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Research Article

Exponential Stability Results On Fractional Order Impulsive Control For Neural Networks Having Time Delay

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Abstract

This paper examines Dirac delta impulse control for Caputo fractional order neural network having time varying delay. With the help of an appropriate convex Lyapunov function and LMI techniques, we give exponential stability conditions for the system. A numerical example is given to show the usefulness of the exponential stability conditions obtained.

Keywords-Impulsive Caputo fractional order neural network, time varying delay, Lyapunov function, exponential stability, Linear Matrix Inequality (LMI).

1 Introduction

A neural network is basically a network of neurons. In modern science, an artificial neural network is a network consisting of artificial neurons or nodes [4]. Initially people studied only integer calculus. But with time fractional calculus got introduced by replacing the integer order with some non-integer order. Even though fractional calculus was up in the air as integer calculus, it got attention among the researchers just recently and is still a great field to work upon. Fractional order differential system can explain fields like neural networks, hydromechanics, mechatronics, electromagnetism, super capacitors, visco-elastic fluid that have materials and processes having memory and hereditary properties more precisely than integer-order ones [3]. Because of greater applications of fractional calculus on different splits of science and industry researchers started paying more attention towards it [1, 16-23]. One of the most important application of fractional calculus is Fractional Order Neural Network (FONN) [6].

In the execution of neural networks, since there is a delay in transmission of signals, the time lag phenomenon is unavoidable and will lead to some stability issues in the network [2]. Oscillation and performance degradation of the system are mainly caused by the time delay [5]. Thus it's very meaningful to study time-delay system. Other than the time-delay effect, impulse effect is also visible in neural networks. In neural networks, a lot of abrupt and peaked changes can occur spontaneously in form of pulses [2]. To make an unstable system into a stable one, any of the control methods can be used. If the degree of stability of the controlled neural network is α then we will say that the neural network is exponentially stable [7]. The non-linear problem is not fully solved like the linear systems where mandatory conditions for stability are provided. Despite of the current efforts, the problem of exponential stability of non-linear, non-autonomous systems can be considered widely open [7]. Since many systems in real life applications like automatic control systems, robotics, artificial intelligence, information science can be designed by non-linear systems, they had been given more attention since the last two decades [8,9,10,11,12]. Thus it's very important to study the stability of the non-linear systems having impulse effects [9,13,14,15]. Lyapunov's method and it's alterations like Lyapunov-Krasovskii function methods and Ruzumikhin type theorems is one of the greatest way to stability of differential systems [7]. Researchers studied many problems like controllability problems [41,42,43], asymptotic stability [24-28], synchronization analysis [38,39,40], Mittage-Leffler stabilization [29], guaranteed cost control [36,37], passivity analysis [34,35], finite-time stability [30-33], exponential stability [57-58] and so on.

A discontinous control method that makes the system chage it's trajectories at discrete times is called an impulsive control and is very cost effective too [3]. Back then, impulsive control was put in to present the integer-order differential system's dynamic control [3, 50-55]. In many cases, some impulsive controllers were modelled using Dirac delta function and based on the properties of the Dirac delta function, the controlled integer-order differential system were changed into the impulsive ones [51-54]. Lately, impulsive control was discerned to explore the dynamics of various fractional-order systems which are more practical like economic models, neural network models and biological models [48,49,55]. Many studies have been made on other control methods like adaptive control [44], sliding-mode control [45], intermittent control and so on [46,47].

So far the study of impulse control on neural network with time-delay was made only when it's integer order. Here we extend it to fractional-order impulsive control neural network having time-delay. So we choose impulsive Caputo fractional-order neural network having time-delay. Then we select an appropriate convex Lyapunov function and use LMI techniques to make the system exponentially stable. By doing so, the convergence rate can be made higher and thus get the best possible result. An example is also included to depict the usefulness of the result acquired.

The structure of this paper is as below: section 2 explains the notations used, section 3 covers some basic concepts and the description of the system considered, section 4 projects the main result and section 5 gives an example that portray the usefulness of the result obtained. We end the paper by a conclusion.

Notations : Here $\mathbb{R}^n$ denotes the set of n-tuple of real numbers and $\mathbb{R}^{n\times m}$ denotes the set of real $n \times m$ matrices. Let 0_n and I_n denote zero matrix and identity matrix of dimension $n \times n$ respectively. sym(X) denotes $X + X^T$ where $X \in \mathbb{R}^{n\times n}$. If a matrix $R \in \mathbb{R}^{n\times n}$ satisfies the conditions $R = R^T$ and $y^T Ry > 0$, $\forall y \in \mathbb{R}^n$, $y \neq 0$ then we will say that R is symmetric positive definite and is denoted by $R > 0$. If a matrix $R \in \mathbb{R}^{n\times n}$ satisfies the conditions $R = R^T$ and $y^T Ry \geq 0$, $\forall y \in \mathbb{R}^n$, $y \neq 0$ then we will say that R is symmetric semi-positive definite and is denoted by $R \geq 0$. Here $\mathbb{A}^n$ and $\mathbb{C}^n$ represents the set of all real symmetric semi-positive definite and the set of all real symmetric positive definite matrices of dimension $n \times n$ respectively. $\mathbb{B}^n$ denotes the set of all positive diagonal matrices, that is, a matrix $Q = \text{diag}\{q_1, ..., q_n\} \in \mathbb{B}^n$ if $q_j > 0$ $(j = 1, 2, ..., n)$.

2 Preliminaries and Model Description

Definition 1 (3). *Let $f : [a, b] \longrightarrow \mathbb{R}$ be a differentiable function. Then the Caputo fractional derivative of order α of f where $\alpha \in (0, 1)$ denoted by ${}^cD_t^\alpha f(t)$ is defined as follows:*

$$ {}^cD_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f^{'}(s)ds}{(t-s)^\alpha} \qquad t \geq t_0. $$

Property 1 (6). *For any constants λ_1,λ_2 and functions $h(t)$, $p(t)$, we have*

$$ {}^cD_\alpha^t(\lambda_1 h(t) + \lambda_2 p(t)) = \lambda_1^c D_t^\alpha h(t) + \lambda_2^c D_\alpha^t p(t) $$

From here on, we will use the notation D_α^t for ${}^cD_\alpha^t$

Let us consider the following Caputo fractional order neural network having time delay:

$$ \begin{aligned} D_t^\alpha y(t) &= -Ay(t) + Bg(y(t)) + Cg(y(t-p(t))) + v(t) \qquad t \geq t_0 \\ y(t) &= \tau(t) \qquad\qquad t \in [-p, 0] \end{aligned} \tag{1} $$

where $\alpha \in (0, 1)$, the neuron state vector $y(t) \in \mathbb{R}^n$, n denotes the number of neurons present in the fractional-order neural network, the control input $v(t) \in \mathbb{R}^n$, the neuron activation function $g(y(t)) = (g_1(y_1(t)), g_2(y_2(t)), \ldots, g_n(y_n(t)))^T$ $\in \mathbb{R}^n$, $A = diag\{a_1, \ldots, a_n\} \in \mathbb{B}^n$, the known constant matrices $B, C \in \mathbb{R}^{n\times n}$, the time delay function $p(t)$ satisfies $0 \leq p(t) \leq p$ where p is a known positive constant, $\tau(t)$ is a continuous vector valued function, $v(t) = H \sum_{k=1}^{\infty} \frac{u(t_k)\delta(t-t_k)}{\Gamma(\alpha+1)}$ $k \in \mathbb{N}$, $H \in \mathbb{R}^{n\times m}$, δ is the Dirac delta function and $t_k < t_{k+1}$ for each $k \in \mathbb{N}$, $\lim_{k\to+\infty} t_k = +\infty$.

When $t \neq t_k$

$$v(t) = H \sum_{k=1}^{\infty} \frac{u(t_k)\delta(t-t_k)}{\Gamma(\alpha+1)} = 0 \tag{2}$$

When $t = t_k$, $\delta(t-t_k) = 1$. Put $u(t_k) = Jy(t_k^-)$ where $J \in \mathbb{R}^{m\times n}$

$$\begin{aligned} \Longrightarrow\ v(t) &= \frac{I_k y(t_k^-)}{\Gamma(\alpha+1)}\ \textit{where}\ I_k = HJ \in \mathbb{R}^{n\times n} \\ y(t_k^+) &= \frac{D_k}{\Gamma(\alpha+1)} y(t_k^-)\ \textit{where}\ D_k \in \mathbb{R}^{n\times n} \end{aligned} \tag{3}$$

From (1),(2) and (3) we can rewrite the system as follows:

$$\begin{aligned} D_t^\alpha y(t) &= -Ay(t) + Bg(y(t)) + Cg(y(t-p(t))) \qquad t \neq t_k, t \geq t_0 \\ y(t_k^+) &= \frac{D_k}{\Gamma(\alpha+1)} y(t_k^-) \qquad t = t_k \\ y(t) &= \tau(t) \qquad t \in [-p, 0] \end{aligned} \tag{4}$$

Assumption 1. [6] *The activation function $g_j(.)$ is a bounded, continuous fnction satisfying the following condition*

$$m_j^- \leq \frac{g_j(u) - g_j(v)}{u - v} \leq m_j^+ \qquad j = 1, 2, \ldots, n$$

where $g_j(0) = 0$ $(j = 1, 2, \ldots, n)$, $u, v \in \mathbb{R}$, $u \neq v$ and m_j^+, m_j^- are known real constants.

Lemma 1. [6] *Let $U : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be a differentiable and convex function with $U(0) = 0$, $y(t)$ be a continuous function in $\mathbb{R}^n$ and $\alpha \in (0, 1]$. Then*

$$D_t^\alpha U(y(t)) \leq \langle \Delta U(y(t)), D_t^\alpha y(t) \rangle \qquad t \geq 0$$

where $\langle , \rangle$ denotes the inner product and $\Delta U(.)$ denotes the gradient of the function U.

Lemma 2. *Let $H(t)$ be a continuous and real valued function on $[b, +\infty)$, $\forall b \in \mathbb{R}$. If there exists a constant k such that $D_t^\alpha H(t) \leq kH(t)$, $0 < \alpha \leq 1$ then*

$$\begin{aligned} H(t) &\leq H(b) e^{\int_b^t \frac{k(t-\tau)^{\alpha-1} d\tau}{\Gamma(\alpha)}} \\ &= H(b) e^{\frac{k(t-b)^\alpha}{\Gamma(\alpha+1)}} \end{aligned}$$

3 Main Result

Theorem 1. *Suppose that the assumption 1 holds. If there exists matrices* $Q, G \in \mathbb{C}^n$, $L \in \mathbb{A}^{3n}$, $\Lambda_i = diag\{\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{in}\} \in \mathbb{B}^n (i = 1, 2, \ldots, n)$, $K_k \in \mathbb{R}$ *satisfying the following LMI's:*

$$p^{\alpha}\alpha^{-1}\Omega^T L\Omega + \sum_{i=1}^{3} \gamma_i - \beta Q < 0 \tag{5}$$

$$-K_k Q + (\frac{D_k}{\Gamma(\alpha+1)})^T Q(\frac{D_k}{\Gamma(\alpha+1)}) < 0 \tag{6}$$

and the condition, if $0 < t_{k+1} - t_k \leq r$,

$$ln(K_k) < \frac{-(\nu - W)r^{\alpha}}{\Gamma(\alpha+1)} \tag{7}$$

where $r, \nu, \beta > 0$ *and*

$\theta_1 = \begin{bmatrix} I_n & 0_n & 0_n & 0_n & 0_n \end{bmatrix}$

$\theta_2 = \begin{bmatrix} 0_n & I_n & 0_n & 0_n & 0_n \end{bmatrix}$

$\theta_3 = \begin{bmatrix} 0_n & 0_n & I_n & 0_n & 0_n \end{bmatrix}$

$\theta_4 = \begin{bmatrix} 0_n & 0_n & 0_n & I_n & 0_n \end{bmatrix}$

$\theta_5 = \begin{bmatrix} 0_n & 0_n & 0_n & 0_n & I_n \end{bmatrix}$

$\pi_1 = diag\{m_1^-, \ldots, m_n^-\}$

$\pi_2 = diag\{m_1^+, \ldots, m_n^+\}$

$\Omega = \begin{bmatrix} \theta_1^T & \theta_2^T & \theta_5^T \end{bmatrix}^T$

$\Omega_1 = \theta_3 - \pi_1\theta_1$

$\Omega_2 = \pi_2\theta_1 - \theta_3$

$\Omega_3 = \theta_4 - \pi_1\theta_2$

$\Omega_4 = \pi_2\theta_2 - \theta_3$

$\Omega_5 = \theta_3 - \theta_4 - \pi_1(\theta_1 - \theta_2)$

$\Omega_6 = \pi_2(\theta_1 - \theta_2) - \theta_3 + \theta_4$

$\gamma_1 = sym(\theta_1^T Q\theta_5 - \theta_1^T G\theta_5 + \theta_1^T GB\theta_3 + \theta_1^T GC\theta_4 - \theta_5^T GA\theta_1 + \theta_5^T GB\theta_3 + \theta_5^T GC\theta_4)$

$\gamma_2 = \theta_1^T[-GA - AG + Q]\theta_1 - \theta_2^T Q\theta_2 + \theta_5^T[-2G]\theta_5$

$\gamma_3 = sym(\Omega_1^T \Lambda_1 \Omega_2 + \Omega_3^T \Lambda_2 \Omega_4 + \Omega_5^T \Lambda_3 \Omega_6)$

then we are able to conclude that the system is exponentially stable.

Proof. Consider the Lyapunov function $U(t) = U(t, y(t)) = y^T(t)Qy(t)$ for our system. Clearly $U(t)$ is a differentiable and convex function on $\mathbb{R}^n$ and also $U(t, 0) = 0$. The Caputo fractional derivative of order α of the system can be calculated using lemma 1 as follows:

$$\begin{aligned} D_t^\alpha U(t, y(t)) &\leq 2y^T(t)QD_t^\alpha y(t) \\ &= \omega^T(t)sym(\theta_1^T Q\theta_5)\omega(t) \end{aligned} \tag{8}$$

where $\omega(t) = \begin{bmatrix} y^T(t) & y^T(t-p(t)) & g^T(y(t)) & g^T(y(t-p(t))) & (D_t^\alpha y(t))^T \end{bmatrix}^T$

The inequality below holds for any $L \in \mathbb{A}^{3n}$

$$p^\alpha \alpha^{-1} \phi^T(t)L\phi(t) - \int_{t-p(t)}^{t} (t-s)^{\alpha-1}\phi^T(t)L\phi(t)ds \geq 0 \tag{9}$$

given $\phi(t) = \begin{bmatrix} y^T(t) & y^T(t-p(t)) & (D_t^\alpha y(t))^T \end{bmatrix}^T$

The following equality can be acquired from our system. For any $G \in \mathbb{C}^n$,

$$[2y^T(t) + 2(D_t^\alpha y(t))^T]G \times [-D_t^\alpha y(t) - Ay(t) + Bg(y(t)) + Cg(y(t-p(t)))] = 0 \tag{10}$$

From Assumption 1, we can say that that for any $\lambda_{ji} > 0$ $(j = 1, 2, 3, i = 1, 2, \ldots, n)$

$$2(g_i(y_i(t)) - m_i^- y_i(t))\lambda_{1i}(m_i^+ y_i(t) - g_i(y_i(t))) \geq 0$$

$$2(g_i(y_i(t-p(t))) - m_i^- y_i(t-p(t)))\lambda_{2i}(m_i^+ y_i(t-p(t)) - g_i(y_i(t-p(t)))) \geq 0$$

$$2(g_i(y_i(t)) - g_i(y_i(t-p(t))) - m_i^-(y_i(t) - y_i(t-p(t))))\lambda_{3i}(m_i^+(y_i(t) - y_i(t-p(t))) - g_i(y_i(t)) + g_i(y_i(t-p(t)))) \geq 0$$

which imply

$$\begin{aligned} &2\omega^T(t)\Omega_1^T\Lambda_1\Omega_2\omega(t) \geq 0 \\ &2\omega^T(t)\Omega_3^T\Lambda_2\Omega_4\omega(t) \geq 0 \\ &2\omega^T(t)\Omega_5^T\Lambda_3\Omega_6\omega(t) \geq 0 \end{aligned} \tag{11}$$

Since $U(t, y(t)) = y^T(t)Qy(t)$, we suppose that for some real number $\rho > 1$

$$U(t+s, y(t+s)) < \rho U(t, y(t)) \qquad \forall s \in [-p, 0]$$

we obtain

$$\rho y^T(t)Qy(t) - y^T(t-p(t))Qy(t-p(t)) > 0 \tag{12}$$

Combining estimates (8)-(12), we obtain

$$D_t^\alpha U(t, y(t)) \leq \omega^T(t)\bar{\gamma}\omega(t) - \int_{t-p(t)}^{t}(t-s)^{\alpha-1}\phi^T(t)L\phi(t)ds \tag{13}$$

where

$\bar{\gamma} = p^\alpha\alpha^{-1}\Omega^T L\Omega + \gamma_1 + \bar{\gamma_2} + \gamma_3$

$\bar{\gamma_2} = \theta_1^T[-GA - AG + \rho Q]\theta_1 - \theta_2^T Q\theta_2 + \theta_5^T[-2G]\theta_5$

Since $\rho > 1$ is an arbitrary parameter and $D_t^\alpha U(t, y(t))$ doesnot depend on ρ, taking $\rho \longrightarrow 1^+$, the inequality (13) becomes

$$D_t^\alpha U(t, y(t)) \leq \omega^T(t)\gamma\omega(t) - \int_{t-p(t)}^{t}(t-s)^{\alpha-1}\phi^T(t)L\phi(t)ds \tag{14}$$

where $\gamma = p^{\alpha}\alpha^{-1}\Omega^{T}L\Omega + \gamma_1 + \gamma_2 + \gamma_3$

$\implies D_t^{\alpha}U(t, y(t)) < \omega^{T}(t)\gamma\omega(t)$

Take $\gamma < \beta Q$, where$\beta \in \mathbb{R}$ and $\beta > 0$

$$\begin{aligned} D_t^{\alpha}U(t, y(t) &< \omega^{T}(t)\beta Q\omega(t) \\ &< \beta\omega^{T}(t)sym(\theta_1 Q\theta_1^{T})\omega(t) \\ &= -Wy^{T}(t)Qy(t) \end{aligned}$$

where $-W \in \mathbb{R}$

Thus,

$$D_t^{\alpha}U(t, y(t)) < -WU(t, y(t)) \tag{15}$$

Take $(\frac{D_k}{\Gamma(\alpha+1)})^{T}Q(\frac{D_k}{\Gamma(\alpha+1)}) \leq K_k Q$

$$\begin{aligned} U(t_k^{+}) &\leq y^{T}(t_k^{-})K_k Qy(t_k^{-}) \\ &\leq K_k y^{T}(t_k^{-})Qy(t_k^{-}) \\ &< K_k\omega^{T}(t_k^{-})sym(\theta_1^{T}Q\theta_1)\omega(t_k^{-}) \end{aligned}$$

Thus,

$$U(t_k^{+}, y(t_k^{+})) \leq K_k U(t_k^{-}, y(t_k^{-})) \tag{16}$$

Hence,

$$\begin{aligned} &D_t^{\alpha}U(t, y(t)) < -WU(t, y(t)) \qquad t \neq t_k \\ &U(t_k^{+}, y(t_k^{+})) \leq K_k U(t_k^{-}, y(t_k^{-})) \quad t = t_k \\ &U(t, y(t)) = U(t_0) \qquad\qquad t = t_0 \end{aligned} \tag{17}$$

For any $t \in [t_0, t_1)$, we have

$$U(t) \leq U(t_0)e^{\frac{-W(t-t_0)^\alpha}{\Gamma(\alpha+1)}}$$

which gives $U(t_1^-) \leq U(t_0)e^{\frac{-W(t_1-t_0)^\alpha}{\Gamma(\alpha+1)}}$

Set $U(t_1^+) = U(t_1)$

For any $t \in [t_1, t_2)$ we have

$$U(t) \leq U(t_1)e^{\frac{-W(t-t_1)^\alpha}{\Gamma(\alpha+1)}}$$

$$\leq U(t_0)K_1 e^{\frac{-W[(t-t_1)^\alpha+(t_1-t_0)^\alpha]}{\Gamma(\alpha+1)}}$$

Similarly for any $t \in [t_k, t_{k+1})$ we have

$$U(t) \leq U(t_k)e^{\frac{-W(t-t_k)^\alpha}{\Gamma(\alpha+1)}}$$

$$\leq U(t_0)K_1 K_2 ... K_k e^{\frac{-W[(t-t_k)^\alpha+(t_k-t_{k-1})^\alpha+...+(t_1-t_0)^\alpha]}{\Gamma(\alpha+1)}}$$

By the condition if $0 < t_{k+1} - t_k \leq r$, $ln(K_k) < \frac{-(\mu-W)r^\alpha}{\Gamma(\alpha+1)}$, where r, $\mu > 0$, we obtain

$$U(t) \leq U(t_0)e^{\frac{-(\mu-W)kr^\alpha}{\Gamma(\alpha+1)}} e^{\frac{-W(k+1)r^\alpha}{\Gamma(\alpha+1)}}$$

$$\leq U(t_0)e^{\frac{-\mu kr^\alpha}{\Gamma(\alpha+1)}} e^{\frac{Wkr^\alpha}{\Gamma(\alpha+1)}} e^{\frac{-Wkr^\alpha}{\Gamma(\alpha+1)}} e^{\frac{-Wr^\alpha}{\Gamma(\alpha+1)}}$$

Thus,

$$U(t) \leq U(t_0)e^{\frac{-(\mu k+W)r^\alpha}{\Gamma(\alpha+1)}} \tag{18}$$

where $\mu k + W > 0$ and $k \longrightarrow \infty$, $r \longrightarrow \infty$ then $U(t) \leq 0$

Thus our system is exponentially stable. $\square$

4 Example

This segment gives an example to depict the usefulness of the result obtained.

We consider a FONN having time delay that can be defined as (4) with the parameters below:

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 0.5 & 3 \end{bmatrix}, C = \begin{bmatrix} 0.2 & -0.3 \\ 0 & 1 \end{bmatrix}, D_k = \begin{bmatrix} 0.7 & 0 \\ 0 & 1 \end{bmatrix}$$

$p(t) = \frac{1}{1+t}$, $t \geq 0$ is choosen as the time varying function.
$g(y(t)) = (tanh\ y_1(t), tanh\ y_2(t))^T$ is chosen as the activation function and $g(y(t - p(t))) = (tanh\ y_1(t - p(t)), tanh\ y_2(t - p(t))) \in \mathbb{R}^2$ is taken as the delay term. Choose $\alpha = 0.99$. Initial condition is taken as $y(0) = (0.2, -0.1)$.

Solution:
The FONN having time varying delay which is described as system (4) with the above parameters satisfies the LMI's (5), (6) and the condition (7). Thus the system is exponentially stable.

5 Conclusion

Here a fractional order neural network having impulses and delay time is considered and its exponential stability is examined. We introduced a convex Lyapunov function for our system and used certain LMI conditions to achieve exponential stability. The future research can be taken forward by broadening the obtained criterion to complex valued neural networks.

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