

The MHD flow of liquid film in the presence of dissipation and thermal radiation through an unsteady stretching sheet by HAM

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ABSTRACT

This paper examines the impact of magnetohydrodynamic (MHD) over a thin film of an unsteady stretching sheet. Prandtl number model of dynamic viscosity and thermal conductivity is examined using Homotopy analysis method. By Making use of appropriate self-similar conversion, the system of model partial differential equations (PDEs) with strong nonlinearity is converted into a non-dimensional set of couple ODEs (Ordinary Differential Equations). Consequently, the system of these transformed equations is analytically explained by implementing Homotopy Analysis Method (HAM). The effects of embedded parameters such as magnetic parameter (M), radiation parameter N_{rand} and Eckert number (Ec) on involved distributions are interpreted graphically to examine the heat transport features for both sorts of unsteadiness parameters $S=0.8$ and $S=1.2$. Moreover, the (C_f) skin friction along with the (Nu) heat transport rate (Nusselt number) is formulated for different values of relevant variables.

Key words: Homotopy analysis method, Magnetohydrodynamics; Non-uniform heat source/sink, Nanofluid .

1. INTRODUCTION

In recent times, a frequent suggestion of liquid film in the arena of engineering and applied sciences has been visualized by a copious number of scholars due to its tremendous applications.

The research of liquid streaming and heat transport analysis under the impact of an induced magnetic field is of great importance due to its extensive ranges of technical, engineering and biotic uses such as crystal growing, freezing of metallic sheets, Hall generator, manufacture of magneto-rheostatic elements (smart liquids), metal molding, cured oil refinement and molten metal freezing blankets for nuclear reactors etc. In MHD mechanism the degree of heat transport can be organized using the MHD stream in electrically conducting liquids and therefore required features of cooling outcome can be attained effortlessly. In aforementioned streaming of liquid, a Lorentzian type of magnetic potential is produced crosswise to the path of the induced magnetic field which is helpful in modifying very high-temperature plasmas, energy instability along with the wetting oscillations.

The studies of liquid film scattering have numerous applications for coating phenomena and have a dynamic part in mechanical engineering. These uses of coating need complete information to link the diffusion of liquid with numerous categories of sheets, discs, wires, and fibers, etc. All coating (varnish) methods mandate a flat polished surface to meet the necessities of the finest look and ideal presentation like roughness, clearness, and strength. The major element which controls the coating procedure is the heat transport rate inside the thin liquid film. Therefore, the analysis of heat transport in thin layer flow over a stretching surface is applicable with respects to the coating mechanism. Wang[1] studied the first time the streaming of thin liquid film over an extending surface. Consequently, many scholars extended the research of Wang to take different working fluids using various assumptions[2-5], Narayana et al[6] examined the numerical computation of thermocapillary stream of thin nano-liquid film through a stretchable surface. More recently, Gul[7] also discussed the mathematical modelling and computation of thin liquid streaming of variable thickness over a nonlinear extending surface. Andersson et al[8] have examined the unsteady flow of a thin film liquid over a stretching surface.

The HAM (homotopy analysis method), which is an asymptotic scheme is used to achieve the series form solution. HAM is a suitable and effective technique used to control the convergence of the estimated solution. Moreover, HAM technique is dependable not only for small variables but also exemplify its usefulness and competency to solve the higher non-linear problem arising in sciences, engineering and modern technologies[9-14].

Noor et al[15-16] examined the heat transfer analysis in a thin film flow over a stretching sheet using the a homotopy analysis method. They observed the magnetic and thermal aspects of the embedding parameters during fluid motion. Liao[17] introduced a new version of this method for the error analysis called optimal homotopy analysis method (OHAM) in its package named BVPh 2.0. Gul and Firdous[18] utilized OHAM BVPh 2.0 package to find the solution of non-linear differential equations involved in the experimental and theoretical models. Guled and Singh[19-20] investigated the effect of MHD flow on heat and mass transfer over stretching sheet by homotopy method. Authors [21-24] have studied various effects of thin liquid film flow over a stretching sheet by numerically.

The major outcome of the present communication is to analyze the combined effects of dissipation and radiation of liquid thin layer and heat transport analysis through a horizontal moving with unsteady stretching plate. The principal equations of a model problem for velocity and thermal field are transformed developing the suitable match quantities. So, the final form of the transformed problem is analytically simplified with help of HAM (Homotopy Analysis Method). The physical developing model variables are depicted through tables and graphs for both $s=0.8$ and $S=1.2$. Furthermore, the surface drag force and heat transport rate are also deliberated.

2. Mathematical model

The governing boundary layer equation of the flow problem is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{16\sigma^* T_\infty^3}{3\rho C_p k^*} \frac{\partial^2 T}{\partial y^2} \quad (3)$$

Where the notations mentioned in (1), (2) and (3) are, velocity components of fluid in flat and perpendicular directions respectively are u and v , the dynamic viscosity- μ , the absorption coefficient- k^* , electrical conductivity- σ , specific heat at constant pressure - C_p , Stefan Boltzman constant - σ^* , temperature- T and fluid density- ρ .

Here, equation (1) shows the conservation law of mass and equation (2) represents the momentum equation. Furthermore, the velocity components in horizontal and vertical directions are denoted by u and v respectively. It is assumed that there is no liquid motion $t \leq 0$.

2.1. Initial and Boundary conditions

Hence we have the following initial conditions:

$$u = v = 0, T = T_w \quad \text{for } t \leq 0 \quad (5)$$

The boundary layer equations (1) to (3) are to be solved in the domain $t > 0$ subject to the boundary conditions:

$$u = U, v = 0, T = T_w \quad \text{at } y = 0 \quad (6)$$

$$\frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0, \quad \frac{\partial h}{\partial t} = v \quad \text{at } y = h \quad (7)$$

Here, we note that the mathematical problem is implicitly formulated only for $x \geq 0$.

Furthermore, it is assumed that, the surface of the planar liquid is smooth so as to avoid the complications due to surface waves. The viscous shear stress and the heat flux vanish at the adiabatic free surface.

2.2. Method of solution

We now introduce the following similarity variables

$$\psi(x, y, t) = \left(\frac{\nu_f b}{1 - \alpha t} \right)^{\frac{1}{2}} x f(\eta) \quad (8)$$

$$T(x, y, t) = T_0 - T_{ref} \left[\frac{bx^2}{2\nu_f} \right] (1 - \alpha t)^{-\frac{1}{2}} \theta(\eta) \quad (9)$$

$$\eta = \left[\frac{b}{\nu_f (1 - \alpha t)} \right]^{\frac{1}{2}} y \quad (10)$$

The Velocity components u and v in terms of the stream function $\psi(x, y, t)$ are given by

$$u = \frac{\partial \psi}{\partial y} = \left(\frac{bx}{1-\alpha t} \right) f'(\eta) \quad (11)$$

$$v = -\frac{\partial \psi}{\partial x} = -\left(\frac{v_f b}{1-\alpha t} \right)^{\frac{1}{2}} f(\eta) \quad (12)$$

Assuming $n = \beta$ (a dimensionless film thickness to be determined as an integral part of the computation) at the free surface and using equation (10), the film thickness can be expressed as

$$\gamma = \left(\frac{b}{v_f} \right)^{\frac{1}{2}} (1-\alpha t)^{-\frac{1}{2}} h, \quad (13)$$

which gives

$$\frac{dh}{dt} = -\frac{\alpha \gamma}{2} \left(\frac{v_f}{b} \right)^{\frac{1}{2}} (1-\alpha t)^{-\frac{1}{2}} \quad (14)$$

Where a prime represents the derivative with respect to η . Using similarity transformation represented in equations (8) to (10), governing boundary layer equation (2)–(3) yield the following point boundary value problem:

$$f''' + \phi_1 \gamma \left(f f'' - \frac{S}{2} \eta f'' - (f')^2 - S f' \right) = 0 \quad (15)$$

$$(1+R)\theta'' + \phi_2 \left(\frac{k_f}{k_{nf}} \right) \gamma \text{Pr} \left\{ \left(f \theta' - \frac{S}{2} \eta \theta' \right) + Ec (f')^2 \right\} = 0 \quad (16)$$

Subject to

$$f(0) = 0, \quad f'(0) = \theta(0) = 1 \quad (17)$$

$$f''(\gamma) = \theta'(\gamma) = 0 \quad (18)$$

$$f(\beta) = \frac{S\gamma}{2} \quad (19)$$

Here $S = \frac{\alpha}{b}$ represents the unsteadiness parameter, $\text{Pr} = \frac{k_f}{k_{nf}}$ represents the Prandtl number of the base liquids, γ is the film thickness and the constants ϕ_1 and ϕ_2 that depends on the volume fractions are represented by

$$\phi_1 = (1-\phi)^{\frac{5}{2}} \left[(1-\phi) + \phi \left(\frac{\rho_s}{\rho_f} \right) \right],$$

$$\phi_2 = (1-\phi) \phi \left\{ \frac{(\rho C_p)_s}{(\rho C_p)_f} \right\}$$

The parameters for engineering interest in heat transfer problems are the skin friction coefficients C_f and Nussult number Nu_x . These parameters characterize the surface drag and heat transfer rates. The shear stress at the stretching surface τ_w is defined as

$$\tau_w = -\mu_{nf} \left[\frac{\partial u}{\partial y} \right]_{y=0}$$

The skin friction coefficient at the stretching surface is given by

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{2}{(1-\phi)^{5/2}} \text{Re}^{-1/2} \{-f''(0)\} \quad (20)$$

The surface heat flux q_w is defined as

$$q_w = -k_{nf} \left[\frac{\partial T}{\partial y} \right]_{y=0}$$

and the local Nusselt number is given by

$$Nu_x = \frac{xq_w}{k_f(T_w - T_0)} = \left(\frac{k_f}{k_{nf}} \right) \text{Re}^{-1/2} \{-\theta'(0)\} \quad (21)$$

3. HAM Solution

To obtain analytical solution of Eqs. (15)-(16) subject to the boundary conditions (17)-(19), we select initial guess approximations as [19-20]

$$f_0(\eta) = 1 - e^{-\eta} \quad (22)$$

$$\theta_0(\eta) = e^{-\eta} \quad (23)$$

and the auxiliary linear operators as

$$\mathcal{L}_f = \frac{\partial^3}{\partial \eta^3} - \frac{\partial}{\partial \eta} \quad (24)$$

$$\mathcal{L}_\theta = \frac{\partial^2}{\partial \eta^2} - 1. \quad (25)$$

The operators in the above equations satisfy

$$\mathcal{L}_f[C_1 + e^\eta C_2 + e^{-\eta} C_3] = 0 \quad (26)$$

$$\mathcal{L}_\theta[e^\eta C_4 + e^{-\eta} C_5] = 0 \quad (27)$$

in which C_i , for $i = 1, 2, \dots, 5$ are arbitrary constants.

By choosing q as an embedding parameter, we construct the zeroth order deformation equations as

$$(1-q)\mathcal{L}_f[\hat{f}(\eta, q) - f_0(\eta)] = q \hbar_f \mathcal{N}_f[\hat{f}(\eta, q)] \quad (28)$$

$$(1-q)\mathcal{L}_\theta[\hat{\theta}(\eta, q) - \theta_0(\eta)] = q \hbar_\theta \mathcal{N}_\theta[\hat{\theta}(\eta, q)] \quad (29)$$

Subject to the boundary conditions

$$\hat{f}(0, q) = 0, \hat{f}'(0, q) = 1, \hat{f}'(\infty, q) = 0 \quad (30)$$

$$\hat{\theta}(0, q) = 1, \hat{\theta}'(\infty, q) = 0 \quad (31)$$

where the prime denotes the partial derivatives w.r.t. η , $\hbar_f$ and $\hbar_\theta$ are non-zero auxiliary parameters. The nonlinear differential operators $\mathcal{N}_f$ and $\mathcal{N}_\theta$ are given by

$$\begin{aligned} \mathcal{N}_f[\hat{f}(\eta, q)] &= \frac{\partial^3 \hat{f}(\eta, q)}{\partial \eta^3} \\ &+ \phi_1 \gamma \left[\hat{f}(\eta, q) \frac{\partial^2 \hat{f}(\eta, q)}{\partial \eta^2} - \frac{S\eta}{2} \frac{\partial^2 \hat{f}(\eta, q)}{\partial \eta^2} \right. \\ &\left. - \left(\frac{\partial \hat{f}(\eta, q)}{\partial \eta} \right)^2 - S \frac{\partial \hat{f}(\eta, q)}{\partial \eta} \right] \end{aligned} \quad (32)$$

$$\begin{aligned} \mathcal{N}_{\hat{\theta}}[\hat{f}(\eta, q)] = & (1 + R) \frac{\partial^2 \hat{\theta}(\eta, q)}{\partial \eta^2} \\ & + \phi_2 \frac{k_f}{k_{nf}} \gamma \Pr \left[\left\{ \hat{f}(\eta, q) \frac{\partial \hat{\theta}(\eta, q)}{\partial \eta} - \frac{S\eta}{2} \frac{\partial \hat{\theta}(\eta, q)}{\partial \eta} \right\} \right. \\ & \left. + Ec \left(\frac{\partial \hat{f}(\eta, q)}{\partial \eta} \right)^2 \right] \end{aligned} \quad (33)$$

Obviously, for $q = 0$ and $q = 1$, we have

$$\hat{f}(\eta, 0) = f_0(\eta), \quad \hat{f}'(\eta, 1) = f(\eta) \quad (34)$$

$$\hat{\theta}(\eta, 0) = \theta_0(\eta), \quad \hat{\theta}'(\eta, 1) = \theta(\eta) \quad (35)$$

By using Taylor's theorem and Eqs. (34) and (35), we have

$$\hat{f}(\eta, q) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) q^m \quad (36)$$

$$\hat{\theta}(\eta, q) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) q^m \quad (37)$$

and

$$f_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \hat{f}(\eta, q)}{\partial \eta^m} \right|_{q=0} \quad (38)$$

$$\theta_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \hat{\theta}(\eta, q)}{\partial \eta^m} \right|_{q=0}. \quad (39)$$

We assume that both $\hbar_f$ and $\hbar_{\theta}$ are properly chosen in such a way that the series (36) and (37), respectively, are convergent at $q = 1$. Then, due to Eqs. (34) and (35), we have

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \quad (40)$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \quad (41)$$

and respectively.

The m -th order deformation equation can be obtained by differentiating the zeroth order deformation equations (28) and (29) m -times w.r.t. q and then dividing by $m!$ and finally setting $q = 0$, we have

$$\mathcal{L}_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_f R_{m,f}(\eta) \quad (42)$$

$$\mathcal{L}_{\theta}[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_{\theta} R_{m,\theta}(\eta) \quad (43)$$

with the boundary conditions

$$f_m(0) = f'_m(0) = f'_m(\infty) = 0 \quad (44)$$

$$\theta_m(0) = \theta_m(\infty) = 0 \quad (45)$$

where

$$\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases}. \quad (46)$$

and

$$\begin{aligned} R_{m,f}(\eta) = & f'''_{m-1} - \phi_1 \gamma \frac{S\eta}{2} f''_{m-1} - \phi_1 \gamma S f'_{m-1} \\ & + \phi_1 \gamma \sum_{k=0}^{m-1} [f_{m-1-k} f''_k - f'_{m-1-k} f'_k] \end{aligned} \quad (47)$$

$$R_{m,\theta}(\eta) = (1 + R)\theta''_{m-1} - \phi_2 \frac{k_f}{k_{nf}} \gamma \text{Pr} \frac{S\eta}{2} \theta'_{m-1} + \phi_2 \frac{k_f}{k_{nf}} \gamma \text{Pr} \sum_{k=0}^{m-1} [f_{m-1-k} \theta'_k - \text{Ec} (f'_{m-1-k} f'_k)] . \quad (48)$$

Here MATHEMATICA is used to solve the linear homogeneous Eqs. (42) and (43) one after the other in the order of $m = 1, 2, 3, \dots$

4. Results and discussion

The exploration of an unsteady uniform flow with thermal radiation of a thin film nanofluid was observed analytically using HAM. The thermo-physical property of a nanofluid was considered for volume fraction. The governing PDE's will transform to ordinary ones by similarity transformations. The obtained nonlinear BVP's which are nonlinear in nature are given by equations (15)-(19) and are solved by Homotopy Analysis Method. The dimensionless film thickness β has been determined by solving an equation (19). The velocity equation (15) is decoupled from the temperature equation (16). The nanoparticle volume fraction ϕ and an unsteadiness parameter S are the two parameters which will affect in the flow and heat transfer. Also Prandtl number was kept constant at 6.2 for the base fluid and same is used throughout our observation.

Fig (1) stands for the variation of unsteadiness parameter S with film thickness γ . With this plot, we conclude that, the film thickness decreases for an increase in S from 0 to 2.

Fig (2) represents the effects of volume fraction ϕ_1 on the axial velocity profile $f'(\eta)$ for two different values of S . With this plot, we conclude that, decreases in velocity profile for an increase in volume fraction parameter ϕ_1 .

Fig (3) represents the effects of volume fraction ϕ_2 on the temperature profile $\theta(\eta)$ for two different values of S . With this plot, we observe that, decreases in the temperature profile for an increase in volume fraction parameter ϕ_2 .

Fig.(4) demonstrate the effect of Prandtl number Pr on the temperature profiles for two different values of unsteadiness parameter S . These plots reveal the fact that for a particular value of Pr the temperature increases monotonically from the free surface temperature to wall velocity. The thermal boundary layer thickness decreases drastically for high values of Pr i.e., low thermal diffusivity.

The effect of radiation parameter R on the horizontal velocity profiles is depicted in Fig. (5) for two different values of unsteadiness parameter S . From both these plots one can make out the increasing values of radiation parameter R decreases the temperature in the fluid film. i.e., increasing value of R contributes in thickening of the boundary layer.

The effect viscous dissipation is found to increase the dimensionless free-surface temperature $\theta(\eta)$ for the fluid cooling case which is depicted in fig(6). The impact of viscous dissipation on $\theta(\eta)$ diminishes in the two limiting cases: $\text{Pr} \rightarrow 0$ and $\text{Pr} \rightarrow \infty$, in which situations $\theta(\eta)$ approaches unity and zero respectively.

The wall shear stress $-f''(0)$ increases for increasing values in volume fraction [ref. fig(7)] and where as the wall heat flux $-\theta'(0)$ also increase for higher negative values [ref. fig.(8)].

The quantity $f''(0)$ curve for special values of ϕ_1 for $S = 1.2$ using 11th - order HAM approximation is shown through fig.(9).

The h-curve for the HAM approximation solution over $-\theta'(0)$ for special values of ϕ_1 for $S = 1.2$ is shown through the fig.(10)

Table 1 represents the analytical results on the effects of the unsteadiness parameter S and nanoparticle volume fraction on the flow and heat transfer. It is noted that, for the range $0 \leq S \leq 2$ the solution exists and for $S \rightarrow 0$, the result come up to analytical solution and observed a thick layer of liquid (i.e., for $\gamma \rightarrow \infty$) while $S \rightarrow 2$ corresponds to $\gamma \rightarrow 0$ i.e., infinitesimal thickness of liquid film. Here HAM solutions were attained for the $0 \leq \phi \leq 0.2$ and $0 \leq S \leq 2$

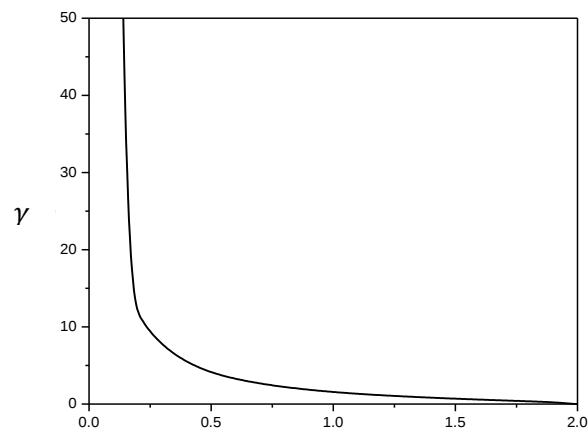


Fig. 1. Film thickness parameter γ vs. unsteadiness parameter S



Fig. 1. Film thickness parameter γ vs. unsteadiness parameter S

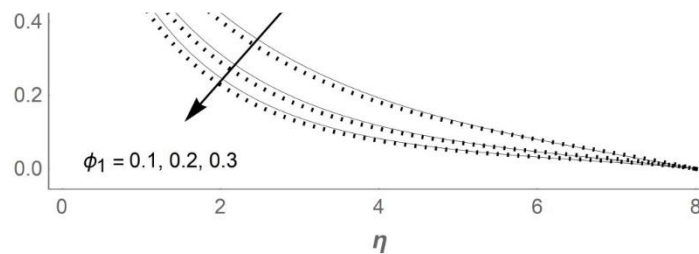


Fig:2: Effects of volume fraction on Velocity Profile

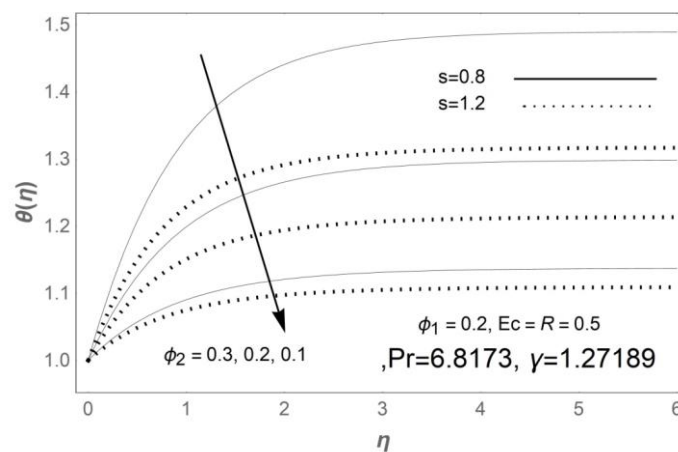


Fig:3: Effects of Volume fraction on temperature Profile

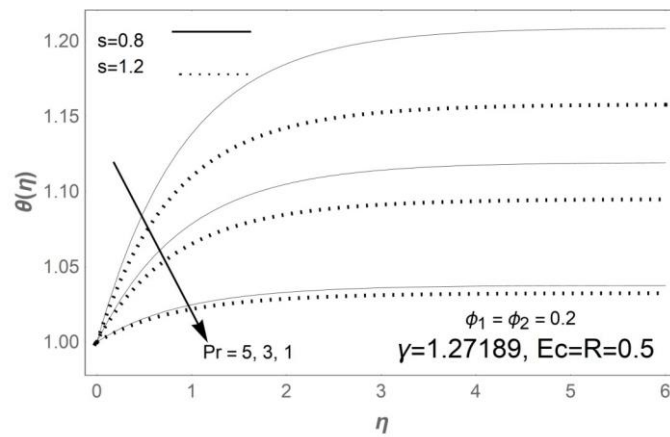


Fig:4: Effects of Prandtl number on temperature Profile

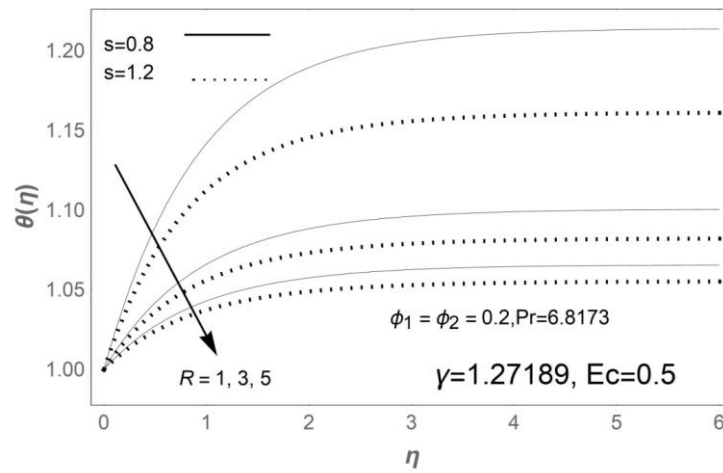


Fig:5: Effects of Radiation parameter on temperature Profile

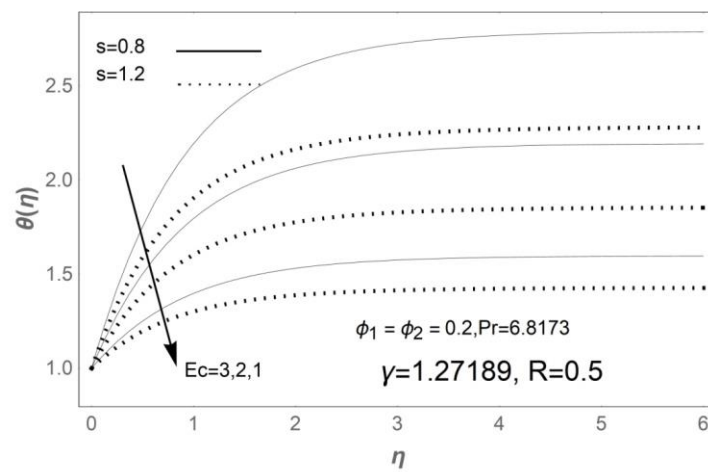


Fig:6: Effects of dissipation parameter on temperature Profile

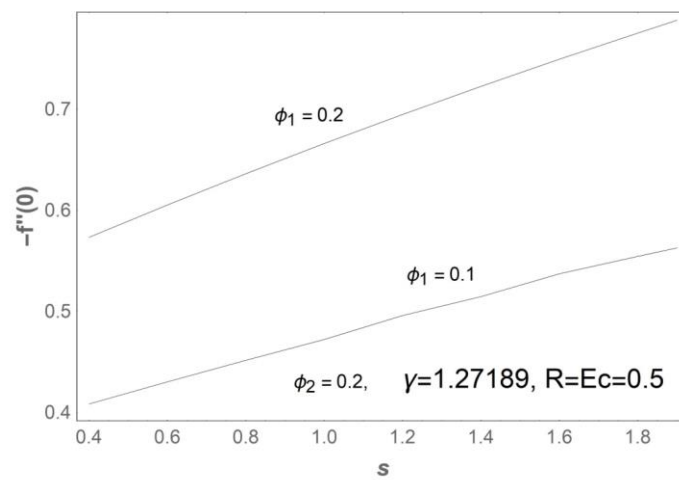


Fig.(7). Wall shear stress $-f''(0)$ vs. S for special values of ϕ_1 and ϕ_2

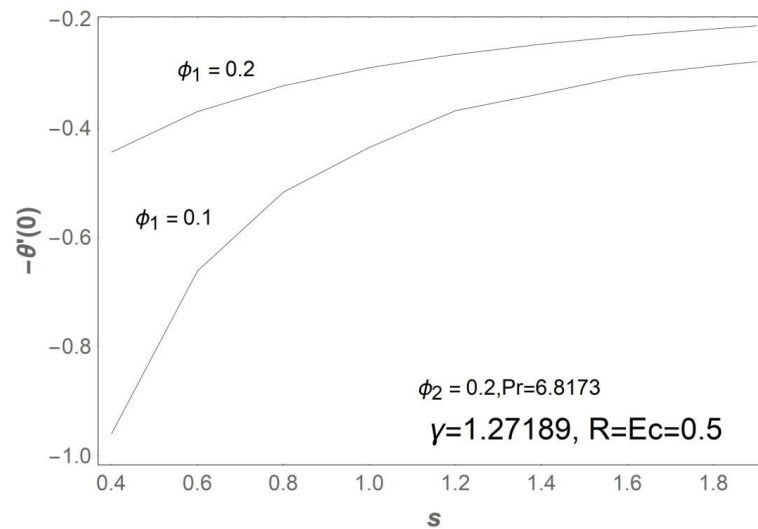


Fig.(8). Wall temperature gradient $-\theta'(\eta)$ vs. S for special values of ϕ_1

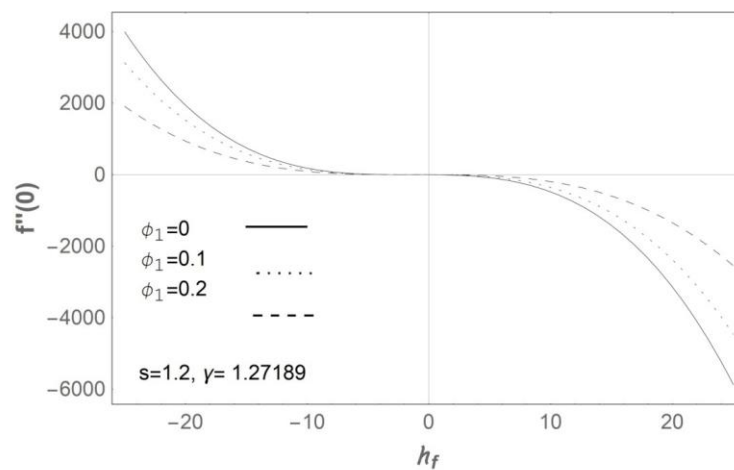


Fig. (9). The h_f curve for special values of volume fraction ϕ_1 for $S = 1.2$ using 11th-order HAM approximation

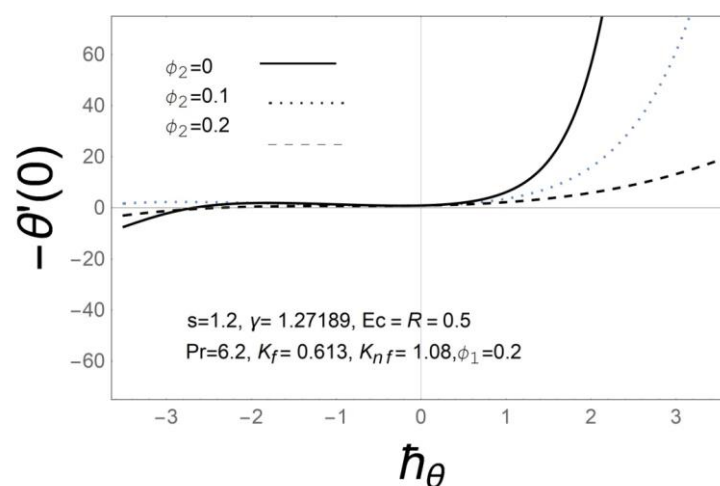


Fig. (10). h_θ -curve for the HAM approximation solution over $-\theta'(0)$ for $S = 1.2$

Table-1: Wall skin friction coefficient $-f''(0)$ and wall temperature gradient $-\theta'(0)$ for different values of an unsteadiness parameter S

S	$-f''(0)$	$-\theta'(0)$
0.4	0.5734	-0.4436
0.6	0.6054	-0.3693
0.8	0.6363	-0.3219
1.0	0.6662	-0.2889
1.2	0.6949	-0.2646
1.4	0.7227	-0.2458
1.6	0.7496	-0.2304
1.8	0.7754	-0.2179
1.9	0.7881	-0.2124

5. Conclusion

The significant conclusions from the current study by HAM technique are as follows.

- The unsteadiness parameter S constantly thickens the velocity boundary layer and nanofluids boundary layer approaches towards thinning.
- An increase in volume fraction parameter ϕ decreases the dimensionless film thickness γ
- An enhancement in ϕ could improve $\theta(\eta)$, hence thickens the thermal boundary layer.
 $-f''(0)$ enhances the volume fraction ϕ
- Dimensionless wall temperature gradient $-\theta'(0)$ declines the volume fraction ϕ

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