Retrial Queue $M^{[x]}/G/C$ With Bernoulli's Vacation, Repair, Service And Re-Service In Orbit

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Abstract: This paper focused on a back - end system for periodic testing with modified Bernoulli's vacation on each service includes additional service after breakdowns and repairs by the method of supplementary variable techniques. Further vacation repair and vacation interruption on queues and orbit is discussed simultaneously. Upon arrival, if the server is busy, serviced, or on vacation, then the entire queue (batch wise) joins in the orbit. In general all the customers require and expect the regular service and appropriate re-service, but only some customers require additional service and re-service. At the end of each service few exceptional case of re-service is available with probability a. This reliability measured was examined through various numerical parameters. To implement these concepts we build a mathematical model using the complementary variable method and obtained a function that gives the probabilities for the number of customers in the system when the server is free or on repair or on vacation.

Keywords: Retrial queue, Selective retrial, Recovery delay, Bernoulli vacation, operational service and operational re-service.

1. Introduction

The recent retrial queuing system has made an important contribution to the fact that the server is busy upon arrival and there is no space to wait. In this case the entire waiting batches joins a reuse group called Orbit, and after a while repeats the same and requests a queue and service. Those customers in the queues are allowed to get service and re-service are often used in telephone exchange systems, telecommunications networks, and computer communications and even in the hospitals. Gao S et al (2014) [8] and Artalejo and Gomez-Corral (2010) [1] were contributed and submitted their detailed discussion of the basic concepts of retrial queues for readers and researchers. B.T. Doshi [3] addressed the queuing systems with vacations in the initial stage. Vacation and Priority Systems of the queues are discussed by Takagi H [15]. The concept of selective re-servicing must be taken into account for the reason that many queues experienced different clients' characteristics [10]. If a service crashes then within the knowledge of orbit, a batch requests a new service again and this creates the new queue and returns the service immediately. But in such case re-service is difficult to systematize and organize. Re-service has many practical applications in situations such as banks, supermarkets, medical clinics, theme parks etc. Baruah et al (2013) [2] and Rajadurai et al (2018) [13] were investigated the re-deployment concepts of queues and further they extended the selective reboots as part of a revised vacation policy that causes server crashes and repair delays. The key point in the queue is regarding server failure [14] . All kinds of servers and service centers are inevitably facing the consequences of server failure [18].

The client's waiting time in the system increases until the faulty server is eliminated or repair in normal time. Fadhil R et al [7] analyzed the $M/G/1$ retrial queue on a server being repaired. In an interrupt queuing system, a server can be unavailable for a specified period of time for a number of reasons, including: maintenance, communications networks, tasks or other queues or interruptions [12]. A detailed study of queuing patterns on vacations can be found in Dimou S et al [6]. Wu J et al [16] discussed Bernoulli's vacation schedule when the server goes on vacation. The server may not be aware of the system client's status when it was under repair or on vacation. S.R. Chakravarthy S et al [4] and Deepa B et al [5] discussed various queuing model with working vacation. T. Jiang et al [9] address this issue in the multi phase server system which selects a vacation with probability p. This concept is known as a modified Bernoulli's vacation. Server crashes and gets repaired during modified Bernoulli's vacation. In this article, we look at analyzing system which freezes after a series of selective recovery attempts of server under repair and under vacation. W.M. Kempa et al [11] and D Y Yang et al [17] discussed finite capacity of queues.

This paper is constructed as follows. Section II focuses retrial queues networks features like arrival process, verification procedure, retrial process, service process, vacation process, recovery process, Ergodicity Condition according to the priority and its steady state distribution. Section III focused the steady state conditions of retrial queue with server breakdown. The governing equations are constructed according to the supplementary variable techniques. Section IV provides the numerical illustrations of Computational probability values of various services. Finally section V concludes the paper.

2. Mathematical description of the model

This section provides a extensive description of the proposed model:

Arrival process:

The customer uses a complex Poisson process of customers' arrival rate λ.

Verification Procedure:

It is assumed that there is no waiting room for the customer when server is busy. When a client detects that the server is processing that there is no waiting room, then the request immediately, reveal that client join a group of another batch of clients (considered in a new group) who were already there to get service, called as Orbit. The sequential iteration time of each client has a random probability distribution $F(a)$ with the corresponding Laplace-Stielejes density $\phi(a)$ and Laplace-Stielejes transformation (LST) function $F^{*}(a)$. The conditional value of Laplace-Stielejes density $\phi(a)$ and Laplace Stielejes transformation function are as follows

$$\phi(a)da = \frac{dF(a)}{1-F(a)}$$

Service Process:

All the customers who entered in the queue should get service on time and service time is usually allocated by the server and sometimes allotted by the orbit (during the vacation or server is under repair). After completion of regular service, the client can request a service update from the previous service without participating in the trajectory with probability p, or turn off the system with probability $\bar{p} = 1 - p$. The service time is unspecified in this case and in such case the server has to follow the general law corresponding to the probability distribution function $I_1(i)$ for providing normal service to the customers, $I_2(i)$ and LST $I_1^*(a), I_2^*(a)$ for selective re-service. The completion level of this service during the vacation process is denoted as:

$$\mu_1(a)da = \frac{dI_1(a)}{1-I_1(a)},\ \mu_2(a)da = \frac{dI_2(a)}{1-I_2(a)}$$

Vacation Process:

After completing servicing for each client, the server can go away for a vacation of duration $M(a)$ period. If there are no pending clients in orbit, I_s assume that the system server is expecting a new client with probability 1 and at the end of the vacation, and is expected to satisfy:

$$\omega(a)da = \frac{dM(a)}{1-M(a)}$$

Recovery process:

During this recovery process period, arrival process of new customer was stopped and services against the existing customers were stopped. Awaiting clients are naturally waiting for the rest of the service to complete before the server crashes. It is assumed that the recovery time, expressed as the server distribution K_1, is randomly distributed using the density functions $K_j(i)$ and LST $K_j^*(b)$, j $= 1,2$. The primitive measure of completion for recovery is as follows:

$$u_j(b)db = \frac{dK_j(b)}{1-K_j(b)}$$

Now let $F^0(n), I_1^0(n), I_2^0(n), M^0(n), K_1^0(n), K_2^0(n)$ be the total elapsed retrial time, normal service time, optional re-service time, vacation time, repair on normal service time and repair on optional re-service time respectively. In the steady state, we assume that

$F(0) = 0, F(\infty) = 1, I_1(0) = 0, I_1(\infty) = 1, I_2(0) = 0, I_2(\infty) = 1, M(0) = 0, M(\infty) = 1$ are obviously continuous at $a = 0$ and $K_j(0) = 0, K_j(\infty) = 1$ are continuous at $b = 0$.

The built-in Markov chain values lies between customer's arrival time and their departure time. There are number of Markov chains were calculated in which the maintenance period ends or a vacation or repair time ends. The probability that the system will be empty at time t is exactly n clients in the orbit at time t_0 (initial arrival time). Consider a is the exact time which includes customers waiting time, service time, server is on vacation and under repair.

$$W(n) = \begin{cases} 0, \text{ if the server is idle at time t} \\ 1, \text{ if the server is busy at time t} \\ 2, \text{ if the server is on re - service at time t} \\ 3, \text{ if the server is repair on normal service at time t} \\ 4, \text{ if the server is repair on optional re-service at time t} \\ 5, \text{ if the server is on vacation at time t} \\ 6, \text{ if the server is working on vacation at time t} \end{cases}$$

Ergodicity Conditions according to the priority

Let $\{n_t/t \in T\}$ be the sequence of epochs at which either a service time completion occurs or a vacation time ends or server on repair time ends. The sequence of random vectors $Q_t = \{W(n_t+t), A(n_t+t)\}$ form a embedded Markov chain for our retrial queuing system. Laplace-Stielejes transformation (LST) function $F^*(\lambda)$, Poisson process of normal service and optional re-service are denoted by β_1 and β_2 respectively. Re-service to the customers without joining the orbit with probability is denoted by the notation u. The embedded Markov chain $\{c_t/t \in T\}$ is ergodic if and only if $\rho < 1$

Where,

$$\psi = (1 - F^*(\lambda)) + \left(\lambda\left[E(I_1)\left(1 + \beta_1 E(K_1)\right) + u\left(E(I_2)\left(1 + \beta_2 E(K_2)\right)\right) + pE(M)\right]\right)$$

1. <u>Steady state distribution</u>

In this section, we developed the stationary distribution for the servers when the server is busy and / or under repair. We define the probability to process $A(i), i \geq 0\}$ this server as:

$P_0(i) = P\{W(i) = 0, A(i) = 0\}$

$P_t(a,i)da = P\{W(i) = 0, A(i) = t, a \leq F^0(i) < a + da\}, t \geq 1$

$\pi_{1,t}(a,i)da = P\{W(i) = 1, A(i) = t, a \leq I_n^0(i) < a + da\}$ for $i \geq 0, a \geq 0, t \geq 0$

$\pi_{2,t}(a,i)da = P\{W(i) = 2, A(i) = t, a \leq I_2^0(n) < a + da\}$ for $i \geq 0, a \geq 0, t \geq 0$

$R_{1,t}(a,b,i)db = P\{W(i) = 3, A(i) = t, b \leq K_1^0(i) < b + db/I_1^0(i) = a\}$ for $i \geq 0, (a,b) \geq 0, t \geq 0$

$R_{2,}(a,b,i)db = P\left\{W(i) = 4, A(i) = t, b \leq K_2^0(i) < b + \frac{db}{I_2^0(i)} = a\right\}$ for $i \geq 0, (a,b) \geq 0, t \geq 0$

$M_t(a,i)da = P\{W(i) = 5, A(i) = t, a \leq M^0(i) < a + da\}, t \geq 0$

The following probabilities are used in the subsequent sections.

$P_0(i)$ is the probability when the system is empty at time t.

$P_t(a,i)$ is the probability that exactly n customers has to undergo retrial service.

$\pi_{1,t}(a,i)$ is the probability that exactly n customers undergoing normal service (there is re-service option).

$\pi_{2,t}(a,i)$ is the probability that exactly n customers has to get re-service.

$R_{1,t}(a,b,i)$ is the probability that exactly n customers undergoes elapsed repair time of server is b.

$R_{2,t}(a,b,i)$ is the probability that exactly n customers undergoes the elapsed re-service time a and the elapsed repair time of server is b.

$M_t(a,i)$ is the probability that customers waiting in the queue when server went on vacation.

The corresponding limiting probabilities related to the ensure its stability conditions are:

$P_0 = \lim_{t\to\infty} P_0(n)$ for $n \geq 0$

$P_t(a) = \lim_{t\to\infty} P_t(a,i)$ for $i \geq 0, a \geq 0, i \geq 1$

$\pi_{1,t}(a) = \lim_{t\to\infty} \pi_{1,t}(a,i)$ for $i \geq 0, a \geq 0, t \geq 0$

$\pi_{2,t}(a) = \lim_{t\to\infty} \pi_{2,t}(a,i)$ for $i \geq 0, a \geq 0, t \geq 0$

$R_{1,t}(a,b) = \lim_{t\to\infty} R_{1,t}(a,i)$ for $i \geq 0$

$R_{2,t}(a,b) = \lim\limits_{t\to\infty} R_{2,t}(a,i)$ for $i \geq 0$

$M_t(a) = \lim\limits_{t\to\infty} M_t(a,i)$ for $i \geq 0$ exist

III Steady state equations

Consider the continuous retrial queue of breakdown where customers were probably join the system when the server is busy, and likely never go to the new system when the server breaks down. Various system behavior under steady state as follows:

Idle state: (empty)

$$\lambda P_0 = \int_0^\infty M_0(a)\omega(a)da + (1-\rho)\bar{u}\int_0^\infty \pi_{1,0}(a)\mu_1(a)da + (1-\rho)\int_0^\infty \pi_{2,0}(a)\mu_2(a)da \quad (3.1)$$

Idle state: (non-empty) retrial case

$$\frac{dP_t(a)}{da} + (\lambda + \phi(a))P_t(a) = 0, t \geq 1 \quad (3.2)$$

Busy state:

$$\frac{d\pi_{1,0}(a)}{da} + \left(\lambda + \beta_1 + \mu_1(a)\pi_{1,0}(a) + \int_0^\infty R_{1,0}(a,b)\mu_1(b)db, t = 0\right) \quad (3.3)$$

$\frac{d\pi_{1,t}(a)}{da} + \left(\lambda + \beta_1 + \mu_1(a)\pi_{1,0}(a) + \lambda\pi_{1t-1}(a) + \int_0^\infty R_{1,t}(a,b)\mu_1(b)db, t \geq 1\right)$ (3.4)

$\frac{d\pi_{2,0}(a)}{da} + \left(\lambda + \beta_2 + \mu_2(a)\pi_{2,0}(a) + \int_0^\infty R_{2,0}(a,b)\mu_2(b)db, t = 0\right)$ (3.5)

$\frac{d\pi_{2,t}(a)}{da} + \left(\lambda + \beta_2 + \mu_2(a)\pi_{2,0}(a) + \lambda\pi_{2t-1}(a) + \int_0^\infty R_{2,t}(a,b)\mu_2(b)db, t \geq 1\right)$ (3.6)

Repair state:

$\frac{dR_{1,0}(a,b)}{db} = (\lambda + u_1(b))R_{1,0}(a,b), t = 0$ (3.7)

$\frac{dR_{1,t}(a,b)}{db} = (\lambda + u_1(b))R_{1,t}(a,b) + \lambda R_{1,t-1}(a,b), t \geq 1$ (3.8)

$\frac{dR_{2,0}(a,b)}{db} = (\lambda + u_2(b))R_{2,0}(a,b), t = 0$ (3.9)

$\frac{dR_{2,t}(a,b)}{db} = \left(\lambda + u_2(b)\right)R_{2,t}(a,b) + \lambda R_{2,t-1}(a,b), t \geq 1$ (3.10)

Vacation state:

$\frac{dM_0(a)}{da} = \left(\lambda + \omega(a)\right)M_0(a), t = 0$ (3.11)

$\frac{dM_t(a)}{da} = (\lambda + \omega(a))M_t(a) + \lambda M_{t-1}(a), t \geq 1$ (3.12)

The steady state boundary conditions are

$\psi_t(a) = \int_0^\infty \mu_t(a)\omega(a)da + (1-\rho)\bar{u}\int_0^\infty \pi_{1,t}(a)\mu_1(a)da + (1-\rho)\int_0^\infty \pi_{2,t}(a)\mu_2(a)da, t \geq 1$ (3.13)

$\pi_{1,0}(0) = \lambda P_0 + \int_0^\infty P_1(a)\phi(a)da$ (3.14)

$\pi_{1,t}(0) = \int_0^\infty P_{t+1}(a)\phi(a)da + \lambda\int_0^\infty P_t(a)da, t \geq 1$ (3.15)

$\pi_{2,0}(0) = u\int_0^\infty \pi_{1,t}(a)\mu_1(a)da, t \geq 0$ (3.16)

$R_{1,t}(a,0) = \beta_1\left(\pi_{1,t}(a)\right), t \geq 0$ (3.17)

$R_{2,t}(a,0) = \beta_2\left(\pi_{2,t}(a)\right), t \geq 0$ (3.18)

$M_0(0) = (1-u)\int_0^\infty \pi_{1,0}(a)\mu_1 da + \int_0^\infty \pi_{2,0}(a)\mu_2(a)da, t = 0$ (3.19)

$M_t(0) = \rho(1-u)\int_0^\infty \pi_{1,t}(a)\mu_1 da + \rho\int_0^\infty \pi_{2,t}(a)\mu_2(a)da$ (3.20)

The normality conditions are:

$\mathrm{p}_0 + \sum_{t=1}^\infty p_t(a)da + \sum_{t=0}^\infty \left[\int_0^\infty \pi_{1,t(a)da+}\int_0^\infty \pi_{2,t(a)da+\int_0^\infty \mu_{t(a)d\theta+}+\iint_0^\infty R_{1,t}(a,b)dadb+\iint_0^\infty R_{2,t}(a,b)dadb}\right]=1$ (3.21)

Multiply the steady state equations and steady state boundary conditions (3.2) to (3.20) by c^t and summing over $t(t = 0,1,2 \ldots)$

$\frac{d}{da}\mathrm{p(a,c)} + (\lambda + \phi(\mathrm{a}))\mathrm{p(a,c)} = 0$ (3.22)

$\frac{d}{da}\pi(\mathrm{a,c}) + (\lambda + \beta_1 + \mu_1(\mathrm{a}))\pi_1(\mathrm{a,c}) = \lambda \mathrm{c}\pi_1(\mathrm{a,c}) + \int_0^\infty R_1(\mathrm{a,b,c})\mu_1(\mathrm{\ b})\mathrm{db}$

$\frac{d}{da}\pi_1(a,c) + (\lambda(1-c) + \beta_1 + \mu_1(a))\pi_1(a,c) = \int_0^\infty R_1(a,b,c)\mu_1(b)db$ (3.23)

$\frac{d}{da}\pi_2(a,c) + (\lambda(1-c) + \beta_2 + \mu_2(a))\pi_2(a,c) = \int_0^\infty R_2(a,b,c)\mu_2(b)db$ (3.24)

$\frac{d}{db}R_1(a,b,c) + (\lambda(1-c) + \mu_1(b)R_1(a,b,c) = 0$ (3.25)

$\frac{d}{db}R_2(a,b,c) + (\lambda(1-c) + \mu_2(b)R_2(a,b,c) = 0$ (3.26)

$\frac{d}{d\theta}(\mu\mathrm{a,c}) + (\lambda(1-\mathrm{c}) + \omega(\mathrm{a})\mu(a,c) = 0$ (3.27)

$\mathrm{P(0,c)} = \int_0^\infty \mu(a,c)\omega(\mathrm{a})da + (1-p)\pi\int_0^\infty \pi_1(\mathrm{a,c})\mu_1(\mathrm{a})\mathrm{da} + (1-p)\pi\int_0^\infty \pi_2(\mathrm{a,c})\mu_2(\mathrm{a})\mathrm{da} - \lambda\mathrm{p}$ (3.28)

$\pi_1(0,\mathrm{c}) = \frac{1}{c}\int_0^\infty p(a,c)\emptyset(\mathrm{a})da + \lambda\int_0^\infty p(\mathrm{a,c})\mathrm{da} + \lambda\mathrm{p}_\mathrm{o}$ (3.29)

$\pi_2(0,\mathrm{c}) = \mu\int_0^\infty \pi_1(a,c)\mu_1(\mathrm{a})\mathrm{da}$ (3.30)

$R_1(\mathrm{a,0,c}) = \beta_1\pi_1(\mathrm{a,c})$ (3.31)

$R_2(\mathrm{a,0,c}) = \beta_2\pi_2(\mathrm{a,c})$ (3.32)

3. Numerical Illustrations

In this section, we show the numerical results. We used Equations (3) to (3.32) to compute the transient probabilities of $, \rho, I_1^0(n), I_2^0(n), M^0(n), k_1^0(n), k_2^0(n)$. Table 1 shows the computation probability values.

Table 1: Computational probability values of various service time (s)

λ	ρ	I_1^0 (n)	I_2^0 (n)	M^0 (n)	k_1^0 (n)	k_2^0 (n)

3.0	1.5	0.0462963	0.615741	0.223148	0.723148	0.152894
3.2	1.6	0.0371933	0.592983	0.210189	0.710189	0.151274
3.4	1.7	0.0303067	0.575767	0.199216	0.699216	0.149902
3.6	1.8	0.0250057	0.562514	0.189735	0.689735	0.148717
3.8	1.9	0.0208619	0.552155	0.181406	0.681406	0.147676
4.0	2	0.0175781	0.543945	0.173991	0.673991	0.146749
4.2	2.1	0.0149436	0.537359	0.167316	0.667316	0.145914
4.4	2.2	0.0128065	0.532016	0.161253	0.661253	0.145157
4.6	2.3	0.0110554	0.527638	0.155703	0.655703	0.144463
4.8	2.4	0.00960739	0.524018	0.150591	0.650591	0.143824
5.0	2.5	0.0084	0.521	0.145857	0.645857	0.143232
5.2	2.6	0.00738551	0.518464	0.141453	0.641453	0.142682
5.4	2.7	0.00652707	0.516318	0.13734	0.63734	0.142168
5.6	2.8	0.00579592	0.51449	0.133486	0.633486	0.141686

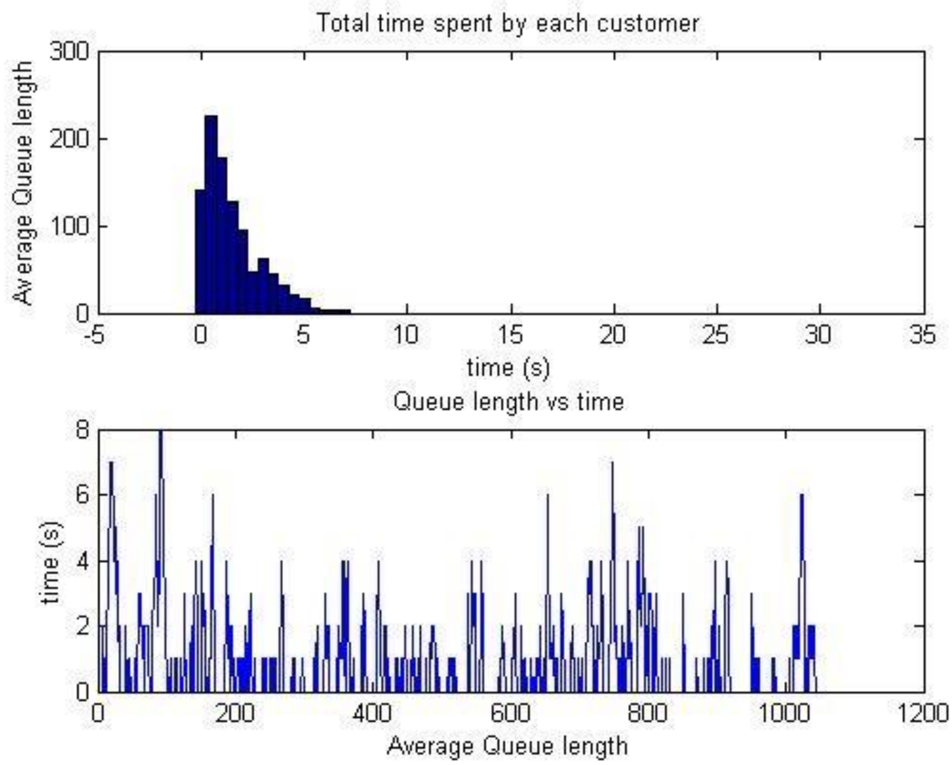

Figure 1: Average Queue length Vs Time (S)

Figure 1 shows that each transient queue-length probability converges to the stationary value. Depending on the server, we process repair, empty, busy, re-service, normal service, additional re-service or leaving the queue without getting service by establishing random values in the Markov process and supplementary variable techniques.

4. Conclusion

We addressed normal and optional re-service for server retrial queue's service when the server is busy, on vacation and under repair with the embedded arrival of customers by the method of supplementary variable techniques. Further we acknowledged that if there are no customs waiting in the orbit the server takes a change towards Bernoulli's vacation. The retrial time, duty time, repair time and vacation are subjective. Moreover, a variety of output metrics are obtained using the additional variables such as queue size, orbit size, server utilization and a chance for an orbit to be empty.

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