Research Article

Exponential Stability ResultsOn Fractional Order Impulsive Control For Neural Networks Having Time Delay

Jisha Ann Abraham

Department Of Mathematics Amrita School Of Arts And Sciences,
Kochi-682024,Indiaemail :
Jisha.Ann15@Gmail.Com

M. Aruny Nandinikkutty

Department Of Mathematics Amrita School Of Arts And Sciences,
Kochi-682024, India

Email: Arunynandinikkutty@Gmail.Com

Article History: Received: 11 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 4 June 2021

Abstract

This Paper Examines Dirac Delta Impulse Control For Caputo Fractional Order Neural Network Having Time Varying Delay. With The Help Of An Appropriate Convex Lyapunov Function And Lmi Techniques, We Give ExponentialStability Conditions For The System. A Numerical Example Is Given To Show The Usefulness Of The Exponential Stability Conditions Obtained.

Keywords-Impulsive Caputo Fractional Order Neural Network, Time Varying Delay, Lyapunov Function, Exponential Stability, Linear Matrix Inequality (Lmi).

Introduction

A Neural Network Is Basically A Network Of Neurons. In Modern Science, An Artificial Neural Network Is A Network Consisting Of Artificial Neurons Or Nodes [4]. Initially People Studied Only Integer Calculus. But With Time Fractional Calculus Got Introduced By Replacing The Integer Order With Some Non-IntegerOrder. Even Though Fractional Calculus Was Up In The Air As Integer Calculus, It Got Attention Among The Researchers Just Recently And Is Still A Great Field To Work Upon. Fractional Order Differential System Can Explain Fields Like Neural Networks, Hydromechanics, Mechatronics, Electromagnetism, Super Capacitors, Visco-Elastic Fluid That Have Materials And Processes Having Memory And Hereditary Properties More Precisely Than Integer-Order Ones [3]. Because Of Greater Applications Of Fractional Calculus On Different Splits Of Science And Industry Researchers Started Paying More Attention Towards It [1, 16-23]. One Of The Most Important Application Of Fractional Calculus Is Fractional Order Neural Network (Fonn) [6].

In The Execution Of Neural Networks, Since There Is A Delay In Transmission Of Signals, The Time Lag Phenomenon Is Unavoidable And Will Lead To Some Stability Issues In The Network [2]. Oscillation And Performance Degradation Of The System Are Mainly Caused By The Time Delay [5]. Thus It's Very Meaningful To Study Time-Delay System. Other Than The Time-Delay Effect, Impulse Effect Is Also Visible In Neural Networks. In Neural Networks, A Lot Of Abrupt And Peaked Changes Can Occur Spontaneously In Form Of Pulses [2]. To Make An Unstable System Into A Stable One, Any Of The Control Methods Can Be Used. If The Degree Of Stability Of The Controlled Neural Network Is A Then We Will Say ThatThe Neural Network Is Exponentially Stable [7]. The Non-Linear Problem Is Not Fully Solved Like The Linear Systems Where Mandatory Conditions For Stability Are Provided. Despite Of The Current Efforts, The Problem Of Exponential Stability Of Non-Linear, Non- Autonomous Systems Can Be Considered Widely Open [7]. Since Many Systems In Real Life Applications Like Automatic Control Systems, Robotics, Artificial Intelligence, Information Science Can Be Designed By Non- Linear Systems, They Had Been Given More Attention Since The Last Two Decades [8,9,10,11,12]. Thus It's Very Important To Study The Stability Of The Non- Linear Systems Having Impulse Effects [9,13,14,15]. Lyapunov's Method AndIt's Alterations Like Lyapunov-Krasovskii Function Methods And Ruzumikhin Type Theorems Is One Of The Greatest Way To Stability Of Differential Systems[7]. Researchers Studied Many Problems Like Controllability Problems [41,42,43], Asymptotic Stability [24-28], Synchronization Analysis [38,39,40], Mittage-Leffler Stabilization [29], Guaranteed Cost Control [36,37], Passivity Analysis [34,35], Finite-Time Stability [30-33], Exponential Stability [57-58] And So On.

A Discontinous Control Method That Makes The System Chage It's Trajectories
At Discrete Times Is Called An Impulsive Control And Is Very Cost Effective Too [3]. Back Then, Impulsive Control Was Put In To Present The Integer-Order Differential System's Dynamic Control [3, 50-55]. In Many Cases, Some Impulsive Controllers Were Modelled Using Dirac Delta Function And Based On The Properties Of The Dirac Delta Function, The Controlled Integer-Order Differential System Were ChangedInto The Impulsive Ones [51-54]. Lately, Impulsive Control Was Discerned To Explore The Dynamics Of Various Fractional-Order Systems Which Are More Practical Like Economic Models, Neural Network Models And Biological Models [48,49,55]. Many Studies Have Been Made On Other Control Methods Like Adaptive Control [44], Sliding-Mode Control [45], Intermittent Control And So On [46,47].

So Far The Study Of Impulse Control On Neural Network With Time-Delay Was Made Only When It's Integer Order. Here We Extend It To Fractional-Order Impulsive Control Neural Network Having Time-Delay. So We Choose Impulsive Caputo Fractional-Order Neural Network Having Time-Delay. Then We Select An AppropriateConvex Lyapunov Function And Use Lmi Techniques To Make The System Exponentially Stable. By Doing So, The Convergence Rate Can Be Made Higher And Thus Get The Best Possible Result. An Example Is Also Included To Depict The Usefulness Of The Result Acquired.

The Structure Of This Paper Is As Below: Section 2 Explains The Notations Used, Section 3 Covers Some Basic Concepts And The Description Of The System Considered, Section 4 Projects The Main Result And Section 5 Gives An Example

That Portray The Usefulness Of The Result Obtained. We End The Paper By A Conclusion.

Notations : Here $\mathbb{R}^n$ Denotes The Set Of N-Tuple Of Real Numbers And $\mathbb{R}^{n\times M}$ Denotes The Set Of Real $N \times M$ Matrices. Let 0_n And I_n Denote Zero Matrix And Identity Matrix Of Dimension $N \times N$ Respectively. Sym(X) Denotes $X + X^t$ Where $X \in \mathbb{R}^{n\times N}$. If A Matrix $R \in \mathbb{R}^{n\times N}$ Satisfies The Conditions $R = R^t$ And $Y^t Ry > 0$, $\forall Y \in \mathbb{R}^n, Y \neq 0$ Then We Will Say That R Is Symmetric Positive Definite And Is Denoted By $R > 0$. If A Matrix $R \in \mathbb{R}^{n\times N}$ Satisfies The Conditions $R = R^t$ And $Y^t Ry \geq 0$, $\forall Y \in \mathbb{R}^n$, $Y \neq 0$ Then We Will Say That R Is Symmetric Semi-Positive Definite And Is Denoted By $R \geq 0$. Here A^n And C^n Represents The Set Of All Real Symmetric Semi-Positive Definite And The Set Of All Real SymmetricPositive Definite Matrices Of Dimension $N \times N$ Respectively. B^n Denotes The Set Of All Positive Diagonal Matrices, That Is, A Matrix $Q = \mathrm{Diag}\{Q_1, ..., Q_n\} \in B^n$ If $Q_j > 0$ $(J = 1, 2, \dots, N)$.

Preliminaries And Model Description

Definition 1 (3). *Let $F : [A, B] \to \mathbb{R}$ Be A Differentiable Function. Then TheCaputo Fractional Derivative Of Order A Of F Denoted By ${}^C d^\alpha f(T)$ Is Defined As Follows:*

$$ {}^C d_t^\alpha f(T) = \frac{1}{\Gamma(1-A)} \int_0^t (T - S)^{-A} F'(S)Ds \quad T \geq T_0. $$

Property 1 (6). *For Any Constants A_1, A_2 And Functions $H(T)$, $P(T)$, We Have*

$$ {}^C d_T^A (A_1 h(T) + A_2 p(T)) = A_1 {}^c d_T^\alpha h(T) + A_2 {}^c d_T^A P(T) $$

From Here On, We Will Use the Notation D_t^α for ${}^C d_t^\alpha$

Let Us Consider the Following Caputo Fractional Order Neural Network HavingTime Delay:

$$ \begin{aligned} D_T^\alpha y(T) &= -Ay(T) + Bg(Y(T)) + Cg(Y(T - P(T))) + V(T) \quad T \geq T_0 \\ Y(T) &= T(T) \quad T \in [-P, 0] \end{aligned} \tag{1} $$

Where $A \in (0, 1)$, The Neuron State Vector $Y(T) \in \mathbb{R}^n$, N Denotes The Number Of Neurons Present In The Fractional-Order Neural Network, The Control Input $V(T) \in \mathbb{R}^n$, The Neuron Activation Function $G(Y(T)) = (G_1(Y_1(T)), G_2(Y_2(T)), \dots, G_n(Y_n(T)))^T \in \mathbb{R}^n$, $A = Diag\{A_1, \dots, A_n\} \in B^n$, The Known Constant Matrices $B, C \in \mathbb{R}^{n\times N}$, The Time Delay Function $P(T)$ Satisfies $0 \leq P(T) \leq P$ Where P Is A Known Positive

Constant, $T(T)$ Is A Continuous Vector Valued Function, $V(T) = H \sum_{K=1}^{\infty}$ =

$K \in \mathrm{N}$, $H \in \mathrm{R}^{n\times M}$, Δ Is The Dirac Delta Function And $T_k < T_{k+1}$ For Each $K \in \mathrm{N}$, $\mathrm{Lim}_{k\to+\infty} T_k = +\infty$.

When $T \neq T_k$

$$V(T) = H \sum_{K=1}^{\infty} \frac{u(t_k)\delta(t-t_k)}{\Gamma(A+1)} = 0 \tag{2}$$

When $T = T_k$, $\Delta(T - T_k) = 1$. Put $U(T_k) = Jy(T_k^-)$ Where $J \in \mathrm{R}^{m\times N}$

$$\Rightarrow V(T) = \frac{I_k y(T_K^-)}{\Gamma(A+1)} \quad \text{Where } I_k = Hj \in \mathrm{R}^{n\times N} \tag{3}$$

$$Y(T_k^+) = \frac{D_k}{\Gamma(\alpha+1)} Y(T_k^-) \quad \text{Where } D_k \in \mathrm{R}^{n\times N}$$

From (1),(2) And (3) We Can Rewrite The System As Follows:

$$\begin{aligned} &D_t^{\alpha} y(T) = -Ay(T) + Bg(Y(T)) + Cg(Y(T - P(T))) \quad T \neq T_k, \quad T \geq T_0 \\ &Y(T_K^+) = \frac{D_k}{\Gamma(A+1)} Y(T_K^-) \qquad T = T_K \\ &Y(T) = T(T) \qquad T \in [-P, 0] \end{aligned} \tag{4}$$

Assumption 1. [6] *The Activation Function* $G_j(.)$ *Is A Bounded, Continuous Fnction Satisfying The Following Condition*

$$M_j^- \leq \frac{G_j(U) - G_j(V)}{U - V} \leq M_j^+ \qquad J = 1, 2, \ldots, N$$

Where $G_j(0) = 0$ $(J = 1, 2, \ldots, N)$, $U, V \in \mathrm{R}$, $U \neq V$ *And* M_J^+, M_J^- *Are Known Real Constants.*

Lemma 1. [6] *Let* $U : \mathrm{R}^n \longrightarrow \mathrm{R}^n$ *Be A Differentiable And Convex Function With* $U(0) = 0$, $Y(T)$ *Be A Continuous Function In* R^n *And* $A \in (0, 1]$. *Then*

$$D_t^{\alpha} u(Y(T)) \leq \langle \Delta U(Y(T)), D_t^{\alpha} y(T) \rangle \quad T \geq 0$$

Where $\langle , \rangle$ *Denotes The Inner Product And* $\Delta U(.)$ *Denotes The Gradient Of TheFunction* U.

Lemma 2. *Let* $H(T)$ *Be A Continuous And Real Valued Function On* $[B, +\infty)$, $\forall B \in$

R. *If There Exists A Constant K Such That* $D^{\alpha}h(T)_t \leq Kh(T)$, $0 < A \leq 1$ *Then*

$$H(t) \leq H(b)e^{\frac{1}{\Gamma(\alpha)}\int_b^T K(T-T)^{A-1} D\tau}$$

$$= H(B)E^{\frac{K(T-B)^A}{\Gamma(A+1)}}$$

Main Result

Theorem 1. *Suppose That The Assumption 1 Holds. If There Exists Matrices* $Q, G \in C^n$, $L \in A^{3n}$, $\Lambda_i = Diag\{\Lambda_{i1}, \Lambda_{i2}, \ldots, \Lambda_{in}\} \in B^n (I = 1, 2, \ldots, N)$, $K_k \in R$ *Satisfying The Following Lmi's:*

$$P^{\alpha}\alpha^{-1}\Omega^t L\Omega + \sum_{I=1}^{3} \Gamma_i - Bq < 0 \qquad (5)$$

$$-K_k q + \left(\frac{D_k}{\Gamma(A+1)}\right)^T Q\left(\frac{Dk}{\Gamma(A+1)}\right) < 0 \qquad (6)$$

And The Condition, If $0 < T_{k+1} - T_k \leq R$,

$$Ln(K_k) < \frac{-(N - W)R^{\alpha}}{\Gamma(A+1)} \qquad (7)$$

Where $R, N, B > 0$ *And*

$\Theta_1 = [I_n \ 0_n \ 0_n \ 0_n \ 0_n]$ $\Theta_2 = [0_n \ I_n \ 0_n \ 0_n \ 0_n]$ $\Theta_3 = [0_n \ 0_n \ I_n \ 0_n \ 0_n]$ $\Theta_4 = [0_n \ 0_n \ 0_n \ I_n \ 0_n]$ $\Theta_5 = [0_n \ 0_n \ 0_n \ 0_n \ I_n]$ $\Pi_1 = Diag\{M_1^-, \ldots, M_N^-\}$
$\Pi_2 = Diag\{M_1^+, \ldots, M_N^+\}$

$$\Omega = [\Theta^t_1 \ T\vartheta_2 \ \Theta^t_5]^T$$

$\Omega_1 = \Theta_3 - \Pi_1\theta_1$ $\Omega_2 = \Pi_2\theta_1 - \Theta_3$ $\Omega_3 = \Theta_4 - \Pi_1\theta_2$ $\Omega_4 = \Pi_2\theta_2 - \Theta_3$
$\Omega_5 = \Theta_3 - \Theta_4 - \Pi_1(\Theta_1 - \Theta_2)$ $\Omega_6 = \Pi_2(\Theta_1 - \Theta_2) - \Theta_3 + \Theta_4$

$\Gamma_1 = Sym(\Theta^t_1 Q\theta_5 - \Theta^t_1 G\theta_5 + \Theta^t_1 Gb\theta_3 + \Theta^t_1 Gc\theta_4 - \Theta^t_5 Ga\theta_1 + \Theta^t_5 Gb\theta_3 + \Theta^t_5 Gc\theta_4)$

$\Gamma_2 = \Theta^t_5 [-Ga - Ag + Q]\Theta_1 - \Theta^t_5 Q\theta_2 + \Theta^t_5 [-2g]\Theta_5$

$\Gamma_3 = Sym(\Omega^t_1 \Lambda_1\Omega_2 + \Omega^t_3 \Lambda_2\Omega_4 + \Omega^t_5 \Lambda_3\Omega_6)$

Then We Are Able To Conclude That The System Is Exponentially Stable.

Proof. Consider The Lyapunov Function $U(T) = U(T, Y(T)) = Y^t(T)Qy(T)$ For Our System. Clearly $U(T)$ Is A Differentiable And Convex Function On R^n And Also $U(T, 0) = 0$. The Caputo Fractional Derivative Of Order A Of The System Can Be Calculated Using Lemma 1 As Follows:

$$D^\alpha_T u(T, Y(T)) \leq 2y^t(T)Qd^\alpha_T y(T) \tag{8}$$

$$= \Omega^t(T)Sym(\Theta^t_1 Q\theta_5)\Omega(T)$$

Where $\Omega(T) = \begin{bmatrix} Y^t(T) & Y^t(T-P(T)) & G^t(Y(T)) & G^t(Y(T-P(T))) & (D^\alpha y(T))^T \end{bmatrix}^T$

The Inequality Below Holds For Any $L \in A^{3n}$

$$p^\alpha \alpha^{-1}\varphi^T(t)L\varphi(t) - \int_{t-p(t)}^{T} (t-s)^{\alpha-1}\varphi^T(t)L\varphi(t)ds \geq 0 \tag{9}$$

Given $\Phi(T) = \begin{bmatrix} Y^t(T) & Y^t(T-P(T)) & (D^\alpha y(T))^T_t \end{bmatrix}^T$

The Following Equality Can Be Acquired From Our System. For Any $G \in C^n$,

$$[2y^t(T)+2(D^\alpha y(T))^T]G\times[-D^\alpha_t y(T)-Ay(T)+Bg(Y(T))+Cg(Y(T-P(T)))] = 0 \tag{10}$$

From Assumption 1, We Can Say That That For Any $\Lambda_{ji} > 0$ $(J = 1, 2, 3, I = 1, 2, \ldots, N)$

$$2(G_i(Y_i(T)) - M^-_i Y_i(T))\Lambda_{1i}(M^+_i Y_i(T) - G_i(Y_i(T))) \geq 0$$

$$2(G_i(Y_i(T-P(T))) - M^-_i Y_i(T-P(T)))\Lambda_{2i}(M^+_i Y_i(T-P(T)) - G_i(Y_i(T-P(T)))) \geq 0$$

$$2(G_i(Y_i(T)) - G_i(Y_i(T-P(T))) - M^-(Y_i(T) - Y_i(T-P(T))))\Lambda_{3i}(M^+(Y_i(T) - Y_i(T-P(T)))$$

$$- G_i(Y_i(T)) + G_i(Y_i(T - P(T)))) \geq 0$$

Which Imply

$$2\omega^t (T)\Omega^t \Lambda_1\Omega_2\omega(T) \geq 0$$

$$2\omega^t (T)\Omega^t \Lambda_2\Omega_4\omega(T) \geq 0 \qquad (11)$$

$$2\omega^t (T)\Omega^t \Lambda_3\Omega_6\omega(T) \geq 0$$

Since $U(T, Y(T)) = Y^t(T)Qy(T)$, We Suppose That For Some Real Number $P > 1$

$U(T + S, Y(T + S)) < Pu(T, Y(T))\ \forall S \in [-P, 0]$

We Obtain

$$Py^t(T)Qy(T) - Y^t(T - P(T))Qy(T - P(T)) > 0 \qquad (12)$$

Combining Estimates (8)-(12), We Obtain

$$D_t^\alpha u(T, Y(T)) \leq \Omega^t(T)\bar{\Gamma}\Omega(T) - \int_{T-P(T)}^{T} (T - S)^{A-1}\varphi^t(T)L\varphi(T)Ds \qquad (13)$$

Where

$$\bar{\Gamma} = P^\alpha\alpha^{-1}\Omega^t L\Omega + \Gamma_1 + \bar{\Gamma}_2 + \Gamma_3$$

$$\bar{\Gamma}_2 = \Theta^t[-G_1 a - Ag + Pq]\Theta_1 - \Theta^t Q\theta_2 + \Theta^t[-2g]\Theta_5$$

Since $P > 1$ Is An Arbitrary Parameter And $D_t^\alpha u(T, Y(T))$ Doesnot Depend On P, Taking $P \longrightarrow 1^+$, The Inequality (13) Becomes

$$D_t^\alpha U(t, y(t)) \leq \omega^T(t)\gamma\omega(t) - \int_{t-p(t)}^{T} (t - s)^{\alpha-1}\varphi^T(t)L\varphi(t)ds \qquad (14)$$

Where $\Gamma = P^{\alpha}\alpha^{-1}\Omega^{t} L\Omega + \Gamma_1 + \Gamma_2 + \Gamma_3$

$\Longrightarrow \; D^{\alpha}_{t} u(T, Y(T)) < \Omega^{t}(T)\Gamma\omega(T)$

Take $\Gamma < Bq$, Where $\beta \in$ R And $B > 0$

$$D^{\alpha}_{t} u(T, Y(T)) < \Omega^{t}(T)Bq\omega(T)$$

$$< B\omega^{t}(T)Sym(\Theta_1 q\theta^{t}_{1})\Omega(T)$$

$$= -Wy^{t}(T)Qy(T)$$

Where $-W \in$ R

Thus,

$$D^{\alpha}_{t} u(T, Y(T)) < -Wu(T, Y(T)) \qquad (15)$$

Take $\left(\frac{D_k}{\Gamma(A+1)}\right)^{T} Q\left(\frac{D_k}{\Gamma(A+1)}\right) \leq K_k q$

$$U(T^{+}_{k}) \leq Y^{t}_{k}(T^{-}_{k})K_k q y(T^{-}_{k})$$

$$\leq K_k y^{t}(T^{-}_{k})Qy(T^{-}_{k})$$

$$< K_k\omega^{t}(T^{-}_{K})Sym(\Theta^{t}_{1} Q\theta_1)\Omega(T^{-}_{K})$$

Thus,

$$U(T^{+}_{k}, Y(T^{+}_{k})) \leq K_k u(T^{-}_{k}, Y(T^{-}_{k})) \qquad (16)$$

Thus,

$$\begin{cases} D^{\alpha}_{t} u(T, Y(T)) < -Wu(T, Y(T)) & T \neq T_k \\ U(T^{+}_{k}, Y(T^{+}_{k})) \leq K_k u(T^{-}_{k}, Y(T^{-}_{k})) & T = T_k \\ U(T, Y(T)) = U(T_0) & T = T_0 \end{cases} \qquad (17)$$

For Any $T \in [T_0, T_1)$, We Have

$$U(T) \leq U(T_0)E_{A}\left(\frac{-W(T-T_0)^{A}}{\Gamma(A+1)}\right)$$

Which Gives

$$U(T_1) \le U(T_0)E^{\frac{-W(T_1-T_0)^A}{\Gamma(A+1)}}$$

Set $U(T^+) =_1 U(T_1)$

For Any $T \in [T_1, T_2)$ We Have

$$U(T) \le U(T_1)E^{\frac{-W(T-T_1)^A}{\Gamma(A+1)}}$$

$$\le U(T_0)K_1 e^{\frac{-W[(T-T_1)^A+(T_1-T_0)^A]}{\Gamma(A+1)}}$$

Similarly For Any $T \in [T_k, T_{k+1})$ We Have

$$U(T) \le U(T_k)E^{\frac{-W(T-T_k)^A}{\Gamma(A+1)}}$$

$$\le U(T_0)K_1k_2\ldots K_k e^{\frac{-W[(T-T_k)^A+(T_k-T_{k-1})^A+\ldots+(T_1-T_0)^A]}{\Gamma(A+1)}}$$

By The Condition If $0 < T_{k+1} - T_k \le R$, $Ln(K_k) < \frac{-(\mu-W)R^\alpha}{\Gamma(\alpha+1)}$, Where $R, \mu > 0$, We Obtain

$$U(T) \le U(T_0)E^{\frac{-(\mu-W)Kr^\alpha}{\Gamma(A+1)}} E^{\frac{-W(K+1)R^\alpha}{\Gamma(A+1)}}$$

$$\le U(T_0)E^{\frac{-\mu kr^\alpha}{\Gamma(A+1)}} E^{\frac{W Kr^\alpha}{\Gamma(A+1)}} E^{\frac{-W Kr^\alpha}{\Gamma(A+1)}} E^{\frac{-W R^\alpha}{\Gamma(A+1)}}$$

Thus, $U(T) \le U(T_0)E$

$$\frac{(\mu k+W)R^{\alpha}}{\Gamma(A+1)} \qquad (18) \qquad =$$

Where $\mu k + W > 0$ And $K \longrightarrow \infty$, $R \longrightarrow \infty$ Then $U(T) \leq 0$

Thus Our System Is Exponentially Stable. □

Example

This Segment Gives An Example To Depict The Usefulness Of The Result Obtained.

We Consider A Fonn Having Time Delay That Can Be Defined As (4) With TheParameters Below:

$$A = \begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 \\ 0.5 & 3 \end{pmatrix}, C = \begin{pmatrix} 0.2 & -0.3 \\ 0 & 1 \end{pmatrix}, Dk = \begin{pmatrix} 0.7 & 0 \\ 0 & 1 \end{pmatrix}$$

$P(T) = \frac{1}{1+t}$, $T \geq 0$ Is Choosen As The Time Varying Function.
$G(Y(T)) = (Tanh\ Y_1(T), Tanh\ Y_2(T))^T$ Is Chosen As The Activation Function And $G(Y(T - P(T))) = (Tanh\ Y_1(T - P(T)), Tanh\ Y_2(T - P(T))) \in \mathrm{R}^2$ Is Taken As The Delay Term. Choose $A = 0.99$. Initial Condition Is Taken As $Y(0) = (0.2, -0.1)$.

Solution:

The Fonn Having Time Varying Delay Which Is Described As System (4) WithThe Above Parameters Satisfies The Lmi's (5), (6) And The Condition (7). ThusThe System Is Exponentially Stable.

Conclusion

Here A Fractional Order Neural Network Having Impulses And Delay Time Is Considered And Its Exponential Stability Is Examined. We Introduced A Convex Lyapunov Function For Our System And Used Certain Lmi Conditions To Achieve Exponential Stability. The Future Research Can Be Taken Forward By Broadening The Obtained Criterion To Complex Valued Neural Networks.

References

[1] Ivanka Stamova : *Global Mittage-Leffler Stability And Synchronization Of Impulsive Fractional-Order Neural Networks With Time-Varying Delays*, Springer(2014)

[2] Zhenjiang Zhao, Qiankun Song : *Global Exponential Stability Of ImpulsiveComplex Valued Neural Networks With Proportional Delays*, Ieee(2019)

[3] Shuai Yang, Cheng Hu, Juan Yu, Haijun Jiang : *Exponential Stability Of Fractional- Order Impulsive Control Systems With Applications In Synchronisation*, Ieee(2019)

[4] Kaizhong Guan, Qisheng Wang*impulsive Control For A Class Of Cellular Neural Networks With Proportional Delay*, Springer (2018)

[5] Zhenjiang Zhao, Qiankun Song : *Stability Of Complex-Valued Cohen- Grossberg Neural Network With Time-Varying Delays*, Springer(2016)

[6] Nguyen Huu Sau, Duong Thi Hong, Nguyen Thi Thanh Huyen, Bui Viet Huong, Mai Viet Thuan : *Delay-Dependent And Oder-Dependent* H_∞ *Control For Fractional-Order Neural Networks With Time-Varying Delay*, Springer(2021)

[7] Rigoberto Medina: *New Conditions For The Exponential Stability Of Nonlinear Differential Equations*, Hindawi(2017)

[8] C. Vinothkumar, J. J. Nietoy, A. Deiveeganz, P. Prakash : *Invariant Solutions Of Hyperbolic Fuzzy Fractional Differential Equations*, WorldScientific(2019)

[9] Pan Mu, Leiwang,Yi An, Yanping Ma : *A Novel Fractional Microbial Batch Culture Process And Parameter Identification*, Springer(2017)

[10] Shuo Zhang, Yongguang Yu, Junzhi Yu : *Lmi Conditions For Global Stability Of Fractional-Order Neural Networks*, Ieee(2017)

[11] Ravi Agarwal, Snezhana Hristova, Donal O'regan : *Lyapunov Functions And Stability Of Caputo Fractional Differential Equations With Delays*,Springer(2015)

[12] Liping Chena, Tingwen Huang,J.A.Tenreiro Machado, António M.Lopes, Yichai, Ranchao Wu : *Delay-Dependent Criterion For Asymptotic Stability Of A Class Of Fractional-Order Memristive Neural Networks With Time-Varying Delays*, Elsevier(2019)

[13] Hai Zhang, Renyu Ye, Jinde Cao, Alsaedi Ahmed, Xiaodi Li, Ying Wan : *Lyapunov Functional Approach To Stability Analysis Of Riemann-Liouville Fractional Neural Networks With Time-Varying Delays*, Asian Journal OfControl (2019)

[14] Hai Zhang, Renyu Ye, Jinde Cao, Ahmed Alsaedi : *Delay-Independent Stability Of Riemann–Liouville Fractional Neutral-Type Delayed Neural Networks*, Springer(2017)

[15] Wenting Chang, Song Zhu, Jinyu Li, Kaili Sun : *Global Mittag–Leffler Stabilization Of Fractional-Order Complex-Valued Memristive Neural Networks*, Elsevier(2018)

[16] Ranchao Wu, Yanfen Lu, Liping Chen : *Finite-Time Stability Of Fractional Delayed Neural Networks*, Elsevier(2015)

[17] Taotao Hu, Zheng He, Xiaojun Zhang, Shouming Zhong : *Finite-Time Stability For Fractional-Order Complex-Valued Neural Networks With Time Delay*, Elsevier(2020)

[18] Pratap Anbalagan, R. Raja, Jehad Alzabut : *Finite-Time Mittag-Leffler Stability Of Fractional-Order Quaternion-Valued Memristive Neural Networks With Impulses*, Springer(2020)

[19] Chongyang Chen, Song Zhu, Wei Yongchang : *Finite-Time Stability Of Delayed Memristor-Based Fractional-Order Neural Networks*, Ieee(2018)

[20] Zhixia Ding, Zhigang Zeng, Hao Zhang, Leimin Wang, Liheng Wang ; *New Results On Passivity Of Fractional-Order Uncertain Neural Networks*, Elsevier(2019)

[21] Mai Viet Thuan, Dinh Cong Huong, Duong Thi Hong : *New Results On Robust Finite-Time Passivity For Fractional-Order Neural Networks With Uncertainties*, Springer(2019)

[22] Mai Viet Thuan, Dinh Cong Huong : *Robust Guaranteed Cost Control For Time-Delay Fractional-Order Neural Networks Systems*, Wiley(2019)

[23] Mai Viet Thuan, Tran Nguyen Binh, Dinh Cong Huong ; *Finite-Time Guaranteed Cost Control Of Caputo Fractional-Order Neural Networks*, Asian Journal Of Control(2020)

[24] Jianmei Zhang, Jianwei Wu, Haibo Bao, Jinde Cao ; *Synchronization Analysis Of Fractional-Order Three-Neuron Bam Neural Networks With Multiple Time Delays*, Elsevier(2018)

[25] A. Pratap, R. Raja, J. Cao, C.P. Lim, O. Bagdasar : *Stability And Pinning Synchronization Analysis Of Fractional Order Delayed Cohen–Grossberg Neural Networks With Discontinuous Activations*, Elsevier(2019)

[26] Ruoxia Li, Xingbao Gao, Jinde Cao : *Quasi-State Estimation And Quasi-Synchronization Control Of Quaternion-Valued Fractional-Order Fuzzy Memristive Neural Networks: Vector Ordering Approach*, Elsevier(2019)

[27] Sanjukta Das, Dwijendra N. Pandey, And N. Sukavanam : *Existence Of Solution And Approximate Controllability For Neutral Differential Equation With State Dependent Delay*, Hindawi(2014)

[28] Kamal Jeet, D. Bahuguna, Rajesh Kumar : *Approximate Controllability Of Finite Delay Fractional Functional Integro-Differential Equations With Nonlocal Condition*, Differential Equations And Dynamical Systems(2016)

[29] T. Sathiyaraj, P. Balasubramaniam : *Null Controllability Of Nonlinear Fractional Stochastic Large-Scale Neutral Systems*, Springer(2016)

[30] Song Liang, Ranchao Wu, Liping Chen : *Adaptive Pinning Synchronization In Fractional-Order Uncertain Complex Dynamical Networks With Delay*,Physica A(2015)

[31] Nooshin Bigdeli, Hossein Alinia Ziazi : *Finite-Time Fractional-Order Adaptive Intelligent Backstepping Sliding Mode Control Of Uncertain Fractional-Order Chaotic Systems*, Elsevier(2017)

[32] Fei Wang, Yongqing Yang : *Intermittent Synchronization Of Fractional Order Coupled Nonlinear Systems Based On A New Differential Inequality*, Elsevier(2018)

[33] Kalin Su: *Control Chaos In Fractional-Order System Via Two Kinds Of Intermittent Schemes*, Elsevier(2015)

[34] Ying Yang, Yong He, Yong Wang, Min Wu : *Stability Analysis For Impulsive Fractional Hybrid Systems Via Variational Lyapunov Method*,Elsevier(2017)

[35] Dong Li, Xingpeng Zhang : *Impulsive Synchronization Of Fractional Order Chaotic Systems With Time-Delay*, Neurocomputing(2016)

[36] Tao Yang, Leon O. Chua : *Impulsive Stabilization For Control And Synchronization Of Chaotic Systems:Theory And Application To Secure Communication*, Ieee(1997)

[37] Zhi-Hong Guan, David J. Hill, Xuemin (Sherman) Shen : *On Hybrid Impulsive And Switching Systems And Applications To Nonlinear Control*

[38] Zhi-Hong Guan, Hao Zhang : *Stabilization Of Complex Network With Hybrid Impulsive And Switching Control*, Elsevier(2008)

[39] Cheng Hu, Haijun Jiang, Zhidong Teng : *Impulsive Control And Synchronization For Delayed Neural Networks With Reaction-Diffusion Terms*,Ieee(2010)

[40] Cheng Hu, Haijun Jiang, Zhidong Teng : *Globally Exponential Stability For Delayed Neural Networks Under Impulsive Control*, Springer(2010)

[41] Ivanka Stamova, Gani Stamov : *Functional And Impulsive Differential Equations Of Fractional Order Qualitative Analysis And Applications*, CrcPress(2017)

[42] Xueyan Yang, Dongxue Peng, Xiaoxiao Lv, Xiaodi Li : *Recent Progress In Impulsive Control Systems*, Elsevier (2018)

[43] A.Vinodkumar, T.Senthilkumar, Zhongmin Liu, Xiaodi Li : *Exponential Stability Of Random Impulsive Pantograph Equations*, Wiley (2021)

[44] A.Vinodkumar, T.Senthilkumar, S.Hariharan, J.Alzabut : *Exponential Stabilization Of Fixed And Random Time Impulsive Delay Differential System With Applications*, Mathematical Biosciences And Engineering (2021)