Simulation for Ruin Probabilities in Insurance with Sequence Autoregessive Dependence Random Variable

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Abstract:
The aim of this paper is used Monte Carlo methods to calculate an apporoximate ruin probabilities for classical risk processes with claim amounts are autoregessive process and generalized risk processes with premiums amounts, claim amountsare autoregessive processes.We build formulas for the algorithm and from there simulate illustrative numerical examples.

Keywords: Ruin probability, Regression, Monte Carlo Methods
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1. Introduction

In risk theory, the premiumsamount U(t) at time t: $U(t) = u + rt - \sum_{i=1}^{N_t} X_i$, where u > 0 is the initial capital of that company, r is the premium rate per a unit of time. The number of claim amounts up to time t, N_t is the pure Poisson process with intensity μ and claim amount series $\{X_i\}$ is a series of independent random variables having the same distribution as the probability distribution function F, have finite expectations μ. The ruin probability with finite time t, denoted $\psi(u, t)$, is defined by:

$$\psi(u,t) = P\{\exists\tau \le t : U(\tau) < 0\} \quad (1.1)$$

Ruin probability with infinite time, denoted $\psi(u)$, is defined by:

$$\psi(u) = \psi(u,+\infty) = \lim_{t\to+\infty} \psi(u,t) \quad (1.2)$$

If there exists a number R > 0 satisfying $\int_0^{+\infty} e^{Rx}(1 - F(x))dx = \frac{r}{\mu}$ (1.3)

Then with every $u \geq 0$ we have $\psi(u) \leq e^{-Ru}$ and if $\int_0^{+\infty} e^{Rx}(1-F(x))dx < +\infty$ then

$$\lim_{u \to +\infty} e^{Ru}\psi(u) = C \tag{1.4}$$

where C là constant. Equation (1.3) iscalledaproximate Cramer – Lundbergvà R iscalledexponential constant Lundberg. (seeGrandell [4]). For thesedependency structure models, It wouldoftenbevery hard to calculate the approximation of exponential constant R. Analyticalresults and numericalresults are oftenunknown. Simulation methodcanprovidetools for calculatingapproximatelyprobabilities $\psi(u), \psi(u,t)$.

The aim of thispaperisusing Monte Carlo simulation method to approximatelycalculateruinprobability $\psi(u,t)$ in two cases: i) the claim amount series is a series of regression independent random variables in classical models; ii) the proceedsseries, The series of claim amounts depending on the regression in the general model does not have effects of interests.

In the second part of the paper, the authorwillintroduce the classical model, the general model that has no effect of interest rates with the assumption of regressiondependence. In Part 3 of the paper, the authorwillintroduce simulation algorithms to calculateruinprobability in the modelsintroduced in part 2 of the paper. In Part 4 of the paper, the authorwillintroduce simulation resultswithdifferentregressiondependentmodels and the conclusions of the paper.

2. Insurance model with a series of regressiondependentrandom variables

2.1. Classical model

In the classical model, we assume that the capital of the insurancecompanyat time t is:

$$U(t) = u + rt - S_t = u + rt - \sum_{k=1}^{N_t} X_k \tag{2.1}$$

Where: u is the initial capital, r is the cost of credit, $X_t$is the claim amount at time t; $N_t$is the number of claims up to time t ($N_t$is the pure Poisson process with intensity μ, the intervalbetweentwo claims, isindependent and co-distributed, following an exponential distribution withparameterμ, expectation $\frac{1}{\mu}$); $X_t$is a sequence of p-levelregressiondependentrandom variables independent of N_t; the total claim amounts up to time t is$S_t = \sum_{k=1}^{N_t} X_k$.

$X_t$follows a P orderautoregressiveprocess, denoted$X_t \sim A(p)$ if satisfies:

$$X_t = a_1X_{t-1} + a_2X_{t-2} + \dots + a_pX_{t-p} + \varepsilon_t \tag{2.2}$$

Constants $a_1, a_2, \dots, a_p$ must satisfythese conditions:

polynomial $a(z) = 1 - \sum_{i=1}^{p} a_i z^i$ must have a solution with a modulusgreaterthan 1 (2.3)

ε_t satisfyingthese conditions: $E(\varepsilon_t) = 0, \text{cov}(\varepsilon_t, \varepsilon_s) = 0 (t \neq s), \text{var}(\varepsilon_t) = \sigma^2$;

denoted: $\varepsilon_t \sim WN(0;\sigma^2)$; ε_t called as white noise.

Ruinprobability to time t isdetermined by:

$$\psi(u,t) = P(\exists\tau \leq t : U(\tau) < 0) \quad (2.4)$$

2.2. The general model where there is no interest rate effect

In the general model where there is no interest rate effect, we assume that the capital of the insurance company at time t is:

$$U(t) = u + \sum_{i=1}^{N_t^1} X_i - \sum_{j=1}^{N_t^2} Y_j \quad (2.5)$$

Where: u is the initial capital; the series of proceeds amounts $X_1, X_2, \ldots, X_n$ depends on regressive level p; series of claim amount $Y_1, Y_2, \ldots, Y_n$ depends on regressive level q(X_t is independent on Y_t); N_t^1 is the number of claims up to time t with N_t^1 is the pure Poisson process with intensity $\mu_1 > 0$ (the time interval between two claims, is independent and co-distributed, following an exponential distribution with parameter μ_1, the expectation is $\frac{1}{\mu_1}$), X_t is independent on N_t^1; N_t^2 is the number of claims to time t with N_t^2 is the pure Poisson process with intensity $\mu_2 > 0$ (the time interval between two claims, is independent and co-distributed, following an exponential distribution with parameter μ_2, the expectation is $\frac{1}{\mu_2}$), Y_t is independent on N_t^2; N_t^1 is independent on N_t^2.

* X_t follows the autoregressive process of order p: $X_t \sim A(p)$

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + \ldots + a_p X_{t-p} + \varepsilon_t; \ \varepsilon_t \sim WN(0;\sigma_1^2) \quad (2.6)$$

Constants $a_1, a_2, \ldots, a_p$must satisfythese conditions:

polynomial $a(z) = 1 - \sum_{i=1}^{p} a_i z^i$ must have a solution with a modulusgreaterthan 1.

(2.7)

* Y_t follows the autoregressive process of level q: $Y_t \sim A(q)$

$$Y_t = b_1 Y_{t-1} + b_2 Y_{t-2} + \ldots + b_q Y_{t-q} + \varepsilon_t; \ \varepsilon_t \sim WN(0;\sigma_2^2) \quad (2.8)$$

Constants $b_1, b_2, \ldots, b_q$must satisfythese conditions:

polynomial $b(z) = 1 - \sum_{i=1}^{q} b_i z^i$ must have a solution with a modulusgreaterthan1

(2.9)

The ruin probability to time t is determined by:

$$\psi(u,t) = P(\exists\tau \le t : U(\tau) < 0) \quad (2.10)$$

3. The Monte Carlo simulation method approximates the ruin probability in the insurance problem

3.1. The Algorithm to simulate a sequence of regression dependent random variables

Algorithm 3.1.

Input: initial values of the autoregression model: $X_1, X_2, ..., X_p$; variance of white noise σ^2.

Output: X_t:X_t follows the autoregressive process of order p: $X_t \sim A(p)$

$$X_t = a_1X_{t-1} + a_2X_{t-2} + ... + a_pX_{t-p} + \varepsilon_t\ ;\ \varepsilon_t \sim WN(0;\sigma_1^2)$$

Steps of the algorithm:

Step 1. Simulate a sequence $\varepsilon_t \sim WN(0;\sigma_1^2)$

Step 1. CalculateX_t: $X_t = a_1X_{t-1} + a_2X_{t-2} + ... + a_pX_{t-p} + \varepsilon_t$

3.2. The Algorithm to simulate ruin probability for the model (2.1)

We see model (2.1) with a series of random variables $\{X_k\}_{k=1}^n$ depends on regressive level p. If we call $\{\tau_i\}_{i\ge1}$ as a series of independent random variables, with same distribution $E\{\mu\}$ (indicates the time between claims $\{T_i\}_{i=1}^{N_t}$), then we have:

$$N_t := \max\left\{k : \sum_{i=1}^{k}\tau_i := T_k \le t\right\}; \tau_o = T_o = 0, \tau_i = -\frac{\ln v_i}{\mu};\ v_i \sim U(0,\ 1)\ (i \ge 1) \quad (3.1)$$

In which, random numbers v_i ($i \ge 1$) is independent.

We, now, consider event A(t) (up to time t) of the problem (2.1):

$$\psi(u,t) = P\{A(t)\}, A(t) := \{\exists s \le t : U(s) < 0\}$$

The basis for simulating event A(t) is the following proposition:

Lemma 3.1. If we set $\psi(u,t) = A(t) := \{\exists s \le t : U(s) < 0\}$ then $A(t) = \bigcup_{i=0}^{N_t}\{U(T_i) < 0\}$

Prove:

Without losing of generality, we assume $N_t \ge 1$, we set

$$< T_{j-1}, T_j > := \begin{cases} (0, T_1) & \text{khi } j = 1, \\ [T_{j-1}, T_j) & \text{khi } j := 2 \div N_t, \\ [T_{N_t}, t] & \text{khi } j = N_t + 1. \end{cases}$$

Thenfrom (4.2) we have:

$$\bigcup_{j=1}^{N_t+1} < T_{j-1}, T_j > = (0,t],\ < T_{j-1}, T_j > \cap < T_{i-1}, T_i > = \varphi(\forall i \ne j)$$

To point out that:

$$U(s) = U(T_{j-1})(\forall s \in < T_{j-1}, T_j >, j = 1 \div N_t + 1),$$

And U(s) = U(T_o) = u > 0, $\forall s \in < T_o, T_1 >$.

Let $A_j(t) := \{\exists s \in < T_{j-1}, T_j >: U(s) < 0\}(\forall j = 1 \div N_t + 1)$. Then

$$A(t) = \bigcup_{j=1}^{N_t+1} A_j(t) = \bigcup_{j=2}^{N_t+1} A_j(t) \text{ because } A_1(t) := \{\exists s \in < T_o, T_1 >: U(s) < 0\} = \varphi.$$

On the other hand:

$$\{U(T_{j-1}) < 0\} \subset A_j(t) \subset \{U(T_{j-1}) < 0\} \Rightarrow A_j(t) = \{U(T_{j-1}) < 0, \forall j = 2 \div N^2(t) + 1\}$$

Then

$$A(t) = \bigcup_{j=2}^{N_t+1} \{U(T_{j-1}) < 0\} = \bigcup_{j=1}^{N_t} \{U(T_j) < 0\} \square.$$

From Lemma 3.1, the ruin probability at (2.4) is estimated as:

$$\psi(u,t) = P\{A(t)\} \approx \frac{M}{N}; A(t) := \{\exists s \le t : U(s) < 0\} = \bigcup_{i=0}^{N_t} \{U(T_i) < 0\} \qquad (3.2)$$

Where M is the number of occurrences of event A(t) in N simulations and M is determined by the following algorithm.

Algorithm 3.2.

Input: initial capital u, cost rate r, time t, number of simulations N, regression level p, parameter μ, autoregressive coefficient: a_1, a_2, ..., a_p; initial values of the autoregression model: x_1, x_2, ..., x_p; variance of white noise σ^2.

Output: Risk probability $\psi(u,t)$

Steps of the algorithm: First of all, assign M = 0, T_o = 0, U(T_o):= u.

Step A. (in the $n = \overline{1, N}$). With each i = 1, 2, … We do it as follows:

A1. Simulate the time to claim: $T_i = T_{i-1} + \tau_i$ with τ_i created according to the formula (3.1) and check inequality:

$$T_i \le t \quad (3.1a)$$

- If (3.1a) is false: terminate the n^{th} simulation of event A(t).
- If (3.1a) is true: move to step A2.

A2. Simulation of claim value X_i according to algorithm 3.1 to calculate (see (2.1)):

$U(T_i) = U(T_{i-1}) + r(T_i - T_{i-1}) - X_i$ and check

inequality: $U(T_i) \ge 0$ (3.1b)

- If (3.1b) is false: terminate the simulation at the n^{th} time of event A(t) and assign M: = M + 1
- If (3.1a) is true: Move back to step A1 with i: = i + 1

Notice that: the loop will stop when i = N_t (xem (3.1)) and finish the n^{th} simulation of event A(t).

Step B. After simulating N times event A(t) (repeat N times step A, approximately calculate the probability of risk: $\Psi(u,t) = \frac{M}{N}$.

3.2. Algorithm to simulate ruin probability for the model (2.2)

To describe the method, we consider the model (2.2) with the assumption that: series of amounts $\{X_i\}_{i\geq 1}$ dependent regression level p and the series of the claim amount $\{Y_j\}_{j\geq 1}$ is regressive dependence of level q.

Let $N_s^k \equiv N^k(s)(k=\overline{1,2})$ the Poisson process with intensity μ_k, represents the number of receiving times (when k = 1) and the number of payments (when k = 2) in period (0, s]. Let T_i^k is the receiving time (when = 1) and claim payment (when k= 2) in the ith time. Then similar to (4.1), we have:

$$N_s^k \equiv N^k(s) := \max\left\{ i : \sum_{j=0}^{i} \tau_j^k := T_i^k \leq s \right\}; \tau_o^k = T_o^k = 0\,(k=\overline{1,2}), \quad (3.3)$$

$$\tau_j^k := \frac{-\ln v_j^k}{\mu_k},\ v_j^k \sim U(0,\,1)\ (\forall j \geq 1, k=\overline{1,2}) \quad (3.4)$$

In which, for each $k=\overline{1,2}$, $v_j^k\ (j \geq 1)$ are independent random numbers. Then we can determine capital process $U(T_j^2)\,(j\geq 1)$ of the insurance company at the time of claim T_j^2, through the following proposition:

Lemma 3.2. With the above assumptions, if $N^2(t) > 0$ và $N^1(T_{j-1}^2) < N^2(T_j^2)\,(\forall j \geq 1)$ then almost sure (a.s) that:

$$\left.\begin{array}{l} 0 < T_1^1 < \ldots < T^1_{N^1(T_1^2)} \leq T_1^2 < T^1_{N^1(T_1^2)+1} < \ldots < T^1_{N^1(T_{j-1}^2)} \leq T_{j-1}^2 < T^1_{N^1(T_{j-1}^2)+1} \\ < \ldots < T^1_{N^1(T_j^2)} \leq T_j^2 < T^1_{N^1(T_j^2)+1} < \ldots < T^1_{N^1(T^2_{N^2(t)})} \leq T^2_{N^2(t)} \leq t \end{array}\right\} \quad (3.5)$$

Thenwe have:

$$U(T_j^2) = U(T_{j-1}^2) + X(T_j^2) - Y_j\ \ (j = 1 \div N^2(t)); U(T_o^2) = u, \quad (3.6)$$

Where

$$X(T_j^2) = \begin{cases} 0 & \text{khi } N^1(T_{j-1}^2) = N^1(T_j^2) \\ \sum_{i=N^1(T_{j-1}^2)+1}^{N^1(T_j^2)} X_i & \text{khi } N^1(T_{j-1}^2) < N^1(T_j^2) \end{cases} \quad (3.7)$$

2) In case $N^2(t) = 0$, we have:

$$U(\tau) \geq 0\,(\forall \tau \leq t) \quad (3.8)$$

Prove:

From the non-trivial properties of random variables $\tau_j^k \sim E(\mu_k)$ $(\forall j \geq 1)$ weinfer: $\tau_j^k > 0$ (h.c.c), $\forall j \geq 1$ thenfrom (3.3) we have:

$$0 < T_o^2 < T_1^2 < ... < T_{j-1}^2 < T_j^2 < ... < T_{N^2(t)}^2 \leq t < T_{N^2(t)+1}^2 \text{ (h.c.c)}. \quad (3.9)$$

Therefore, whenconsidering the definition of $N^1(s)$ (in (3.3)) with, respectively, value $s = T_j^2$ $(j = 1 \div N^2(t))$,weeasilyobtain (3.5).

Also, whenusing (3.3) with k = 2 and $s = T_j^2$, wealso have:

$$T_{N^2(T_j^2)}^2 = T_j^2 \Rightarrow N^2(T_j^2) = j \ (j = 1 \div N^2(t)). \quad (3.10)$$

On this basis we have the representation of $U(\tau)$ in (2.10) with $\tau = T_j^2$ in the form:

$$U(T_j^2) = u + \sum_{i=0}^{N^1(T_j^2)} X_i - \sum_{i=0}^{j} Y_i \ \ (1 \leq j \leq N^2(t)). \quad (3.11)$$

Whenreplacing j in the above formula by $j-1 \geq 1$, we have

$$U(T_{j-1}^2) = u + \sum_{i=0}^{N^1(T_{j-1}^2)} X_i - \sum_{i=0}^{j-1} Y_i \ \ (2 \leq j \leq N^2(t)). \quad (3.12)$$

For each $j = 2 \div N^2(t)$, werely on equations (3.6) and (3.12) to represent (3.11) in the form:

$$U(T_j^2) = \begin{cases} U(T_{j-1}^2) + X(T_j^2) - Y_j & \text{khi } N^1(T_{j-1}^2) < N^1(T_j^2) \\ U(T_{j-1}^2) - Y_j & \text{khi } N^1(T_{j-1}^2) = N^1(T_j^2) \end{cases}$$

Whichmeansthatwe have (3.6) for all $j = 2 \div N^2(t)$. Moreover, since $T_o^2 = 0, N^1(T_o^2) = 0$ (see (3.3)) so $U(T_o^2) = U(0) = u$. Then since $X_o = 0$ so when considering (3.11) with j =1, we can rely on (3.5) to infer:

$$U(T_1^2) = U(T_o^2) + \sum_{i=1}^{N^1(T_1^2)} X_i - Y_1 = U(T_o^2) + \sum_{i=N^1(T_o^2)+1}^{N^1(T_1^2)} X_i - Y_1 \ \text{ when } N^1(T_o^2) < N^1(T_1^2)$$

And $U(T_1^2) = U(T_o^2) - Y_1 = U(T_o^2) - Y_1$ when $N^1(T_o^2) = N^1(T_1^2)$ s

and we get (3.6) in both the case j = 1.

Finally, we consider the case: $N^2(t) = 0$. Since $0 \leq N^2(\tau) \leq N^2(t), \forall \tau \leq t$ (see (3.3), $N^2(\tau) = 0 \ (\forall \tau \leq t)$. Then formula u_τ in (2.10) has the form:

$$U(\tau) = u + \sum_{i=0}^{N^1(\tau)} X_i - Y_o = u + \sum_{i=0}^{N^1(\tau)} X_i \ (\forall \tau \leq t)$$

Since u > 0 and $X_i \sim E(\overline{\mu}_i)\,(i \geq 1)$ are non-negative random variables, from the above formula, we directly deduce (3.8)□.

Now we consider the risky event A(t) (up to time t) of problem (2.2):

$$\psi(u,t) = P\{A(t)\},\ A(t) := \{\exists s \leq t : U(s) < 0\} \tag{3.13}$$

The basis for simulating event A(t) is the following proposition:

Lemma 3.3. In the conditions of Lemma 3.2, we have the following conclusions:

1. If $N^2(t) \geq 1$, then

$$A(t) = B(t) := \bigcup_{j=1}^{N^2(t)} \{U(T_j^2) < 0\}. \tag{3.14}$$

Then event A(t) will not occur, if:

$$U(T_j^2) \geq 0\,(\forall j = 1 \div N^2(t)). \tag{3.15}$$

2- Event A(t) also does not occur, if:

$$N^2(t) = 0 \Leftrightarrow \tau_1^2 = \frac{-\ln v_1^2}{\mu_2} > t,\ (v_1^2 \sim U(0,1)). \tag{3.16}$$

Prove:

In the case of $N^2(t) \geq 1$, we assign

$$< T_{j-1}^2, T_j^2 > := \begin{cases} (0, T_1^2) & \text{khi } j = 1 \\ [T_{j-1}^2, T_j^2) & \text{khi } j := 2 \div N^2(t), \\ [T_{N^2(t)}^2, t] & \text{khi } j = N^2(t) + 1. \end{cases} \tag{3.17}$$

Thenfrom (3.9) we have:

$$\bigcup_{j=1}^{N^2(t)+1} < T_{j-1}^2, T_j^2 > = (0,t],\ < T_{j-1}^2, T_j^2 > \cap < T_{i-1}^2, T_i^2 > = \varphi\,(\forall i \neq j) \tag{3.18}$$

To show that:

$$U(s) \geq U(T_{j-1}^2)\,(\forall s \in < T_{j-1}^2, T_j^2 >, j = 1 \div N^2(t) + 1), \tag{3.19}$$

Firstly, we consider the case j= 1 meaning that (see (3.17)): $0 < s < T_1^2$. In this case, we have (see (3.3), (3.9)):

$$N^1(s) \geq 0, 0 = T_o^2 \leq N^2(s) \leq s < T_1^2 \Rightarrow N^2(s) = 0.$$

Therefore, from (2.10) we get:

$$U(s) = u + \sum_{i=0}^{N^1(s)} X_i \geq u = U(0) = U(T_o^2) > 0(\forall s \in < T_o^2, T_1^2 >). \tag{3.20}$$

Which means that we obtained (3.19) with j = 1. Next, we consider case $j = 2 \div N^2(t)$, in which (see (3.17)): $T^2_{j-1} \le s < T^2_j$. Then from (3.9) and (3.3) we have: $N^2(s) = N^2(T^2_{j-1}) = j-1, N^1(s) \ge N^1(T^2_{j-1})$. Therefore, from (2.0), (3.10) and (3.12) we deduce:

$$U(s) \ge u + \sum_{i=0}^{N^1(T^2_{j-1})} X_i - \sum_{i=0}^{j-1} Y_i = U(T^2_{j-1}) \, (\forall s \in < T^2_{j-1}, T^2_j >)$$

And obtain (3.19) with all $j = 2 \div N^2(t)$. Finally, case $j = N^2(t)+1$, where $s \in [T^2_{N^2(t)}, t]$. When $T^2_{N^2(t)} = t$ then (3.19) is obvious. When $T^2_{N^2(t)} < t$ then from (3.9) we have $T^2_{N^2(t)} \le s \le t < T^2_{N^2(t)+1}$ and similar to the above case, we obtain (3.19) in both cases. Then the formula (3.19) is completely proved.

To prove (3.14), firstly, we let:

$$A_j(t) := \left\{ \exists s \in < T^2_{j-1}, T^2_j >: U(s) < 0 \right\} (\forall j = 1 \div N^2(t)+1) \qquad (3.21)$$

In which (see (3.20)): $A_1(t) := \left\{ \exists s \in (0, T^2_1) : U(s) < 0 \right\} = \varphi$. Then from (3.13) và (3.18), it is easy to see that: $A(t) = \bigcup_{j=1}^{N^2(t)+1} A_j(t) = \bigcup_{j=2}^{N^2(t)+1} A_j(t)$ (3.22)

But from (3.19) and (3.21) we also find:

$$\left\{ U(T^2_{j-1}) < 0 \right\} \subset A_j(t) \subset \left\{ U(T^2_{j-1}) < 0 \right\} \Rightarrow A_j(t) = \left\{ U(T^2_{j-1}) < 0, \forall j = 2 \div N^2(t)+1 \right\},$$

On this basis and (3.22) we get:

$$A(t) = \bigcup_{j=1}^{N^2(t)+1} A_j(t) = \bigcup_{j=2}^{N^2(t)+1} \left\{ U(T^2_{j-1}) < 0 \right\} = \bigcup_{j=1}^{N^2(t)} \left\{ U(T^2_j) < 0 \right\},$$

Means (3:14) is proven. When letting:

$$B_j(t) := \left\{ U(T^2_j) < 0 \right\} \Leftrightarrow \overline{B_j(t)} := \left\{ U(T^2_j) \ge 0 \right\} (\forall j = 1 \div N^2(t)+1),$$

We rely on (3.14) and the D' Morgan duality rule to infer:

$$\overline{A(t)} = \overline{B(t)} = \bigcap_{j=1}^{N^2(t)} \overline{B_j(t)} = \left\{ U(T^2_j) \ge 0, \forall j = 1 \div N^2(t) \right\}.$$

Therefore, in condition (3.15) event A(t) will not occur and conclusion number 1 is completely proved.

To prove the rest, we rely on (3.4) and (3.5) to deduce the equivalence of the following events:

$$\left\{ N^2(t) = 0 \right\} = \left\{ \tau^2_1 = \frac{-\ln v^2_1}{\mu_2} > t \right\}, \; v^2_1 \sim U(0, 1).$$

When the above event has occurred, from (3.8) and (3.13) we find that event A(t) will not happen and we get the conclusion number 2.

Since random variables $U(T_j^2)$ can be simulated by Lemma 3.2, so random event A(t) can also be simulated according to Lemma 3.3. Therefore, we can approximate the solution of problem (2.10) in the following form:

$$\psi(u,t) = P\{A(t)\} \approx \frac{M}{N} \tag{3.23}$$

Where M is the number of occurrences of event A(t) in N simulations and determined by the following algorithm:

Algorithm 3.3.

Input: initial capital u, time t, number of simulations N, parameter μ_1, parameter μ_2, variance of white noises σ_1^2, σ_2^2.

+ Data of X_t: Regression level p, autoregression coefficient: $a_1, a_2, ..., a_p$; initial values of the autoregressive model: $x_1, x_2, ..., x_p$;

+ Data of Y_t: Regression level q, autoregression coefficient: $b_1, b_2, ..., b_q$; initial values of the autoregressive model: $y_1, y_2, ..., y_q$.

Output: Risk probability $\psi(u,t)$

Comment: For the problem of determining the risk probability of this model, we only need to calculate and check the condition that capital receives negative values at the time of claim as in Lemma 4.2 and Lemma 3.3.

Steps of the algorithm:

Firstly, let M = 0, $T_o^2 = T_o^1 = 0, U(T_o^2) = u$

Step A. With each j =1,2, … we perform the following steps:

A1. Simulate the time to claim T_j^2 (after the time of claiming T_{j-1}^2 in the previous time) by this formula: $T_j^2 := T_{j-1}^2 - \frac{\ln v_j^2}{\mu_2}$, $v_j^2 \sim U(0, 1)$, and check the inequality:

$$T_1^2 \le t \tag{3.23a}$$

* If (3.23a) is false: terminate the n^th^ time simulation of event A(t).

* If (3.23a) is true: simulate Y_j depending on regression according to algorithm 3.1 and we move to step A2.

A2. Simulate the time to claim T_i^1 $(i = N^1(T_{j-1}^2) + 1 \div N^1(T_j^2))$ according to the iterative formula:

$$T_i^1 := T_{i-1}^1 - \frac{\ln v_i^1}{\mu_1},\ v_i^1 \sim U(0, 1)$$

Where $N^1(T_j^2) = N^1(T_{j-1}^2)$ when $T^1_{N^1(T_{j-1}^2)+1} > T_j^2$. Otherwise, $N^1(T_j^2)$ is selected from the condition:

$T^1_{N^1(T_{j-1}^2)} < T^1_{N^1(T_{j-1}^2)+1} < ... < T^1_{N^1(T_j^2)} \le T_j^2 < T^1_{N^1(T_j^2)+1}$.

A3. Stimulate X_i depending on regression according to algorithm 3.1 ($i = N^1(T_{j-1}^2)+1 \div N^1(T_j^2)$), so as to:

A4. Calculate $U(T_j^2)$ according to formula (3.6) and check inequality:

$$U(T_j^2) \ge 0 \qquad (3.23b)$$

- If (3.23a) is true: Move back to step A1, with j : = j + 1.

- If (3.23b) is false: terminate the n^{th} time simulation of event A(t) and assign M: = M +1.

Step B: After simulating N times event B(t) (repeat N times step A), approximately calculate the ruin probability: $\Psi(u,t) = \frac{M}{N}$.

Notice 3.1. The aforementioned loop will stop with $j = N^2(t): T^2_{N^2(t)} \le t < T^2_{N^2(t)+1}$. Then we finish the n^{th} time simulation of event A(t) (see (3.15)). In case $N^2(t) = 0$ (see (3.16), the n^{th} time simulation of event A(t) will end immediately at step A1 with j = 1.

4. Numerical experiment results

4.1. Simulation results of the model's ruin probability (2.1)

With input data: initial capital takes values: u = 2; u = 3; u = 4; u = 5; u = 6; u = 7; time t gets values: t = 4, t = 6, t = 10; number of simulations N = 1000; interest rate r = 0,088; Poisson distribution time series with mean μ = 2,5.

*The claim process follows the autoregressive process level p = 1:

$$X_t = 0,59X_{t-1} + \varepsilon_t;\ \varepsilon_t \sim WN(0,\ \sigma^2) \text{ with } \sigma^2 = 0,37^2 \qquad (4.1)$$

We have compiled calculation software in Maple environment to demonstrate algorithm 3.2, when running this program on PC - Pentium 4 we obtain simulation results of ruin probability for model (2.1) with hypothesis (4.1) given in table 4.1 below:

Initial capital	Number of simulations	Interest rate	Parameters	Deviation of WN	Regression level	Initial value of X_t	Regression coefficient	Probability of bankruptcy $\psi(u,t)$		
u	N	r	M	Σ	P	x	a	t = 4	t = 6	t = 10
2	1000	0.088	2.5	0.37	1	0.79	0.59	0,5620	0,6450	0,6950
3								0,3180	0,4220	0,5290
4								0,1750	0,2800	0,3650
5								0,0740	0,1600	0,2490
6								0,0330	0,1050	0,1690
7								0,0160	0,0440	0,1300

Table 4.1. Simulating the ruin probability of the model (2.1) with assumption (4.1)

* The claim process follows the autoregressive process level p = 2:

$$X_t = 0,59X_{t-1} + 0,07X_{t-2} + \varepsilon_t\ ;\ \varepsilon_t \sim WN(0,\ \sigma^2) \text{ with } \sigma^2 = 0,37^2 \qquad (4.2)$$

We have compiled calculation software in Maple environment to demonstrate algorithm 3.2, when running this program on PC - Pentium 4, we obtain simulation results of ruin probability for model (2.1) with hypothesis (4.2) given in table 4.2 below:

Initial capital	Number of simulations	Interest rate	Parameters	Deviation of WN	Regression level	Initial value of X_t	Regression coefficient	Probability of bankruptcy $\psi(u,t)$		
u	N	r	μ	Σ	p	x	a	t = 4	t = 6	t = 10
2	1000	0.088	2.5	0.37	2	0.79	0.59	0,8020	0,8450	0,8870

3						0.53	0.07	0,5460	0,5930	0,7280
4								0,2990	0,4300	0,5400
5								0,1530	0,2800	0,3810
6								0,0980	0,1860	0,2700
7								0,0430	0,1070	0,2090

Table 4.2. Simulating the ruin probability of the model (2.1) with assumption (4.2)

* The claim process follows the autoregressive process level p = 3:

$$X_t = 0,59X_{t-1} - 0,07X_{t-2} + 0,017X_{t-3} + \varepsilon_t\ ;\ \varepsilon_t \sim WN(0,\ \sigma^2)\text{ with } \sigma^2 = 0,37^2 \quad (4.3)$$

We have compiled calculation software in Maple environment to demonstrate algorithm 3.2, when running this program on PC - Pentium 4, we obtain simulation results of ruin probability for model (2.1) with hypothesis (4.3) given in table 4.3 below:

Initial capital	**Number of simulations**	**interest rate**	**Parameters**	**Deviation of WN**	**Regression level**	**Initial value of X_t**	**Regression coefficient**	**Probability of bankruptcy $\psi(u,t)$**		
u	N	r	μ	σ	p	X	A	t = 4	t = 6	t = 10
2	1000	0.088	2.5	0.37	3	1.24	0.59	0,8970	0,8990	0,9200
3						0.62	-0.07	0,4970	0,6010	0,6900
4						0.48	0.017	0,2430	0,3520	0,4830
5								0,1080	0,1820	0,3090
6								0,0420	0,0830	0,2300
7								0,0140	0,0450	0,1000

Table 4.3. Simulating the ruin probability of the model (2.1) with assumption (4.3)

4.2. Simulation results of the model's ruin probability (2.5)

With input data: u = 2; u = 3; u = 4; u = 5; u = 6; u = 7; time t gets values: t = 4, t = 6, t = 10; number of simulations N = 1000; the time series of premium claim amounts with a Poisson distribution with mean $\mu_1 = 4$; the time series of premium claim amounts with a Poisson distribution with mean $\mu_2 = 2$;

* X_t follows the autoregressive process level p =2:

$$X_t = 0,79X_{t-1} + 0,07X_{t-2} + \varepsilon_t;\ \varepsilon_t \sim WN(0,\ \sigma_1^2)\text{ với }\sigma_1^2 = 0,17^2 \quad (4.4a)$$

Y_t follows the autoregressive process level q = 2:

$$Y_t = 0,46Y_{t-1} + 0,21Y_{t-2} + \varepsilon_t;\ \varepsilon_t \sim WN(0,\ \sigma_2^2)\text{với }\sigma_2^2 = 0,13^2 \quad (4.4b)$$

The data of processes X_t, Y_t are given by the following table 4.4:

Data of X_t			**Data of Y_t**		
Level	**Initial value**	**Coefficient**	**Level**	**Initial value**	**Coefficient**
p = 2	0,85	0,79	q = 2	1,12	0,46
	1,2	0,07		0,54	0,21

Table 4.4. The data of regression processes X_t, Y_t

We have compiled calculation software in Maple environment to demonstrate algorithm 3.3, when running this program on PC - Pentium 4, we obtain simulation results of bankruptcy probability for model (2.5) with hypothesis (4.4a) and (4.4b) given in table 4.3 below:

Initial capital	**Ruin Probability** $\psi(u,t)$		
u	t = 4	t = 6	t = 10
2	0,037	0,076	0,165
3	0,003	0,018	0,079

4	0,001	0,014	0,053
5	0,000	0,008	0,052
6	0,000	0,003	0,045
7	0,000	0,003	0,022

Table 4.5. Simulating the risk probability of the model (2.5) with assumptions (4.4a), (4.4b)

* X_t follows the autoregressive process level p =3:

$$X_t = 0,59X_{t-1} + 0,07X_{t-2} + 0,017X_{t-3} + \varepsilon_t \,;\ \varepsilon_t \sim WN(0,\ \sigma_1^2) \text{ with } \sigma_1^2 = 0,17^2 \quad (4.5a)$$

Y_t follows the autoregressive process level q =4:

$$Y_t = 0,57Y_{t-1} + 0,13Y_{t-2} - 0,31Y_{t-3} + 0,019Y_{t-4} + \varepsilon_t \,;\ \varepsilon_t \sim WN(0,\ \sigma_2^2) \text{ with } \sigma_2^2 = 0,13^2 \quad (4.5b)$$

The data of processes X_t, Y_t are given by the following table 4.6:

Data of X_t			**Data of Y_t**		
Level	**Initial value**	**Coefficient**	**Level**	**Initial value**	**Coefficient**
p = 3	0,9	0,59	q = 4	1,24	0,57
	1,06	0,07		0,76	0,13
	0,31	0,017		0,94	-0,31
				1,3	0,019

Table 4.6. The data of regression processes X_t, Y_t

We have compiled calculation software in Maple environment to demonstrate algorithm 3.3, when running this program on PC - Pentium 4, we obtain simulation results of ruin probability for model (2.5) with hypothesis (4.5a) and (4.5b) given in table 4.7 below:

Initial Capital	Ruin Probability $\psi(u,t)$		
u	t = 4	t = 6	t = 10
2	0,6240	0,7050	0,7720
3	0,2840	0,4000	0,5000
4	0,0900	0,1930	0,2820
5	0,0210	0,0077	0,1740
6	0,0050	0,0230	0,1110
7	0,0010	0,0110	0,0480

Table 4.7. Simulating the ruin probability of the model (2.5) with assumption (4.5a), (4.5b)

* X_t follows the autoregressive process level p = 4:

$$X_t = 0,59X_{t-1} + 0,07X_{t-2} - 0,017X_{t-3} + 0,0012X_{t-4} + \varepsilon_t \tag{4.6a}$$

$\varepsilon_t \sim WN(0, \sigma_1^2)$ with $\sigma_1^2 = 0,17^2$.

Y_t follows the autoregressive process level q =5:

$$Y_t = 0,57Y_{t-1} + 0,13Y_{t-2} - 0,31Y_{t-3} + 0,019Y_{t-4} + 0,008X_{t-5} + \varepsilon_t \tag{4.6b}$$

$\varepsilon_t \sim WN(0, \sigma_2^2)$ with $\sigma_2^2 = 0,13^2$.

The data of processes X_t, Y_t are given by the following table 4.8:

Data of X_t			Data of Y_t		
Level	**Initial value**	**Coefficient**	**Level**	**Initial value**	**Coefficient**
p = 4	0,9	0,59	q = 5	1,24	0,57
	1,06	0,07		0,76	0,13

	0,31	- 0,017		0,94	- 0,31
	0,12	0,0012		1,32	0,019
				0,52	0,008

Table 4.8. The data of regression processes X_t, Y_t

We have compiled calculation software in Maple environment to demonstrate algorithm 3.3, when running this program on PC - Pentium 4, we obtain simulation results of ruin probability for model (2.5) with hypothesis (4.6a) and (4.6b) given in table 4.9 below:

Initial capital	**Ruin Probability** $\psi(u, t)$		
u	t = 4	t = 6	t = 10
2	0,704	0,7620	0,8450
3	0,306	0,4370	0,5520
4	0,108	0,2160	0,3300
5	0,027	0,0810	0,2120
6	0,005	0,0330	0,1190
7	0,002	0,0140	0,0560

Table 4.9. Simulating the ruin probability of the model (2.5) with (2.15a), (2.15b)

5. Conclusion

The paper has built the theoretical basis of lemma 3.1, lemma 3.2, lemma 3.3, from which, It has built algorithms 3.2 and 3.3 to simulate ruin probability for model (2.1) and model (2.5) with a series of regression dependent random variables. From the results of approximately calculating the ruin probability for model (2.1) given in table 4.1, table 4.2, table 4.3 and model (2.5) given in table 4.5, table 4.7, table 4.9 shows the conformity of the results of quantitative research with qualitative research, specifically:

When increasing the initial capital u of insurance companies, the ruin probability. For each level of capital u, as time t increases, the ruin probability will increase.

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