Exponential Synchronization on Impulsive Fractional Order Neural Networks with Time Delay

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Abstract

This paper examines a class of impulsive Caputo fractional order neural networks(FONNs) having time varying delays.We investigate exponential synchronization for these FONNs. For this, we take into account an appropriate Lyapunov function and obtain the synchronization results' conditions as Linear Matrix Inequalities (LMI). The efficiency of the outcome we have obtained is verified using an example.

Keywords— impulsive Caputo fractional order neural network · exponential synchronization · time-varying delays · linear matrix inequality · Lyapunov function

1 Introduction

Artificial neural networks, or simply, neural networks, whose performance is relatively established on how animal neurons function, are interconnected association of simple processing units. The processing ability of the network is acquired by reviewing from a set of training patterns. This ability is stored in the inter-unit connection strengths, usually known as weight [11]. The last few decades have seen an increase in the importance of neural networks due to its broad range of applications like pattern recognition, independent component analysis, weather prediction, handwriting recognition, autopilot, robotics and so on [1][2][20].As new results are established for neural networks its different dynamic behaviors are thoroughly studied on many kinds of neural networks. For instance, the stability [4-8],synchronization [27-29] passivity [15], exponential stabilization [3][9][12][37],exponential synchronization [16-20], Mittag-Leffler stability and/or synchronization [30-32], impulsive synchronization [21]-[26],H_∞ control problem[33],bifurcation [10][36] and so on. We came into fractional calculus by simply altering the usual integer order to non-integer order.[12]. Fractional-order

model is preferred over integer-order model due to different advantages.The behavior of real-world situations is accurately noted and practical processes are analyzed by fractional-order model.[5][12][27]
Usually, one faces time-delay in many engineering structures such as airliner, control systems, communication systems and so on. Time delay results in instability, divergence, chaos or other poor conducts of the system[20][14][8].In [15][31][32][34] consequences of time delays in FONNs is discussed.
Synchronization primarily signify the dynamic behavior of a system designed to simulate another. This simply means that the state trajectory of the said systems are similar in the end[20].Many applications of synchronization are seen in several areas namely parallel image processing, secure communication , transmission of digital signals, [22] and so on. In order to achieve fast synchronization, exponential synchronization is used[16].

In [12] exponential synchronization of fractional-order Cohen–Grossberg neural networks is considered as a function of stabilization of fractional order impulse control systems.Along with this, we also consider many synchronization conditions subsequent to impulsive control. From [34], we only take the case where zero disturbance is taken for FONNs and we find solution to the stabilization of H_∞ control for the system.

Motivated by the aforementioned comments, this paper examines a class of impulsive Caputo FONNs having time varying delays. Its corresponding response system is taken and hence the error system is found. Our aim is to establish exponential synchronization of this system. The focus is on achieving the convergence in order for the system to be exponentially synchronized faster than the studies done by now. For that, we employ a convex Lyapunov function for our system and obtain the synchronization results' conditions as LMIs .
The remnant sections are along these lines; section 2 comprises of some preliminaries required for the study. Also, our FONNs, its corresponding response system and the resulting error system is introduced here.Section 3 covers the main results that we have obtained: i.e. the conditions for the system taken to reach exponential synchronization in terms of certain LMIs. Section 4 contains a numerical example which verifies the result that was obtained in section 3. Section 5 sums up the paper and shows the direction in which the forthcoming study in this topic can be taken.

Notations: Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ imply the set of n-dimensional real vectors and $n \times m$ matrices, respectively. I_n and 0_n stands for the $n \times n$ dimensional identity and zero matrix, respectively. $0_{n\times m}$ designate the $n \times m$ dimensional zero matrix. consider matrices $A, B \in \mathbb{R}^{n\times n}$. $sym(A)$ stands for $A + A^T$. Matrix A is symmetric positive definite, with notation $A > 0$ if $A = A^T$ and $x^T Ax > 0 \;\forall\; x \in \mathbb{R}^n$. Matrix B is symmetric semi-positive-definite, with notation $B \geq 0$ if $B = B^T$ and $x^T Bx \geq 0 \;\forall\; x \in \mathbb{R}^n$. Set $\mathbb{P}^n$ and $\bar{\mathbb{P}}^n$ to be the set of symmetric semi-positive definite and set of symmetric positive definite matrices in $\mathbb{R}^{n\times n}$ and $\mathbb{G}^n$ the set of positive diagonal matrices i.e. a matrix $\Lambda = diag\{\lambda_1, ...\lambda_n\} \in \mathbb{G}^n$ if $\lambda_i > 0(i = 1, 2, ..., n)$.

2 Preliminaries and Model Description

Definition 1 (12). *The Caputo fractional derivative* ${}^C D_t^{\mu} f(t)$ *for differentiable* $f : [a, b] \to \mathbb{R}, \mu \in (0, 1)$ *is defined as;*

$$ {}^C D_t^{\mu} f(t) = \frac{1}{\Gamma(1-\mu)} \int_0^t \frac{f'(s)}{(t-s)^{\mu}} \, ds \qquad t \geq 0 $$

Here $\Gamma(.)$ *denotes the gamma function;*

$$ \Gamma(s) = \int_0^{\infty} e^{-t} t^{s-1} dt, s > 0 $$

For our convenience we will also use $D_t^{\mu} f(t)$ to denote the Caputo fractional derivative.

Property 1 (34). *For any constants* λ_1 *and* λ_2 *and two functions* $f_1(t)$ *and* $f_2(t)$,

$$ D_t^{\mu}(\lambda_1 f_1(t) + \lambda_2 f_2(t)) = \lambda_1 D_t^{\mu} f_1(t) + \lambda_2 D_t^{\mu} f_2(t) $$

Examine the subsequent fractional order neural network having a time varying delay:

$$ \begin{aligned} D_t^{\mu} x(t) &= -Lx(t) + Mh(x(t)) + Nh(x(t - g(t))) \qquad t \neq t_k, t > 0 \\ x(t_k^+) &= (\frac{D_k}{\Gamma(\mu+1)}) x(t_k^-) \qquad\qquad t = t_k \\ x(t_0) &= \phi(t) \qquad\qquad t \in [-g, 0] \end{aligned} \tag{1} $$

where $\mu \in (0, 1)$, $x(t) \in \mathbb{R}^n$ is the neuron state vector, n is the number of neurons in the fractional order neural networks,
$h(x(t)) = (h_1(x_1(t)), h_2(x_2(t)), \ldots, h_n(x_n(t)))^T \in \mathbb{R}^n$ denotes the neuron activation function, $L = diag\{m_1, \ldots, m_n\} \in \mathbb{G}^n$, $M, N \in \mathbb{R}^{n \times n}$ are known constant matrices, $g(t)$ is a time varying delay satisfying $0 \leq g(t) \leq g$ where g is a known positive constant, $\phi(t)$ is a vector valued continuous function, $t_k < t_{k+1}$ for each $k \in \mathbb{N}$, and $D_k \in \mathbb{R}^{n \times n}$ and $D_k > 0$ and $\lim_{k \to +\infty} t_k = \infty$.
If we take (1) to be the drive system its response system is taken as

$$ \begin{aligned} D_t^{\mu} y(t) &= -Ly(t) + Mh(y(t)) + Nh(y(t - g(t))) \qquad t \neq t_k, t > 0 \\ y(t_k^+) &= (\frac{D_k}{\Gamma(\mu+1)}) y(t_k^-) \qquad\qquad t = t_k \\ y(t_0) &= \varphi(t) \qquad\qquad t \in [-g, 0] \end{aligned} \tag{2} $$

We take

$$e(t) = y(t) - x(t)$$

to be the synchronization error, then the error system is obtained from (1) and (2) as

$$\begin{aligned} D_t^{\mu} e(t) &= -Le(t) + Mh(e(t)) + Nh(e(t-g(t))) \qquad t \neq t_k, t > 0 \\ e(t_k^+) &= (\frac{D_k}{\Gamma(\mu+1)}) e(t_k^-) \qquad\qquad t = t_k \\ e(t_0) &= \psi(t) = \varphi(t) - \phi(t) \end{aligned} \tag{3}$$

Assumption 1 (34)**.** *Any activation function $h_i(.)$ is continuous, bounded and fulfills the condition*

$$l_i^- \leq \frac{h_i(c) - h_i(d)}{c-d} \leq l_i^+ \qquad i = 1, 2, \ldots, n$$

where $h_i(0) = 0$, l_i^+, l_i^- are known real constants ($i = 1, 2, \ldots, n$) , $c, d \in \mathbb{R}$ and $c \neq d$.

Lemma 1 (34)**.** *Given $\mu \in (0, 1]$, $e(t) \in \mathbb{R}^n$ be a continuous function and $U : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be a convex and differentiable function on $\mathbb{R}^n$ such that $U(0) = 0$. We have*

$$D_t^{\mu} U(e(t)) \leq \langle \Delta U(e(t)), D_t^{\mu} e(t) \rangle \qquad t \geq 0$$

where $\Delta U(.)$ is the gradient of U and $\langle , \rangle$ is the inner product.

Lemma 2. *For real valued function $U(t)$ on $[a, \infty)$ and $a \in \mathbb{R}$, if there exists a constant θ in a manner that*

$$D_t^{\mu} U(t) \leq \theta U(t), 0 < \mu \leq 1$$

then

$$\begin{aligned} U(t) &\leq U(a) e^{\int_a^t \frac{\theta (t-\tau)^{\mu-1} d\tau}{\Gamma(a)}} \\ &= U(a) e^{\frac{\theta (t-a)^{\mu}}{\Gamma(a+1)}} \end{aligned}$$

3 Main Result

Theorem 1. *Consider the case where Assumption 1 is true. System (3) is exponentially synchronized if there exists* $Q, K \in \bar{\mathbb{P}}^n$*,* $Y \in \mathbb{P}^{3n}$*,* $\Lambda_i = diag\{\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{in}\} \in \mathbb{G}^n (i = 1, 2, \ldots, n), K_k \in \mathbb{R}$ *satisfying the subsequent LMI*

$$g^{\mu}\mu^{-1}\Upsilon^T Y \Upsilon + \sum_{i=1}^{3} \Phi_i - \beta Q < 0 \tag{4}$$

$$-K_k Q + (\frac{D_k}{\Gamma(\mu+1)})^T Q (\frac{D_k}{\Gamma(\mu+1)}) \leq 0 \tag{5}$$

and the condition, if $0 < t_{k+1} - t_k \leq \tau$*,*

$$ln(K_k) < \frac{-(\nu - Z)\tau^{\mu}}{\Gamma(\mu+1)} \tag{6}$$

$$\tau, \nu, \beta > 0, Z \in \mathbb{R}$$

$\Pi_1 = \begin{bmatrix} I_n & 0_n & 0_n & 0_n & 0_n \end{bmatrix}$

$\Pi_2 = \begin{bmatrix} 0_n & I_n & 0_n & 0_n & 0_n \end{bmatrix}$

$\Pi_3 = \begin{bmatrix} 0_n & 0_n & I_n & 0_n & 0_n \end{bmatrix}$

$\Pi_4 = \begin{bmatrix} 0_n & 0_n & 0_n & I_n & 0_n \end{bmatrix}$

$\Pi_5 = \begin{bmatrix} 0_n & 0_n & 0_n & 0_n & I_n \end{bmatrix}$

$\Theta_1 = diag\{l_1^-, \ldots, l_n^-\}$

$\Theta_2 = diag\{l_1^+, \ldots, l_n^+\}$

$\Upsilon = \begin{bmatrix} \Pi_1^T & \Pi_2^T & \Pi_5^T \end{bmatrix}^T$

$\Upsilon_1 = \Pi_3 - \Theta_1 \Pi_1$

$\Upsilon_2 = \Theta_2 \Pi_1 - \Pi_3$

$\Upsilon_3 = \Pi_4 - \Theta_1\Pi_2$

$\Upsilon_4 = \Theta_2\Pi_2 - \Pi_4$

$\Upsilon_5 = \Pi_3 - \Pi_4 - \Theta_1(\Pi_1 - \Pi_2)$

$\Upsilon_6 = \Theta_2(\Pi_1 - \Pi_2) - \Pi_3 + \Pi_4$

$\Phi_1 = sym(\Pi_1^T Q\Pi_5 - \Pi_1^T Ke_5 + \Pi_1^T KM\Pi_3 + \Pi_1^T KN\Pi_4 - \Pi_5^T KL\Pi_1 + \Pi_5^T KM\Pi_3 + \Pi_5^T KN\Pi_4)$

$\Phi_2 = \Pi_1^T[-KL - LK + Q]\Pi_1 - \Pi_2^T Q\Pi_2 + \Pi_5^T[-2K]\Pi_5$

$\Phi_3 = sym(\Upsilon_1^T\Lambda_1\Upsilon_2 + \Upsilon_3^T\Lambda_2\Upsilon_4 + \Upsilon_5^T\Lambda_3\Upsilon_6)$

Proof. First we will look at the case when $t \neq t_k$. Let

$$U(t) = U(t, e(t)) = e^T(t)Qe(t)$$

be the Lyapunov function for system (3)

We observe that $U(t)$ is a differentiable and convex function on $\mathbb{R}^n$ and $U(t, 0) = 0$. Hence on applying Lemma 1, Caputo fractional derivative of the system (3) of order μ is calculated in this way:

$$\begin{aligned} D_t^\mu U(t, e(t)) &\leq 2e^T(t)QD_t^\mu e(t) \\ &= \gamma^T(t)sym(\Pi_1^T Q\Pi_5)\gamma(t) \end{aligned} \tag{7}$$

where

$$\gamma(t) = \begin{bmatrix} e^T(t) & e^T(t-g(t)) & h^T(e(t)) & h^T(e(t-g(t))) & (D_t^\mu e(t))^T \end{bmatrix}^T$$

For any matrix $Y \in \mathbb{P}^{3n}$, the following holds

$$g^\mu \mu^{-1}\zeta^T(t)Y\zeta(t) - \int_{t-g(t)}^{t}(t-s)^{\mu-1}\zeta^T(t)Y\zeta(t)ds \geq 0 \tag{8}$$

where

$$\zeta(t) = \begin{bmatrix} e^T(t) & e^T(t-g(t)) & (D_t^\mu e(t))^T \end{bmatrix}^T$$

Then again, for any $K \in \bar{\mathbb{P}}^n$, the following equation can be attained from the system (3)

$$[2e^T(t) + 2(D_t^\mu e(t))^T]K \times [-D_t^\mu e(t) - Le(t) + Mh(e(t)) + Nh(e(t - g(t)))] = 0 \quad (9)$$

From Assumption 1, we get that for any $\lambda_{ji} > 0$ ($i = 1, 2, \ldots, n, j = 1, 2, 3$)

$$2(h_i(e_i(t)) - l_i^- e_i(t))\lambda_{1i}(l_i^+ e_i(t) - h_i(e_i(t))) \geq 0$$
$$2(h_i(e_i(t - g(t))) - l_i^- e_i(t - g(t)))\lambda_{2i}(l_i^+ e_i(t - g(t)) - h_i(e_i(t - g(t)))) \geq 0$$

$$\begin{aligned} 2(h_i(e_i(t)) - h_i(e_i(t - h(t))) - l_i^- (e_i(t) - e_i(t - h(t))))\lambda_{3i}(l_i^+ (e_i(t) \\ - e_i(t - h(t))) - h_i(e_i(t)) + h_i(e_i(t - g(t)))) \geq 0 \end{aligned}$$

which imply

$$\begin{aligned} 2\gamma^T(t)\Upsilon_1^T \Lambda_1 \Upsilon_2 \gamma(t) \geq 0 \\ 2\gamma^T(t)\Upsilon_3^T \Lambda_2 \Upsilon_4 \gamma(t) \geq 0 \\ 2\gamma^T(t)\Upsilon_5^T \Lambda_3 \Upsilon_6 \gamma(t) \geq 0 \end{aligned} \quad (10)$$

Since

$$U(t, e(t)) = e^T(t)Pe(t)$$

taking some real number $\sigma > 1$ we presume that

$$U(t + s, e(t + s)) < \sigma U(t, e(t)) \qquad \forall s \in [-g, 0]$$

which gives

$$\sigma e^T(t)Qe(t) - e^T(t - g(t))Qe(t - g(t)) > 0 \quad (11)$$

Combining estimates (7-11),

$$D_t^\mu U(t, e(t)) \leq \gamma^T(t)\bar{\Phi}\gamma(t) - \int_{t-g(t)}^{t} (t - s)^{\mu-1}\zeta^T(t)Y\zeta(t)ds \quad (12)$$

is attained, where

$$\bar{\Phi} = g^\mu \mu^{-1}\Upsilon^T X\Upsilon + \Phi_1 + \bar{\Phi_2} + \Phi_3$$

$$\bar{\Phi_2} = \Pi_1^T[-KL - LK + \sigma Q]\Pi_1 - \Pi_2^T Q\Pi_2 - \Pi_5^T[-2K]\Pi_5$$

Since $\sigma > 1$ is arbitrary and does not affect $D_t^\mu U(t, e(t))$, as $\sigma \longrightarrow 1^+$, the inequality (12) becomes

$$D_t^\mu U(t, e(t)) \leq \gamma^T(t)\Phi\gamma(t) - \int_{t-g(t)}^{t} (t-s)^{\mu-1}\zeta^T(t)Y\zeta(t)ds \tag{13}$$

$$\Longrightarrow D_t^\mu U(t, e(t)) < \gamma^T(t)\Phi\gamma(t)$$

Take $\Phi < \beta Q$ where $\beta \in \mathbb{R}$ and $\beta > 0$

$$\begin{aligned} D_t^\mu U(t, e(t)) &< \gamma^T(t)\beta Q\gamma(t) \\ &< \beta\gamma^T(t)sym(\Pi_1 Q\Pi^T)\gamma(t) \\ &= -Ze^T(t)Qe(t) \end{aligned}$$

where $Z \in \mathbb{R}$
Hence

$$D_t^\mu U(t, e(t)) < -ZU(t, e(t)) \tag{14}$$

Let

$$\begin{aligned} (1 + \frac{d_k}{\Gamma(\mu+1)})^T Q(1 + \frac{d_k}{\Gamma(\mu+1)}) &\leq K_k Q \\ U(t_k, e(t_k) &\leq e^T(t_k^-)K_k Qe(t_k^-) \\ &= K_k e^T(t_k^-)Qe(t_k^-) \end{aligned}$$

Thus

$$U(t_k) \leq K_k U(t_k^-)$$

Hence we have

$$\begin{aligned} D_t^\mu U(t, e(t)) &< -ZU(t, e(t)) \qquad t \neq t_k \\ U(t_k^+, e(t_k^+) &\leq K_k U(t_k, e(t_k)) \qquad t = t_k \\ U(t, e(t)) &= U(t_0) \qquad t = t_0 \end{aligned} \tag{15}$$

For any $t \in [t_0, t_1)$ we have from lemma 2

$$U(t) \leq U(t_0) e^{\frac{Z(t-t_0)^\mu}{\Gamma(\mu+1)}}$$

which leads to

$$U(t_1^-) \leq U(t_0) e^{\frac{Z(t_1-t_0)^\mu}{\Gamma(\mu+1)}}$$

Set

$$U(t_1^+) = U(t_1)$$

Considering any $t \in [t_1, t_2)$ we get

$$\begin{aligned} U(t) &\leq U(t_1) e^{\frac{Z(t-t_0)^\mu}{\Gamma(\mu+1)}} \\ &\leq U(t_0) K_1 e^{\frac{Z((t-t_1)^\mu + (t_1-t_0))^\mu)}{\Gamma(\mu+1)}} \end{aligned}$$

Similarly for any $t \in [t_k, t_{k+1})$,

$$\begin{aligned} U(t) &\leq U(t_1) e^{\frac{Z(t-t_k)^\mu}{\Gamma(\mu+1)}} \\ &\leq U(t_0) K_1 K_2 ... K_k e^{\frac{Z((t-t_k)^\mu + (t_k - t_{k-1}))^\mu + ... + (t_1-t_0)^\mu)}{\Gamma(\mu+1)}} \end{aligned}$$

By the condition if

$$0 < t_{k+1} - t_k \leq \tau, \ ln(K_k) < \frac{-(\nu - Z)\tau^\mu}{\Gamma(\mu+1)}$$

where $\tau, \nu > 0$, we obtain

$$\begin{aligned} U(t) &\leq U(t_0) e^{\frac{-(\nu-Z)k\tau^\mu}{\Gamma(\mu+1)}} e^{\frac{-Z(k+1)\tau^\mu}{\Gamma(\mu+1)}} \\ &\leq U(t_0) e^{\frac{-\nu k\tau^\mu}{\Gamma(\mu+1)}} e^{\frac{-Zk\tau^\mu}{\Gamma(\mu+1)}} e^{\frac{-Z\tau^\mu}{\Gamma(\mu+1)}} \end{aligned}$$

Thus,

$$U(t) \leq U(t_0) e^{-(\nu k + Z)\tau^\mu} \tag{16}$$

where

$$\nu k + Z > 0$$

As $k \to \infty$ and $\tau \to \infty$ we have

$$U(t) \leq 0$$

This proves the exponential synchronization of the system (3). □

4 Example

Given below is an example to depict the results proved;

Consider the following parameters for system (1);
$n = 2, \mu = 0.1$, L=$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, M=$\begin{bmatrix} 2 & -0.1 \\ -5 & 0.75 \end{bmatrix}$ and N=$\begin{bmatrix} 0.15 & 0.1 \\ 0 & 0.3 \end{bmatrix}$.
D_k=$\begin{bmatrix} 0.95 & 0 \\ 0 & 0.95 \end{bmatrix}$
We take the time varying function as

$$g(t) = \frac{t}{1+t}, t \geq 0$$

Activation function and delay term are given by

$$h(x(t)) = (\frac{1}{1+e^{-x_1(t)}}, \frac{1}{1+e^{-x_2(t)}}) \in \mathbb{R}^2$$

$$h(x(t-g(t)) = (\frac{1}{1+e^{-x_1(t-g(t)}}, \frac{1}{1+e^{-x_2(t-g(t))}})$$

Initial condition is $x(0) = (0.15, 0.1)$
The corresponding response system (2) has the same parameters as above with initial condition $y(0) = (0.45, 0.3)$
Hence we get the error system as (3), the same parameters as above and initial condition $e(0) = (0.3, 0.2)$

Solution:

Upon considering the above parameters and conditions for system (3) and substituting theses values in the conditions given in the theorem we can see that these conditions are satisfied and hence we get that the system is exponentially synchronized.

5 Conclusions

In view of this paper, we explored the exponential synchronization of impulsive neural networks having fractional order having a time delay.We introduce a convex Lyapunov function for our system and use certain LMI conditions to achieve synchronization .The research in the same problem can be lead forward by broadening the result obtained into complex valued neural networks.

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