Research Article

Video Rate Control Using Fourier Transform

Tashmanov E.B., Ochilov I.N., Mastanov N.T., Begimkulov F.X., Jumaniyozov J.R.
(Applicants for the Military Technical Institute of the National Guard of the Republic of Uzbekistan)

Article History: Received: 11 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 4 June 2021

Abstract: This paper studies the control of the video data transmission rate by the compression parameters based on the Fourier transform. The Dirichlet problem for the Poisson equation is considered.

Keywords: image, compression, encoding, signal, transform

Introduction

A single-color discrete image can be described by a certain matrix, the elements of which (image points, also called pixels) represent the samples of the function obtained as a result of some spatial sampling, which describes the distribution of brightness on a continuous image. A digital image is a matrix obtained as a result of element-wise quantization (with a finite number of levels) of the sample values of a discrete image [1,15].

Methods for processing discrete (digital) images can be divided into two large groups: direct and spectral. In direct methods, the original pixels of the image are processed, while the spectral methods are based on the use of various discrete unitary transformations (Fourier, Walsh, Haar, etc.) and they process not the original points of the image, but the transformation coefficients (elements of the discrete spectrum). Moreover, the transformations used in spectral methods are usually two-dimensional, due to the two-dimensionality of the processed signals (digital or discrete images) [2.69].

The formulation of the question of the application of spectral methods for image compression became possible due to the appearance in 1965 of the work of Cooley and Tukey [3,297], which contained a description of an algorithm for fast computation of the discrete Fourier transform. The idea of replacing a single-color image as a direct encoding object with samples of its two-dimensional spectrum of the discrete Fourier transform (DFT) was put forward in 1968 [4,677; 5,488]. DFT encoding is based on the fact that for most natural images, the values of many DFT coefficients are relatively small. Such coefficients can often be discarded altogether, or a small number of bits can be allocated for their encoding, without the risk of introducing any significant distortions.

The idea of the fast Fourier transform has been repeatedly expressed, but only recently was an algorithm outlined that led to a significant decrease in the number of operations required, which stimulated a lot of interest in the method. Let there be a function of a discrete argument, where is a parameter. We represent this function in the form of a finite series (ie, sum) Fourier [see. 6.37]

$$f(k) = \sum_{n=0}^{N-1} A(n) W^{kn} \,,$$

$$A(n) = \frac{1}{N} \sum_{n=0}^{N-1} f(k) W^{-kn}. \qquad (1)$$

Here we have introduced the following notation for the principal root of the ith degree of unity $W = e^{i2\pi / N}$.

Let's call an operation the execution of two consecutive actions in complex arithmetic, namely addition and multiplication. Then it follows from (1) that for given $A(n)$ и W^{kn} used operations to find $f(k)$.

The idea is that if it is not simple, which can significantly reduce the number of operations, it will represent (1) as a multiple of the sum. Indeed, consider the case $N = N_1 \cdot N_2$, where N_1 и N_2- integers. We also represent in the form

$$k = k_1 N_2 + k_2; \quad k_1 = 0,1,\dots,\ N_1 - 1; \quad k_2 = 0,1,\dots,\ N_2 - 1; \qquad (2)$$

$$n = n_1 + n_2 N_1; \quad n_1 = 0,1,\dots,\ N_1 - 1; \quad n_2 = 0,1,\dots,\ N_2 - 1;$$

As $W^{k_1 n_2 N_1 N_2} = (W^N)^{k_1 n_2} = 1$, то $W^{k n_2 N_1} = W^{k_2 n_2 N_1}$ and

$$f(k) = f(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \left[\sum_{n_2=0}^{N_2-1} A(n_1, n_2) W^{k_2 n_2 N_1} \right] W^{k n_1}. \qquad (3)$$

Therefore, finding the sum of series (1) is reduced to finding the double sum (3) or, what, to sequentially finding the sums of the series

$$A_1(n_1, k_2) = \sum_{n_2=0}^{N_2-1} A(n_1, n_2) W^{k_2 n_2 N_1}, \qquad (4)$$

$$f(k_1, k_2) = \sum_{n_1=0}^{N_1-1} A_1(n_1, k_2) W^{k n_1}. \qquad (5)$$

But it follows from (2) and (4) that operations are required to find. Knowing, with the help of (5) we find by applying operations. Therefore, everything will be required $N(N_1 + N_2)$ operations. The more, the more the number of operations decreases.

It is easy to see that if is a prime and is a composite number, then this transformation can be applied to the sum (4), in which it is a parameter, and the number of operations can be further reduced by presenting it as a product. And in general, if $N = N_1 \times N_2 \times \ldots \times N_m$, then instead of N^2 operations we will come to $N = N_1 + N_2 + \ldots + N_m$ operations, and the largest decrease is obtained at or 4. If, for example, $N = 256 = 2^8$, then the number of operations will decrease by a factor of 256 / (8 x 2) = 16 times, and for $N = 243 = 3^5$ - в 243/(5 x 3) =16,2 times.

From a programming point of view, the most convenient case is $N_i = 2 (i = 1, 2, \ldots, m)$, although there are economical options for others. Consider this case: $N = 2^m$. To obtain the corresponding formulas, we can put $N_1 = 2,\ N_2 = 2^{m-1}$ and get sums like (7.4) and (7.5), and then continue this process. We have

$$k = \overline{k_{m-1} k_{m-2} \ldots k_1 k_0} \equiv k_{m-1} 2^{m-1} + k_{m-2} 2^{m-2} + \ldots + k_2 2 + k_0,$$

$$n = \overline{n_{m-1} n_{m-2} \ldots n_1 n_0} \equiv n_{m-1} 2^{m-1} + n_{m-2} 2^{m-2} + \ldots + n_1 2 + n_0,$$

where and are equal to 0 or 1. Then

$$f(k_{m-1}, \ldots, k_0) = \sum_{n_0=0}^{1} \left\{ \sum_{n_1=0}^{1} \left[\ldots \sum_{n_{m-1}=0}^{1} \left(A(n_{m-1}, \ldots, n_0) W^{k n_{m-1} 2^{m-1}} \right) \ldots W^{k n_1 2} \right] W^{k n_0} \right\}. \qquad (6)$$

As

$$W^{k n_{m-1} 2^{m-1}} = W^{k_0 n_{m-1} 2^{m-1}},$$

$$W^{k n_{m-2} 2^{m-2}} = W^{\overline{k_1 k_0} n_{m-2} 2^{m-2}}$$

etc., then finding the multiple sum (6) is reduced to the sequential calculation of m sums

$$A_1(k_0, n_{m-2}, \ldots, n_0) = \sum_{n_{m-1}=0}^{1} A(n_{m-1}, \ldots, n_0) W^{k_0 n_{m-1} 2^{m-1}},$$

$$A_2(k_1, k_0, n_{m-3}, \ldots, n_0) = \sum_{n_{m-2}=0}^{1} A_1(k_0, n_{m-2}, \ldots, n_0) W^{\overline{k_1 k_0} n_{m-2} 2^{m-2}}, \qquad (7)$$

. . . .

$$A_m(k_{m-1}, \ldots, n_0) = \sum_{n_0=0}^{1} A_{m-1}(k_{m-2}, \ldots, k_0, n_0) W^{\overline{k_{m-1} \ldots k_0} n_0},$$

$$f(k) = A_m(k_{m-1}, \ldots, k_0).$$

It should be noted that the fast Fourier transform is very effectively used in the correlation analysis of processing statistical data for random variables.

Let us now consider the Dirichlet problem for the equation $f(k)\ (k = 0, 1, \ldots, N-1)$.

Let us now consider the Dirichlet problem for the equation

$$-\Delta\varphi + \mu\varphi = f \text{ в } D,$$
$$\varphi = 0 \text{ на } \partial D. \quad (8)$$

Here μ - a given constant, and f - given in $D = \{0 \le x \le 1,\ 0 \le x \le 1\}$ a function that has the necessary smoothness. Let us associate problem (8) with its difference analog

$$\frac{4\varphi_{k,l} - \varphi_{k-1,l} - \varphi_{k+1,l} - \varphi_{k,l-1} - \varphi_{k,l+1}}{h^2} + \mu\varphi_{k,l} = f_{k,l} \text{ in } D_h,$$
$$\varphi_{k,l} = 0 \text{ fot } \partial D_h, \quad (9)$$

$$0 \le k \le \frac{1}{h} = N, \qquad 0 \le l \le \frac{1}{h} = N.$$

if $\mu \ge 0$, then solutions to problems (8) and (9) exist and are unique. When $\mu < 0$ the requirements for the existence of solutions to problems (8) and (9) impose additional restrictions on μ and f. Suppose that solutions to problems (8) and (9) exist and are unique. Let us introduce the notation

$$\varphi_l = \begin{Vmatrix} \varphi_1, l \\ .\ .\ .\ . \\ \varphi_{n-1,l} \end{Vmatrix}, \qquad f_l = \begin{Vmatrix} f_{1,l} \\ .\ .\ .\ . \\ f_{N-1,l} \end{Vmatrix}, \qquad l = 1, 2, \ldots, N-1,$$

$$A = \begin{Vmatrix} 2 & -1 & 0 & \ldots & 0 & 0 & 0 \\ -1 & 2 & -1 & \ldots & 0 & 0 & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & \ldots & -1 & 2 & -1 \\ 0 & 0 & 0 & \ldots & 0 & -1 & 2 \end{Vmatrix},$$

where is a matrix of order -1. Let us denote by the identity matrix of the same order. We rewrite problem (9) in the following form

$$B\varphi_1 - \varphi_2 = h^2 f_1,$$
$$-\varphi_{l-1} + B\varphi_l - \varphi_{l+1} = h^2 f_l, \quad l = 2, \ldots, N-2 \quad (10)$$
$$-\varphi_{N-2} + B\varphi_{N-1} = h^2 f_{N-1},$$

where $B = A + (2 + \mu h^2)E$.

Note that the matrices and have a common basis of eigenvectors and a solution to the complete eigenvalue problem

$$Au^{(m)} = \lambda_m(A)u^{(m)}$$

has the form

$$\lambda_m(A) = 2(1 - \cos\frac{m\pi}{N}, \qquad u_k^{(m)} = \sqrt{\frac{2}{N}} \sin\frac{m\pi k}{N},$$

where $u_k^{(m)}$ - component with eigenvector number $u^{(m)}$, $k = 1, 2, \ldots, N-1;$ $m = 1, 2, \ldots, N-1.$ Factor $\sqrt{2/N}$ introduced from the normalization condition

$$\left\| u^{(m)} \right\|^2 = \sum_{k=1}^{N-1} (u_k^{(m)})^2 = 1.$$

Since vectors $u^{(m)}$ form an orthonormal basis in $(N-1)$ - dimensional space, then the vectors φ_l и $f_l\ (l = 1, 2, \ldots, N-1)$ then the vector

$$\varphi_l = \sum_{m=1}^{N-1} \Phi_{m,l} u^{(m)}, \quad f_l = \sum_{m=1}^{N-1} F_{m,l} u^{(m)}. \qquad (11)$$

Substituting these expressions into system (10) and multiplying both sides by the vector $u^{(m)}$, we obtain for each fixed a system of equations with a tridiagonal matrix

$$\begin{aligned} \lambda_m \Phi_{m,1} - \Phi_{m,2} &= F_{m,1}, \\ -\Phi_{m,l-1} + \lambda_m \Phi_{m,l} - \Phi_{m,l+1} &= F_{m,l}, \quad l = 2, \ldots, N-2, \\ -\Phi_{m,N-2} + \lambda_m \Phi_{m,N-1} &= F_{m,N-1}. \end{aligned} \qquad (12)$$

here $\lambda_m = \lambda_m(B) = \lambda_m(A) + 2 + \mu h^2$.

Thus, in order to solve system (10), it suffices to calculate $N-1$ times the Fourier coefficients of the vectors f_l, decide $N-1$ system with tridiagonal matrices of the form (12) determining the Fourier coefficients of the vectors $\varphi_l \, (l = 1,2,\ldots,N-1)$, and calculate φ_l accordingly (11). The Fourier series expansion can be performed using the Fast Fourier Transform. For this, the formulas that determine the Fourier coefficients $F_{m,l}$ vector f_l, can be written as follows

$$F_{m,l} = \sqrt{\frac{2}{N}} \sum_{n=1}^{N-1} f_{n,l} \sin \frac{m\pi n}{N} = \sqrt{\frac{2}{N}} \sum_{n=0}^{2N-1} f_{n,l} \sin \frac{2m\pi n}{2N},$$

where $f_{0,l} = f_{N,l} = \ldots = f_{2N-1,l} = 0$. We denote by the value of the principal root of the degree $M = 2N$ from one, then

$$F_{m,l} = \sqrt{\frac{2}{N}} \, \mathrm{Im}\left(\sum_{n=0}^{M-1} f_{n,l} \varpi^{nm} \right), \quad m = 1, 2, \ldots, \; N-1,$$

and the described algorithm can be applied directly to calculate the sums. The calculation of the vectors φ_l.

REFERENCES

1. Gonzalez R., Woods R. Digital image processing. - M., Technosphere, 2005 .-- 1072 p.
2. Belyaev E.A., Tyurlikov A.M. Control of the coding rate and error in the system of compression and transmission of video information with restrictions on the memory of the transmitting and receiving devices // Computer Optics. Moscow.-2007.-№2.- P.69-76.
3. Cooley J.W., Tukey J.W. An algorithm for machine computation of complex Fourier series // Mach. Comput. - 1965 .-- V.19. - P. 297-301.
4. Andrews H.C., Pratt W.K. Fourier transform coding of images // Hawaii International Conference on System Science, January 1968. - P. 677-679.
5. Anderson J.B., Huang T.S. Piecewise Fourier transformation for picture bandwidth compression // IEEE Trans. Commun. -1972.- V. COM-20 - No. 3. - P.488-491.
6.E.B. Tashmanov. Video information transfer rate control using wavelet transform // Actual problems of the humanities and natural sciences. Moscow. - 2011. - No. 11. - P.37-40.