamic behavior that finds enormous applications in the diverse field. Circuit components are commercially available in isolated form or in integrated circuits, such as resistors and capacitors. But it is difficult to produce inductors with appreciable inductance in small sizes, so they are usually not used in integrated circuits [5]. There are many applications in which there is no replacement for inductors. Relays, delays, sensing systems, radio and TV receivers, power supplies, and loud speakers are used these inductors regularly [11].

Together, the three features of capacitors and inductors make them very useful in electrical circuits. The energy storage ability makes them useful as sources of temporary current or voltage [15]. In particular, in a short period of time, they can be used to produce a large amount of current or voltage [8, 14]. Capacitors oppose abrupt voltage changes, whereas inductors oppose abrupt current changes. This function makes inductors useful for suppression of sparks or arcs and for transforming pulsating DC (Direct Current) voltage into relatively smooth DC voltage [1, 7]. Inductors and capacitors are sensitive to frequency. This property is useful for discrimination based on frequency [4, 9]. An inductor functions as a short circuit to DC and we know from the differential equation explaining the current-voltage relationship in an inductor that when the current is constant, the voltage across an inductor is zero [6, 13].

The current does not charge instantaneously through an inductor, there is an opposition to the shift in current flowing through it. The ideal inductor does not dissipate electricity and at a later time, the retained energy can be used. Existing models such as SPICE use current source models from Euler's and Trapezoidal that are first and second order accurate and critically damped circuit is quickly reached zero without any oscillation. Higher order strategies are beneficial since greater steps can be taken in time [2, 10,19,20]. There are valuable computations and information stored at each time stage when simulating an electric circuit over a meaningful period of time. Few researchers are identified the direct implicit Euler's techniques provided us with high order precision and the ability to deal with ideal diodes' stiff problems [3, 12,16,17,18].

The paper is structured as follows: Section II dealt with necessary and basic concepts of Euler's identity in RLC Circuits. In Section III, RLC Circuit Equation Framework was analyzed for Inference of the explicit Euler method - resistor based. The ability to measure Euler time based on inductor is introduced. Implicit midpoint methods based on conductor with local time steps are analyzed. Finally section IV concludes the paper.

II Preliminaries of Euler's Identity

Initially, the functions of time with complex numbers are defined and, in most cases, these functions need to be separated and incorporated with respect to the actual variable. The differentiation and integration operations for these complex variables are described through suitable mechanism that is enabled according to various constraints in the execution of derivative or integration operations for real variables, complex variables and complex constants, which are treated in the same and similar way as real variables or real constants. That is, if $g(x)$ is a complex function of time, then

$$g(x) = p\, e^{t/2} cos\, (\omega t + \theta) + \, i\, q e^{-t/2} \sin(\omega t + \theta) \qquad (1)$$

p and q are constants, further its integral constant is completely complex.

While we use the significant basic relationship as Euler's identity and this identity is by the way of representing complex numbers in non-rectangular form. The notation used for this complex number is D, where $D = u + iv$, equalizing the real parts in an usual manner , we get
$u = B\cos\phi$ and the imaginary parts $v = B\sin\phi$, then $B = \sqrt{u^2 + v^2}$ and $\phi = tan^{-1}\frac{v}{u}$. Now determine B and ϕ, by knowing u and v. For instance, let us take $6 + i8$ then define u as 6, v as 8 , $B = 10$ and $\phi = = 53.1°$. Then, $D = 10\cos 53.1° + \, i\, 10 \sin 53.1°$ and $D = Be^{i\phi} = 53.1 e^{i53.1°} = 10e^{i0.926}$.

The actual multiplication factor of B is known as the magnitude (amplitude) and the real number ϕ occurring in the exponent. Here ϕ value must be taken in such a way that $cos\phi$ must be positive, where $6 = 10\, cos\phi$, and $sin\phi$ must be negative, and $-8 = 10 sin\phi$. Hence, we obtain $\phi = -53.1°, 413.1°, -306.9°$ and so on.

It is often easier to work with negative numbers than complex numbers, avoiding angles greater than 90. For instance, let us take $K = -3 + i4 = -(3 - i4)$ and convert to exponential format, we get:

$$K = -B\, e^{i\phi}$$

since

$$B = \sqrt{25} = 5\; and\; \phi = tan^{-1}\frac{-4}{3} = -53.1^{o}$$

Hence, the exponential form for K is given by,

$$K = -5\, e^{-i\, 53.1°}$$

As we can see in the complex plane sketch, the angle can be removed from the complex number by increasing or decreasing it by 180^{o}. So we can write $K = 5\, e^{-i\, 126.9°}$.
This concept has extended to the following value:

$$Z\ =\ [9 - 7.5 * j\ - 6 + 4 * j;\ \ -6 + 5 * j\ 16 + 3 * j]$$

In the above equation, the

phasor current i1, magnitude: 0.513516

phasor current i1, angle in degree: 17.197914

phasor voltage Vc, magnitude: 6.393443

phasor voltage Vc, angle in degree: -38.793975

Two complex numbers with same amplitude and are in exponential form if and only if they have the same magnitude and the same angle. An equivalent angle is an angle other than a multiple of 360^{o}. For instance, if $D = Be^{i\phi}\; and\; G = Ee^{i\phi}$, also $D = G$, it is mandatory that $B = E$ and $\phi = \psi \pm 360° r$, since $r = 0,1,2,3$ .... At zero, we consider the capacitor voltage to be zero and hence we would have the following equation from the equation of the current in an inductance, the voltage in a capacitor and resolving KVL (kirchhoff's voltage law) in the loop where KVL = Capacitor voltage + inductance voltage + Resistor voltage = zero.

III RLC Circuit Equation Framework

The development of a mathematical model for complex circuits (static and dynamic) are simplified the application of loop analysis, resulting in a linear equation method.

Explicit Euler Method

We can construct a polygonal method by considering the geometry of the uv plane, which is an extended state space model and can solve via explicit Euler's method through the following steps:

Phase I:

Step 1: Observe the tangent field and solution curve of the concern geometry

Step 2: Follow tangent for a short periodical time

Step 3: Calculate the Time step according to continuous evolution of intermediate wave

$$[x_{y-1}, x_y]\ , y = 1,2 \dots P, x_P := W.$$

Step 4: Approximate the solution $[x_{y-1}, x_y]$ of the tangent to the solution path in terms of (x_{y-1}, X_{y-1}).

Step 5: Tangent slope $= j(x_0, X_0)\ X_1$ is used as the initial value then go to the step 1. Figure 1 represents the Comparison of Implicit and Explicit Euler Method on RLC inputs of complex values.

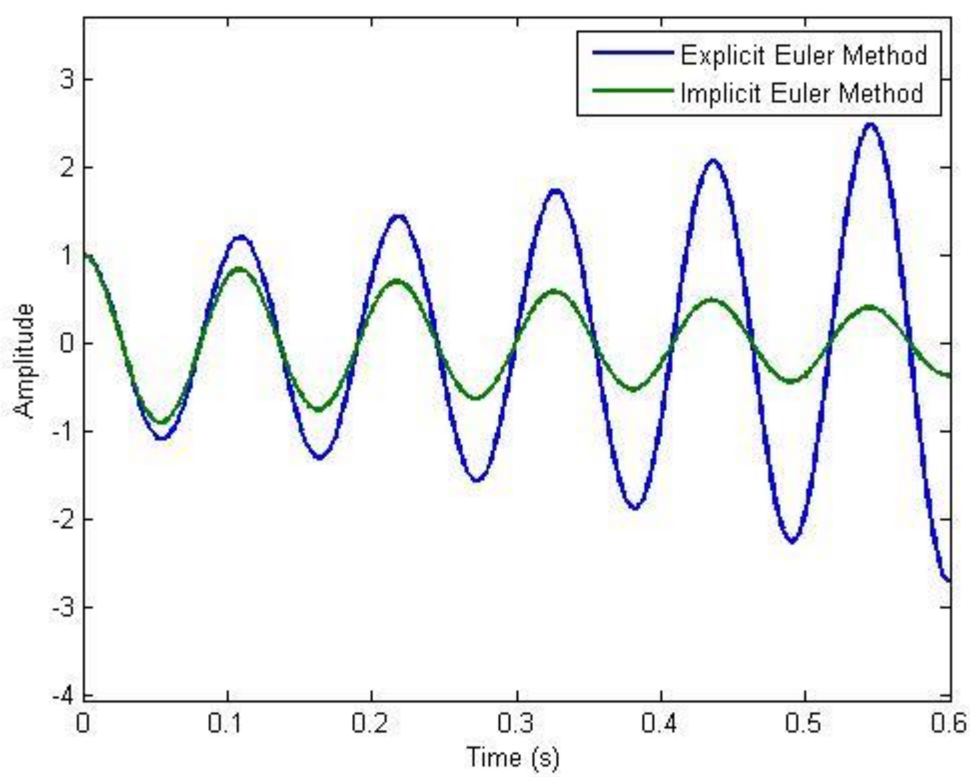

Figure 1: Comparison of Implicit and Explicit Euler Method on RLC inputs

Euler's method explicitly generates a sequence when applied to the entire IVP $\dot{v} = j(x, v), v(x_0) = v_{0,}$, $(X_y)_{y=0}^{P}$ is a sequence generated by the explicit Euler method using the recursion

$$X_{y+1} = X_y + r_y j(x_y, v_y), y = 0, \dots, P-1, \qquad (2)$$

using local time step (measure)

$$r_y := x_{y+1} - x_{y.}$$

Phase II: (Another scheme of explicit Euler's method)

You can get (2) by referring to the derivative $\frac{d}{dx}$ in a (temporary) network, the forward difference coefficient $Z := \{x_0, x_1, \ldots, x_P\}$:

$$\dot{v} = j(x, v) \leftrightarrow \frac{X_{y+1} - X_y}{r_y} = j\left(x_y, X_y(x_y)\right), y = 0, \ldots, P - 1. \quad (3)$$

The different circuits follow simple guidelines for distinguishing differential equations. Replace all derivatives with the difference coefficient associated with the solution value for a set of discrete (grid) points. The Current flows of RLC circuit based on Euler's implicit and explicit formula are shown in figure 2.

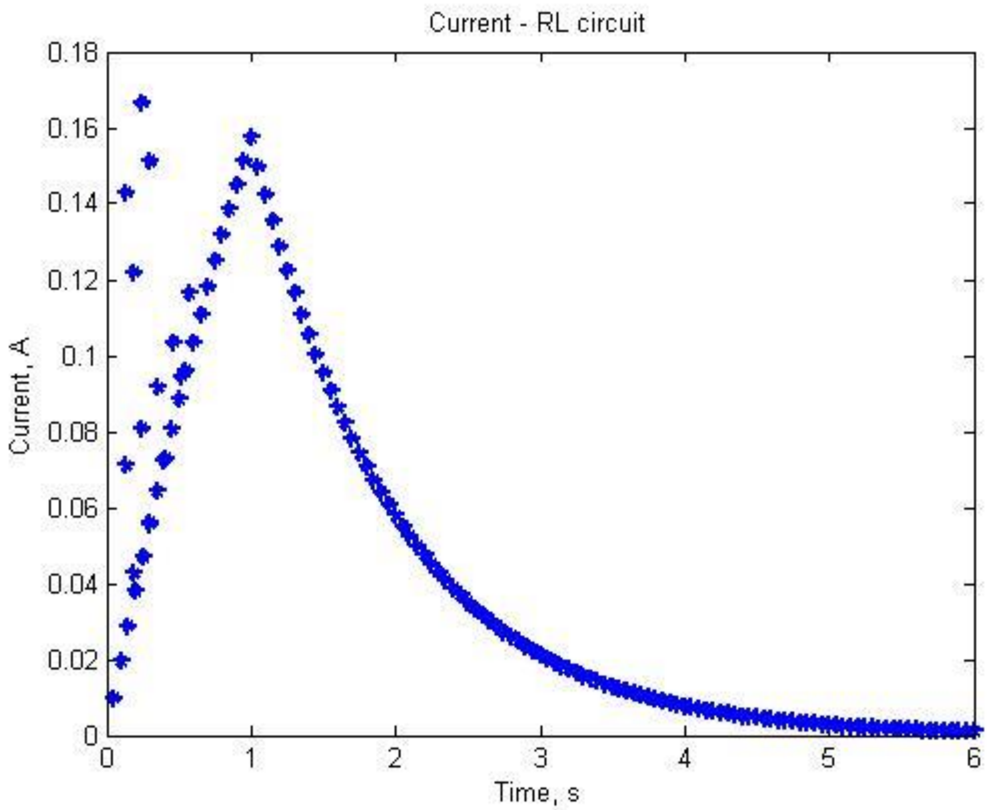

Figure 2 : Current flow of RLC circuit based on Euler's implicit and explicit formula

Phase III : (Inference of the explicit Euler method)

Euler's recursion (2) gives the sequence $v_0, \ldots, v_P$ state and the geometric understanding of this sequence is

$$v_y \approx v(x_y).$$

The path $x \to v(x)$ is approximated as a linear function of the part ("Euler's polygon") and the initial values are $[x_0, x_P]$, consecutive voltage inputs are $v_y\ [x_y, x_{y+1}]$

$$v_y: [x_0, x_P] \to S^g, v_y(x) := v_y \frac{x_{y+1} - x}{x_{y+1} - x_y} + v_{y+1} \frac{x - x_y}{x_{y+1} - x_y} \text{ for all } x \in [x_y, x_{y+1}] \quad (4)$$

This function can easily scan all networks at $[x_0, x_P]$. In fact, this is a linear interpolation of numerical values of $(x_y, v_y), y = 0, \ldots, P$. The same conditions are applied to the implicit Euler's methods and are described as follows.

Phase IV: Implicit Euler method – resistor based:

The coefficient of difference is not the inverse coefficient. Let's consider the (temporary) network $\mathrm{Z} \coloneqq \{\mathrm{x}_0, \mathrm{x}_1, \dots, \mathrm{x}_\mathrm{P}\}$, we get backward difference quotient through implicit Euler's method as follows

$$\dot{\mathrm{v}} = \mathrm{j}(\mathrm{x}, \mathrm{v}) \leftrightarrow \frac{\mathrm{v}_{\mathrm{y}+1} - \mathrm{v}_\mathrm{y}}{\mathrm{r}_\mathrm{y}} = \mathrm{j}\left(\mathrm{x}_{\mathrm{y}+1}, \mathrm{v}_\mathrm{r}(\mathrm{x}_{\mathrm{y}+1})\right), \mathrm{y} = 0,1, \dots, \mathrm{p}-1. \quad (5)$$

This leads to the following system like

$$\mathrm{v}_{\mathrm{y}+1} = \mathrm{v}_\mathrm{y} + \mathrm{r}_\mathrm{y}\, j\left(\mathrm{x}_{\mathrm{y}+1}, \mathrm{v}_{\mathrm{y}+1}\right), \mathrm{k} = 0,1, \dots \mathrm{P}-1 \quad (6)$$

according to local time steps according to the resistor are $r_y \coloneqq x_{y+1} - x_y$ and (6) is the implicit Euler's method. To get v_{y+1}, we need to solve a system of equations in nonlinear probability in the form of implicit Euler's method. The geometry of the implicit Euler method, to approximate the solution from $[x_0, v_0]$ to (x_0, v_1) is classified in the following steps:

Step 1: construct a straight line passing through (x_0, v_0)

Step 2: find the slope at $j(x_1, v_1)$

Sep 3: again set path from (x_0, v_0) to (x_1, v_1)

Step 4: set tangential path to (x_1, v_1).

Phase V: (Implicitly - the ability to measure Euler time based on inductor)

Consider an autonomous function j that can be sequentially differentiated from the right $j \in K^q(G, S^g)$, and consider (6) as a system of nonlinear equations depending on r and this nonlinear equation is being investigated.

$$v_{y+1} = v_y + r_y j\left(x_{y+1}, v_{y+1}\right) \Leftrightarrow \mathrm{I}\left(r, v_{y+1}\right) = 0 \; with \; \mathrm{I}(r, m) \coloneqq m - rj\left(x_{y+1}, m\right) - v_y.$$

Observing the partial derivative of the inductor I: Further, $E\,(0, v_y) = 0$.

$$\frac{dE}{dm}(r, m) = M - rG_v\, j\left(x_{y+1}, m\right) \Rightarrow \frac{dE}{dm}(0, m) = M.$$

Also, I $(0, v_y) = 0$. This leads to the implicit function form.

Phase VI : Implicit midpoint method based on conductor

In addition that while using the difference coefficients, the derivative can also be approximated by a symmetric difference coefficient represents in the form of conductor.

$$\mathrm{c}(\dot{x}) \approx \frac{v(x+r) - v(x-r)}{2r} \quad (7)$$

Use r to change this formula to $x = \frac{1}{2}\left(\mathrm{x}_\mathrm{y} + \mathrm{x}_{\mathrm{y}+1}\right)$ along with $r = \frac{\mathrm{r}_\mathrm{y}}{2}$,

$$\dot{\mathrm{c}} = \mathrm{j}(\mathrm{x}, \mathrm{c}) \leftrightarrow \frac{\mathrm{v}_{\mathrm{y}+1} - \mathrm{v}_\mathrm{y}}{\mathrm{r}_\mathrm{y}} = \mathrm{j}\left(\frac{1}{2}\left(\mathrm{x}_\mathrm{y} + \mathrm{x}_{\mathrm{y}+1}\right), \mathrm{v}_\mathrm{r}\left(\frac{1}{2}\left(\mathrm{x}_{\mathrm{y}+1} + \mathrm{x}_{\mathrm{y}+1}\right)\right)\right), \mathrm{y} = 0,1, \dots \mathrm{P}-1 \quad (8)$$

The problem is that $\mathrm{v}_\mathrm{r}\left(\frac{1}{2}\left(\mathrm{x}_{\mathrm{y}+1} + \mathrm{x}_{\mathrm{y}+1}\right)\right)$ value used to estimate the path for $x \rightarrow \mathrm{v}_\mathrm{r}(x)$ and is assumed to be linear in the slice $\mathrm{v}_\mathrm{r}\left(\frac{1}{2}\left(\mathrm{x}_{\mathrm{y}+1} + \mathrm{x}_{\mathrm{y}+1}\right)\right) = \frac{1}{2}\left(\mathrm{v}_\mathrm{y}\left(\mathrm{x}_\mathrm{y}\right) + \mathrm{v}_\mathrm{y}(\mathrm{x}_{\mathrm{y}+1}\right)$. Then the equation (2) and (6) is analogy which gives the recursion formula for the implicit midpoint method.

$$\mathrm{v}_{\mathrm{y}+1} = \mathrm{v}_\mathrm{y} + \mathrm{r}_\mathrm{y}\mathrm{j}\left(\frac{1}{2}\left(\mathrm{x}_\mathrm{y} + \mathrm{x}_{\mathrm{y}+1}\right), \left(\frac{1}{2}\left(\mathrm{v}_\mathrm{y} + \mathrm{v}_{\mathrm{y}+1}\right)\right)\right), \mathrm{y} = 0,1, \dots, \mathrm{P}-1 \quad (9)$$

at the same time when time step is local then, $\mathrm{c}_\mathrm{y} \coloneqq \mathrm{j}\, \mathrm{x}_{\mathrm{y}+1} - (\mathrm{j}+1)\mathrm{x}_\mathrm{y}$.

Phase VII : Implicit midpoint method with local time steps

Geometric view: approach the trajectory from $(\mathrm{x}_0, \mathrm{v}_0)$to $(\mathrm{x}_0, \mathrm{x}_1)$ through:

Step 1: Straight line through $(\mathrm{x}_0, \mathrm{v}_0)$

Step 2: Slope of the trajectory $\mathrm{j}(\mathrm{x}^*, \mathrm{v}^*)$, as $\mathrm{x}^* \coloneqq \frac{1}{2}(\mathrm{x}_0 + \mathrm{x}_1), \mathrm{v}^* = \frac{1}{2}(\mathrm{v}_0 + \mathrm{v}_1)$.

Step 3: Initial path against $(\mathrm{x}_0, \mathrm{v}_0)$

Step 4: Final path against $(\mathrm{x}^*, \mathrm{v}^*)$

Step 5: tangential line from $(\mathrm{x}^*, \mathrm{v}^*)$

As in case (6), it is assumed that path (9) also solves the (nonlinear) system of equations to obtain v_{y+1}, implicitly the ability to measure Euler's time and also applicable in this case, if r is small and there is a unique solution v_{y+1} defined recursively well.

IV Result and discussion

All aspects of the various methods of Euler have been tested for initial values of RLC circuits. Implicit and Explicit Euler's identities are used to compare the RLC circuit's issues. It is observed that it is relatively short when the integration range is relatively short, and minor step sizes can be used without additional processing time. The Current flows of RLC circuit based on Euler's implicit and explicit formula are identified. When Euler's forward algorithm is embedded in the Backward Method of Euler, relatively large step sizes are used. Phasor current is found with magnitude and degree angle. Phasor voltage with magnitude and angle in degree has been numerically arrived in a similar manner. For the explicit Euler process-resistor-based inference, the RLC Circuit Equation Structure was analysed. The ability to calculate Euler time is implemented on the basis of an inductor. Implicit conductor-based midpoint approaches of local time steps are analyzed.

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