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Research Article

Analysis of the Boundary Layer Flow of Thermally Conducting Jeffrey Fluid over a Stratified Exponentially Stretching Sheet

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Abstract: In this analysis, the Jeffrey fluid flow over boundary layer magnetohydrodynamic exponentially enlarged sheet with thermally stratified medium in existence of suction is studied. The equations of governing flow together with boundary conditions are renewed into a non-similar arrangement with appropriate similarity transformations and are computed with the aid of MATLAB solver bvp4c. The things of several constraints on velocity and temperature are studied graphically. The computational outcomes of skin-friction and the heat transfer rate are studied pictorially. This article describes that the rate of heat transfer on the surface rises with effect of thermal stratification. The fluid velocity diminishes through growing in magnetic and Jeffrey parameters.

Keywords: Jeffrey fluid; MHD flow; exponentially stretching sheet; Prandtl number.
MSC 2020 No.: 76D05, 76M55, 76W05.

1. Introduction

Flow in the boundary layer with an enlarging surface is very significant owed to its realistic utilizations in engineering, polymer refining technology and electrochemistry. Productions of sheet materials are involved into many industrial mechanized processes and take account of together polymer and metal sheets. Viscous dissipation transforms the temperature profiles by singing a task resembling energy sources, which direct to moving the rate of heat transport. Flow in boundary-layer performance on incessant surfaces is investigated by Sakiadis (1961). Erickson (1966) deliberated this investigation to the case in which slanting velocity is transforming with non-zero surface of the special effects with heat and mass reassign is to be considered. Danberg and Fansler (1979), taking non-similar solution method, deliberate the stream within boundary layer past a wall that is prolonged with a velocity relative to the distance flanking the partition.

Sucking / injecting (blowing) fluid flow with a binding area can considerably alter the stream field. Evidently, suction leads to enlarge the friction factor coefficient, whereas injection performs in the conflicting way. Wang (1989) discussed free convective on an upright extending surface. Elbashbeshy (2001) studied the analysis of temperature transport ended an exponentially extending incessant surface through suction. He acquired resemblance results of laminar boundary stratum equations recitation heat and flow in an inactive liquid ambitious by exponentially extending surface through suction. The procedures of blowing /suction have also significance in numerous technical behaviors such as intend of force manner and thermal oil recovery and radial diffusers is investigated by Bhattacharyya (2011). Suction is functional to chemical procedures to eliminate reactants by Mukhopadhyay (2012).

The applications of MHD fluid flows on an enlarging sheet have accomplished lot significance in engineering and industries nowadays. Such applications include the boundary layer fluid flow during the liquid film in the deliberation procedure, the fluid covering on photogenic films and aerodynamic extrusion of synthetic sheets. In accumulation, a broad variety of appliances on MHD boundary layer stream can be originate in many fields such as electronic cooling, geothermal structures, heat insulation and metal extrusion, nuclear process, boilers, groundwater systems, micro-MHD pumps, energy storage units, high heat plasmas, thermal energy storage strategies and biological hauling. Nadeem et al. (2012) explored the boundary layer flow of a MHD Casson fluid ended an exponentially porous enlarged sheet.

Learning of non-Newtonian liquids is of significance due to its technical and industrial applications. However, the Navier Stokes equations are no longer valid to precisely depict the geographical assets of non-Newtonian liquids. Due to dissimilarities among Newtonian liquids, numerous replicas of non-Newtonian liquids to be suggested. The most familiar and easiest assortment of non-Newtonian liquid is the Jeffrey fluid, which has a time derivative as an alternative of a convicted derivative, which is used by most liquid replicas. Newly, this model of liquid has prompted active discussion. Maryam Aleem et al. (2020) have studied stream of Jeffrey liquid through porous medium in two hot similar plates, one of them is in motion with variable velocity and another one is rigid and both are embedded with effect of magnetic field. Babu et al. (2018) investigated magnitohydrodynamic Jeffrey fluid flow through a past vertical plate in presence of porous medium, heat transfer and rotation. Jeffrey fluid flow in rotating channel with the presence of Couette flow is to be examined by Sreenadh et al. (2016). Reddappa et al. (2015) concentrate on convective Couette flow of a Jeffrey liquid in an inclined channel while the walls are afford with porous coating. Some of the studies can be carried in Ahmad and Ishak (2015), Prasad et al. (2015), Nallapu & Radhakrishnamacharya (2014) and Shehzad et al. (2013).

Nadeem et al. (2011) explored the encrusted boundary layer flow of a Jeffrey fluid over an exponential enlarged surface with radiation effects.

In view of the above discussions, the intend of present manuscript is to examine boundary layer magnetohydrodynamic stream of Jeffrey fluid over an exponentially enlarging sheet entrenched in a thermally stratified medium focus to suction is studied. The leading equations and corresponding frontier circumstances are changed into a non-similar arrangement utilizing suitable similarity transformations and are resolved by using MATLAB solver bvp4c. The vital parameters on velocity as well as temperature have been shown pictorially.

2. Mathematical Formulation

A stable two-dimensional conductive, invisible viscous flow of Jeffrey fluid on a horizontal heated sheet is considered. The flow is restricted to positive side of y-axis. Two identical and conflicting forces are implemented along with x- axis, so that the wall is enlarged with origin rigid (Figure 1). A homogeneous magnetic force $B(\mathrm{x}) = B_0 e^{\frac{\mathrm{x}}{2\mathrm{L}}}$ is applied in the direction of x- axis which is normal to the sheet. Here $\tilde{T}_w(\mathrm{x})$ is the temperature of the sheet and is entrenched in a thermally stratified medium of changeable ambient temperature $\tilde{T}_\infty(\mathrm{x})$ where $\tilde{T}_w(\mathrm{x}) > \tilde{T}_\infty(\mathrm{x})$. It is assumed that $\tilde{T}_w(\mathrm{x}) = \tilde{T}_0 + be^{\frac{\mathrm{x}}{2\mathrm{L}}}$ and $\tilde{T}_\infty(\infty) = \tilde{T}_0 + ce^{\frac{\mathrm{x}}{2\mathrm{L}}}$ here $b > 0,\ c \geq 0$ are constants and $\tilde{T}_0$ is the reference temperature.

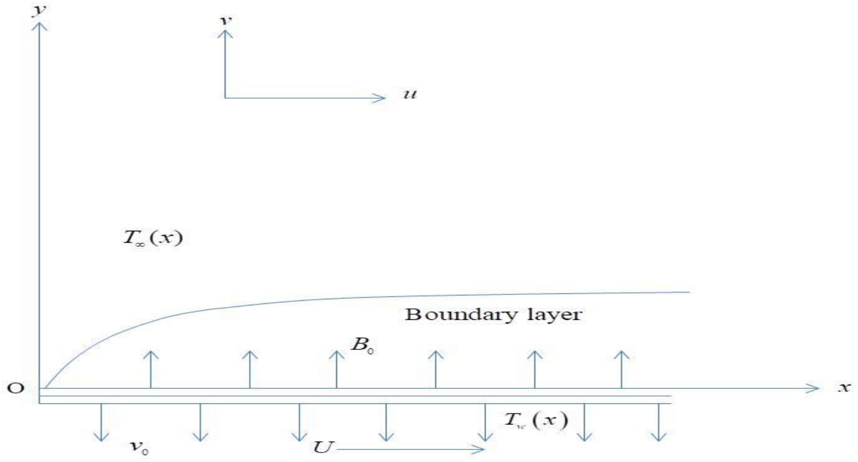

Fig1. Physical configuration.

The governing flow equations are

$$\frac{\partial \tilde{u}}{\partial \mathrm{x}} + \frac{\partial \tilde{v}}{\partial \mathrm{y}} = 0$$

(1)

$$\frac{\partial \tilde{u}}{\partial \mathrm{x}}\tilde{u} + \frac{\partial \tilde{v}}{\partial \mathrm{y}}\tilde{v} = \frac{\nu}{(1+\lambda_1)}\frac{\partial^2 \tilde{u}}{\partial \mathrm{y}^2} - \frac{\sigma B^2}{\rho}\tilde{u}$$

(2)

$$\frac{\partial \tilde{T}}{\partial \mathrm{x}}\tilde{u} + \frac{\partial \tilde{T}}{\partial \mathrm{y}}\tilde{v} = \frac{k}{\rho c_p}\frac{\partial^2 \tilde{T}}{\partial \mathrm{y}^2}$$

(3)

The subsequent boundary conditions

At $\mathrm{y} = 0$: $\tilde{u} = \tilde{U},\ \tilde{v} = -\tilde{V},\ \tilde{\mathrm{T}} = \tilde{\mathrm{T}}_w$

(4)

As $y \to \infty$: $\tilde{u} = 0,\ \tilde{\mathrm{T}} \to \tilde{\mathrm{T}}_\infty(x)$

(5)

where $\tilde{u}$ & $\tilde{v}$ - Velocity components in x & y directions, $\tilde{T}$ - Fluid temperature, k - Thermal conductivity of fluid, c_p - Specific heat at stable pressure, σ - Electrical conductivity of the fluid, $\nu = \frac{\mu}{\rho}$ - Kinematic viscosity, μ - Dynamic viscosity, ρ - Fluid density, B_0 - Strength of magnetic field functional in the y -direction, the persuaded magnetic field being neglected. $\tilde{U} = \tilde{U}_0 e^{\frac{x}{L}}$ - Enlarging velocity, $\tilde{U}_0$ - Reference velocity, $\tilde{V} < 0$ -

Velocity of blowing, $\tilde{V} > 0$ - Velocity of suction, $\tilde{V} = \tilde{V}_0 e^{\frac{x}{2L}}$ - Special kind of velocity at the wall is measured, $\tilde{V}_0$ - Primary potency of suction.

3. Method of solution

We introduce the similarity variables as

$$\eta = \sqrt{\frac{\tilde{U}_0}{2\nu L}} e^{\frac{x}{2L}} y, \;\; \tilde{u} = \tilde{U}_0 e^{\frac{x}{L}} f'(\eta), \;\; \tilde{v} = -\sqrt{\frac{\nu}{2L}} e^{\frac{x}{2L}} \left\{ f(\eta) + \eta f'(\eta) \right\}, \;\; \frac{\tilde{T} - \tilde{T}_\infty}{\tilde{T}_w - \tilde{T}_0} = \theta(\eta)$$

(6)

The accomplishment of above variables leads to the following expressions

$$f''' + (1+\lambda_1) ff'' - 2(1+\lambda_1) f'^2 - (1+\lambda_1) Mf' = 0$$

(7)

$$\theta'' + (f\theta' - f'\theta)\mathrm{Pr} - St\,\mathrm{Pr}\, f' = 0$$

(8)

with the boundary conditions

$$f(0) = S, \; f'(0) = 1, \; \theta(0) = -St + 1$$

(9)

$$f'(\infty) = 0, \;\; \theta(\infty) = 0$$

(10)

Where λ_1 - Jeffrey factor, $M = \frac{2\sigma B_0^2 L}{\rho \tilde{U}_0}$ - Magnetic factor, σ - Electrical conductivity of the fluid, $St = c/b$ - stratification factor, θ - Non-dimensional temperature, $\mathrm{Pr} = \frac{\mu c_p}{k}$ - Prandtl number and $S = \tilde{V}_0 \times \sqrt{\frac{2L}{\nu \tilde{U}_0}} > 0\,(\text{or} < 0)$ - suction (or blowing) factor. Here St is greater than zero states that a steadily stratified situation and $St = 0$ states that an unstratified situation.

Physical quantity like friction factor coefficient C_f and Nusselt number Nu represented as follows

$$C_f = -f''(0)$$

(11)

$$Nu = -\theta'(0)$$

(12)

4. Results and discussion

The resulting nonlinear ordinary differential expressions (7) and (8) are solved numerically with corresponding boundary conditions through the help of bvp4c with MATLAB package. This section provides the effect of various physiological parameters on velocity, temperature, friction factor coefficient and heat transfer rate, for fixed values of $M = 1$, $\lambda_1 = 0.3$, $S = 0.5$, $St = 0.5$, $Pr = 0.7$ through Figs. 2 -19.

We observe from Figs. 2, 3 and 4 that the velocity diminishes with the enlarge in λ_1, M and S. From Figs. 5 and 6 we observe that the temperature enhances with the raise in the M and λ_1. From Figs. 7, 8 and 9 we examine that the temperature diminishes with the raise in S, St and Pr. From Figs. 10, 11 and 12, we notice that the shear stress increases with enhance in λ_1, M and S. Evidently, when considering wall suction ($S > 0$), this cause a shrink in thickness of the boundary layer. $S = 0$ represents the case of non-porous extended sheet. Figs. 13, 14, 15 and 16 are correspondingly the pictorial illustrations of temperature gradient silhouettes $\theta'(\eta)$ for diverse values of λ_1 and M for the porous ($S > 0$) and nonporous ($S = 0$) sheet. This is originate that the temperature gradient diminishes with an enhance in the parameters λ_1 and M. Figs. 17 and 18 are plotted to learn the outcome of the Stratification parameter St we see from this figures the temperature gradient enhances with rising St for both porous ($S > 0$) and nonporous ($S = 0$) sheet. The temperature gradient enhances with rise in the suction parameter S, such characteristic is shown in Fig. 19.

Tables 1 and 2 presented the outcome of various emerging thermo physical factors for friction factor coefficient and Nusselt number. In Table 1, it can be renowned that the coefficient of skin friction enhances on augmenting M, λ_1 and S. Table 2 presents the coefficient of Nusselt number for different values of M, St, λ_1, S and Pr. It is observed that the Nusselt number reduces on augmenting M and λ_1 whereas the conflicting behavior is noticed for growing values of St, S and $\Pr$. Table 3 depicts the comparison of $-\theta'(0)$ for $K = E = S = St = M = \lambda_1 = 0$ with Bidin & Nazar (2009) and Swati Mukhopadhyay (2013) for some values of $\Pr$. It is noticed that Nusselt number rises on augmenting $\Pr$. An enhancing in $\Pr$, diminishes the thermal boundary layer thickness. Fluids with inferior $\Pr$ will acquire superior thermal conductivities, in order that heat can disseminate from the sheet quicker than for superior $\Pr$.

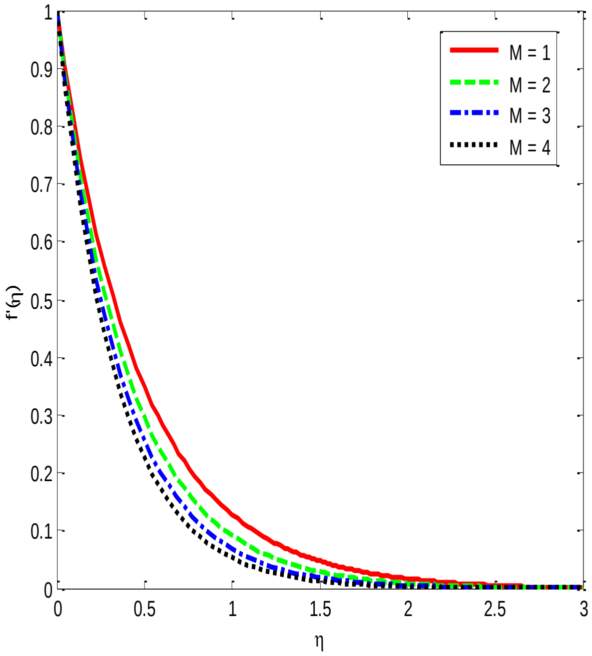

Fig. 2. Difference of magnetic field M on $f'(\eta)$.

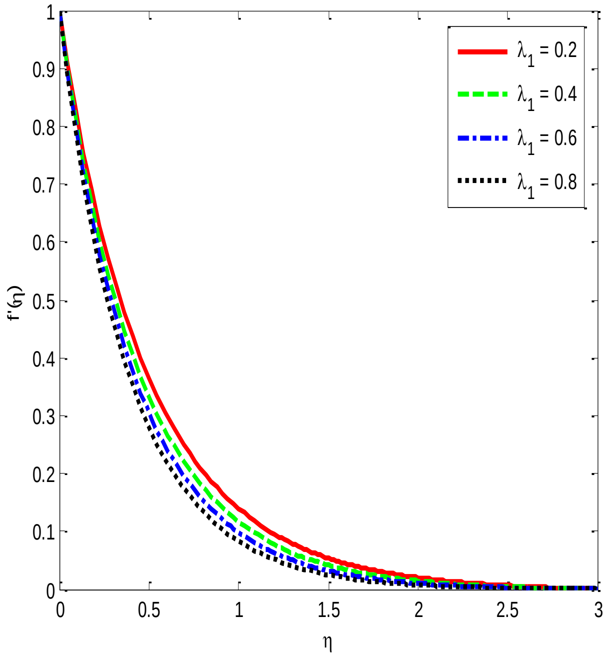

Fig. 3. Difference of Jeffrey parameter λ_1 on $f'(\eta)$.

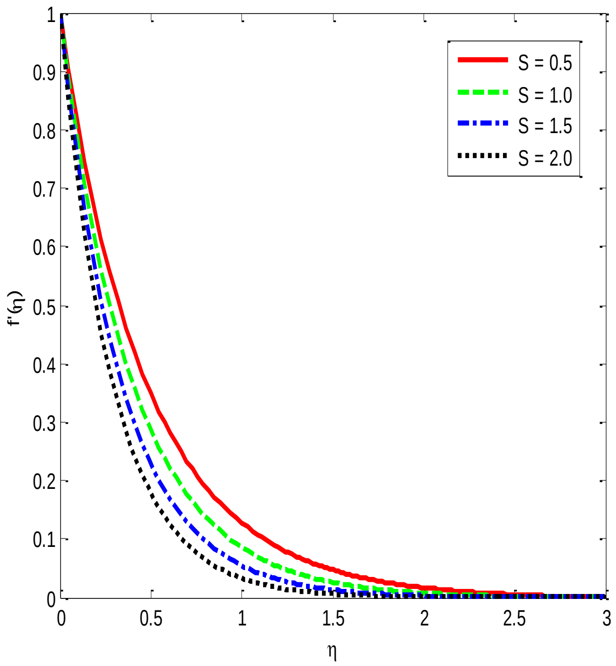

Fig. 4. Difference of Suction parameter S on $f'(\eta)$.

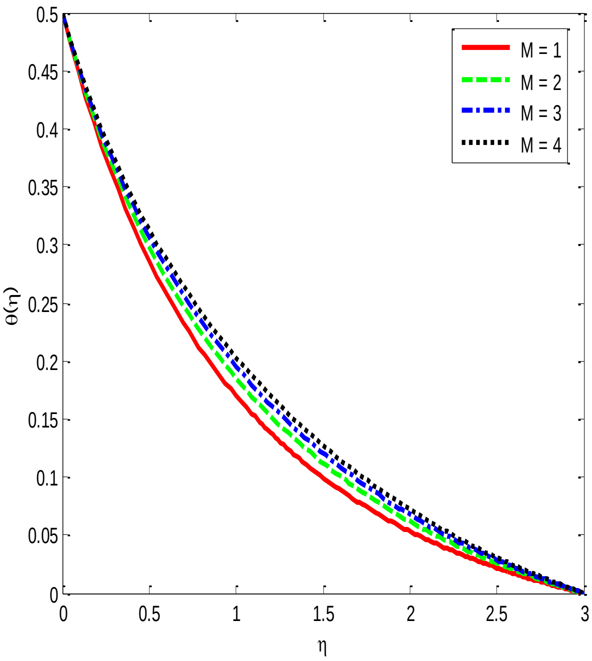

Fig. 5. Difference of magnetic field M on $\theta(\eta)$.

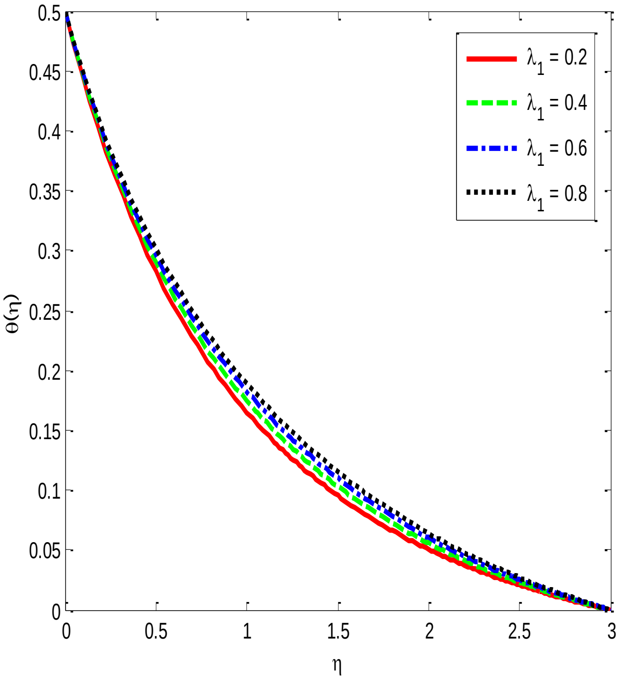

Fig. 6. Difference of Jeffrey parameter λ_1 on $\theta(\eta)$.

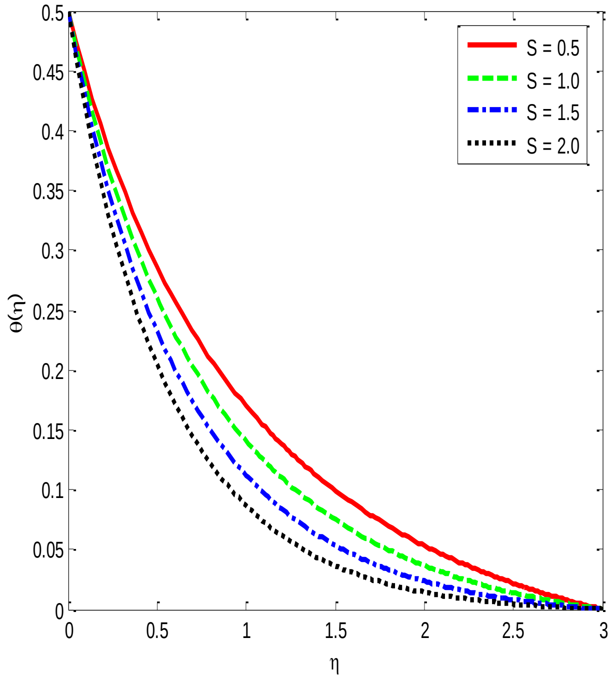

Fig. 7. Difference of Suction parameter S on $\theta(\eta)$.

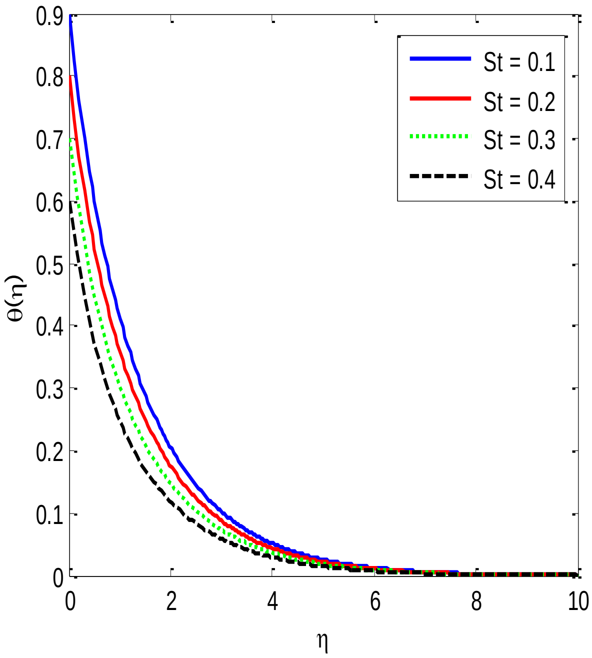

Fig. 8. Difference of Stratification parameter St on $\theta(\eta)$.

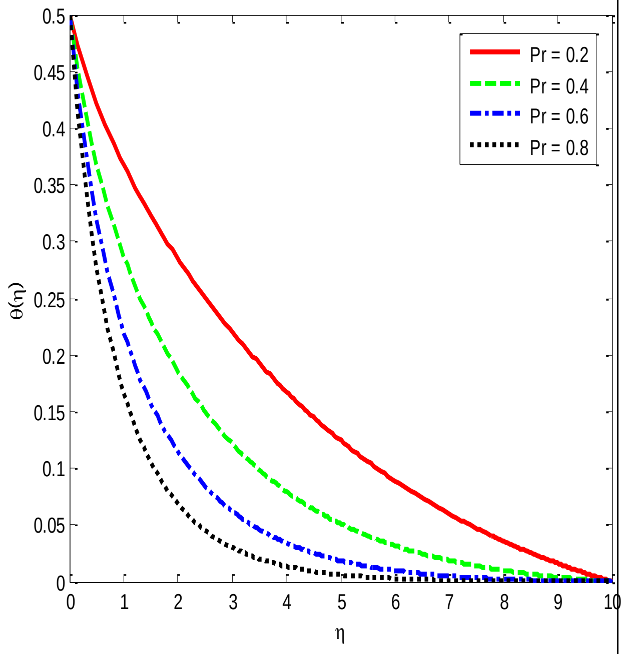

Fig. 9. Difference of Prandtl number $\Pr$ on $\theta(\eta)$.

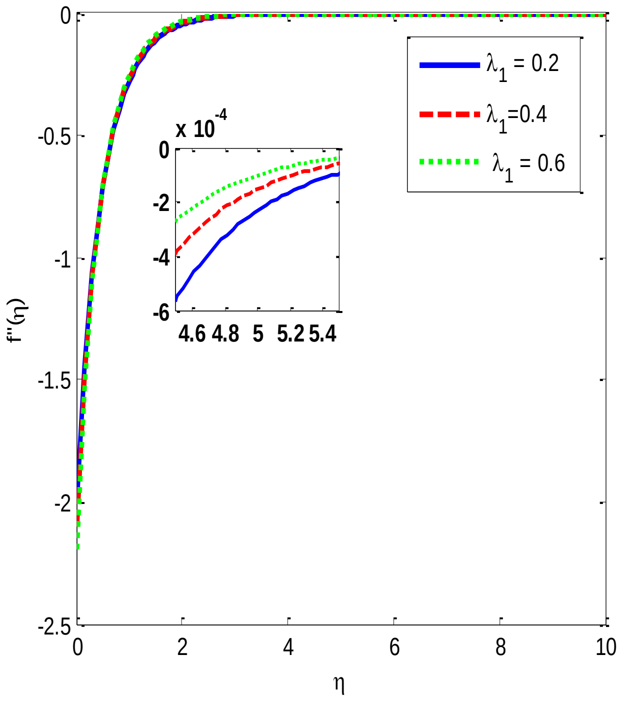

Fig. 10. $f''(\eta)$ **with** η **for distinct values of** λ_1 **.**

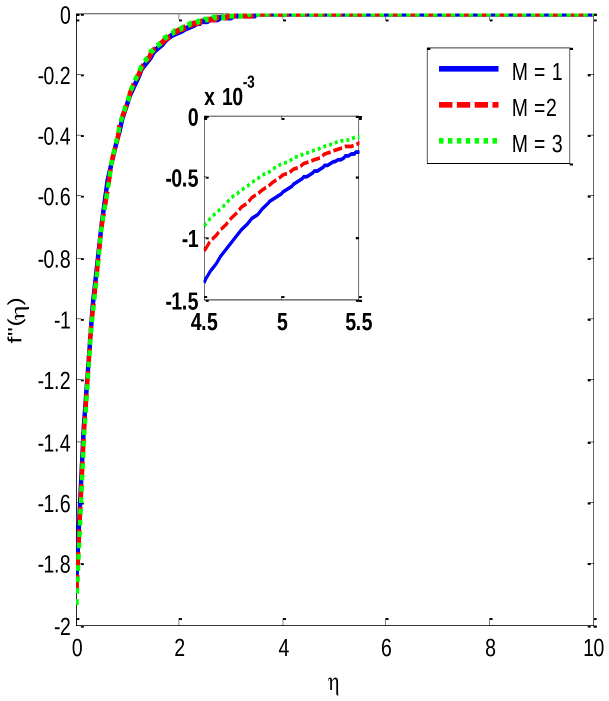

Fig. 11. $f''(\eta)$ **with** η **for distinct values of** M **.**

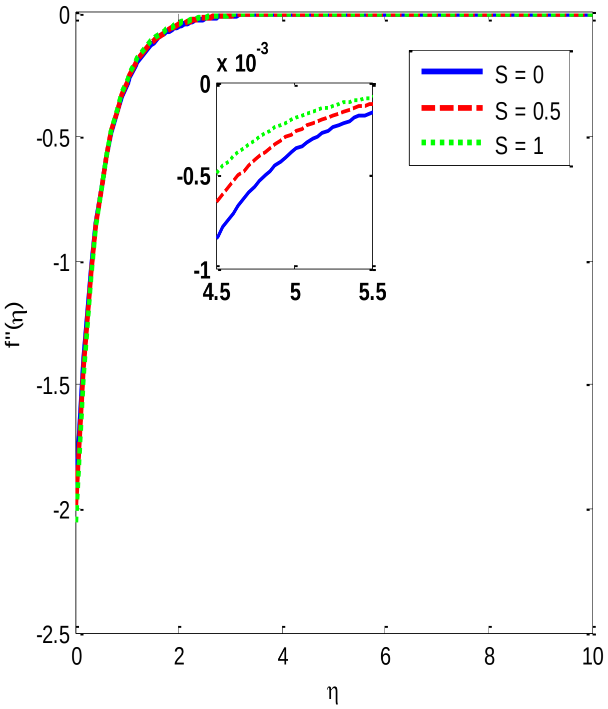

Fig. 12. $f''(\eta)$ **with** η **for distinct values of** S **.**

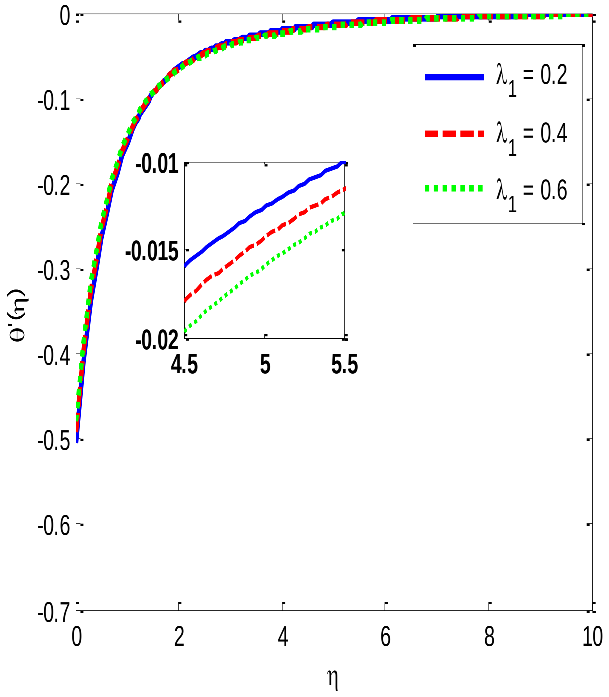

Fig. 13. $\theta'(\eta)$ **with** η **for distinct values of** λ_1 **in the absence of** S **.**

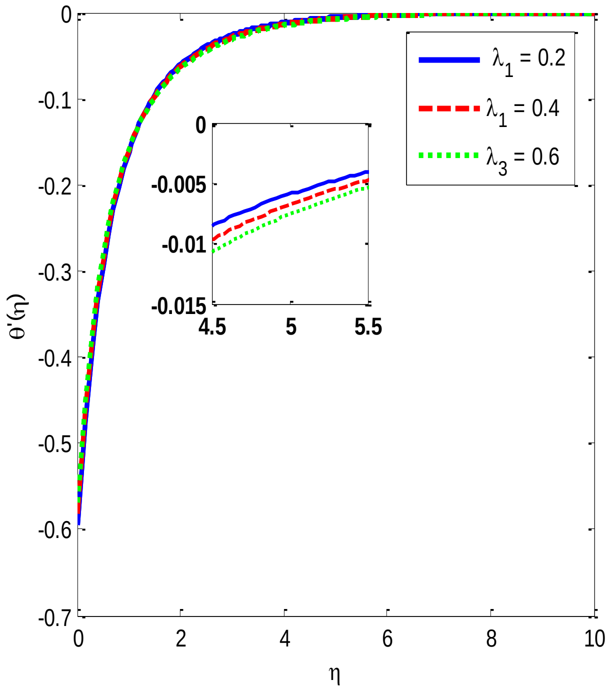

Fig. 14. $\theta'(\eta)$ **with** η **for distinct values of** λ_1 **in the presence of** S **.**

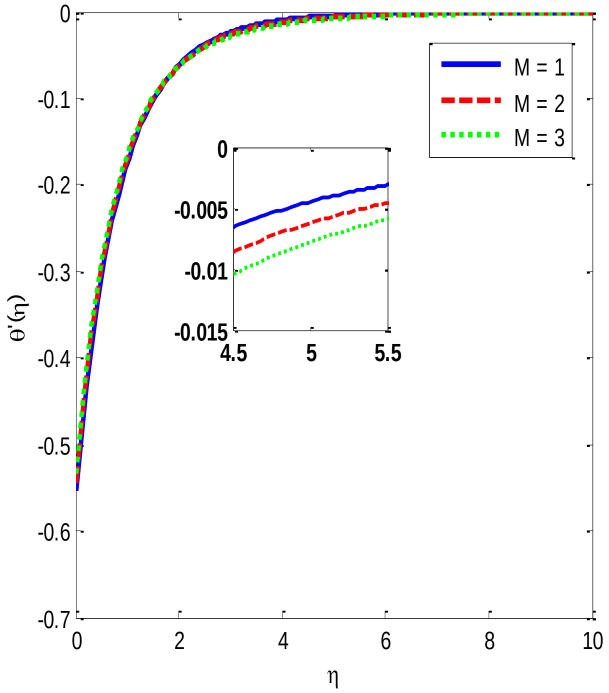

Fig. 15. $\theta'(\eta)$ **with** η **for distinct values of** M **in the absence of** S **.**

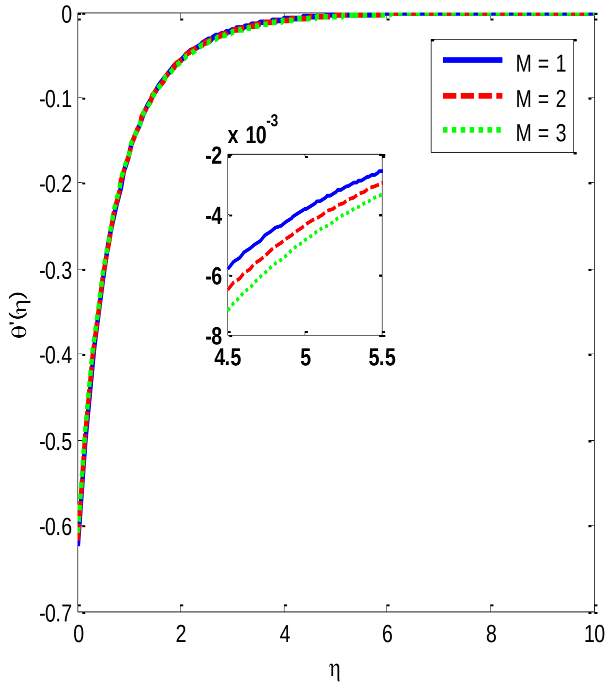

Fig. 16. $\theta'(\eta)$ **with** η **for distinct values of** M **in the presence of** S **.**

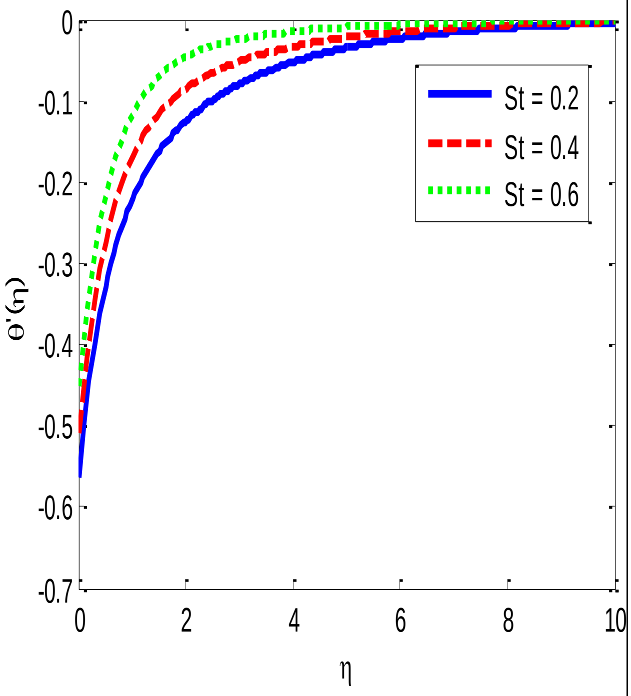

Fig. 17. $\theta'(\eta)$ **with** η **for distinct values of** St **in the absence of** S **.**

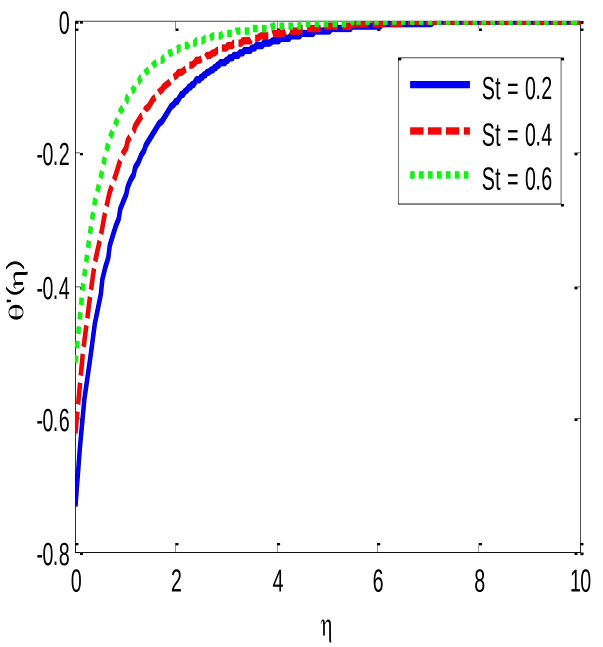

Fig. 18. $\theta'(\eta)$ **with** η **for distinct values of** St **in the presence of** S **.**

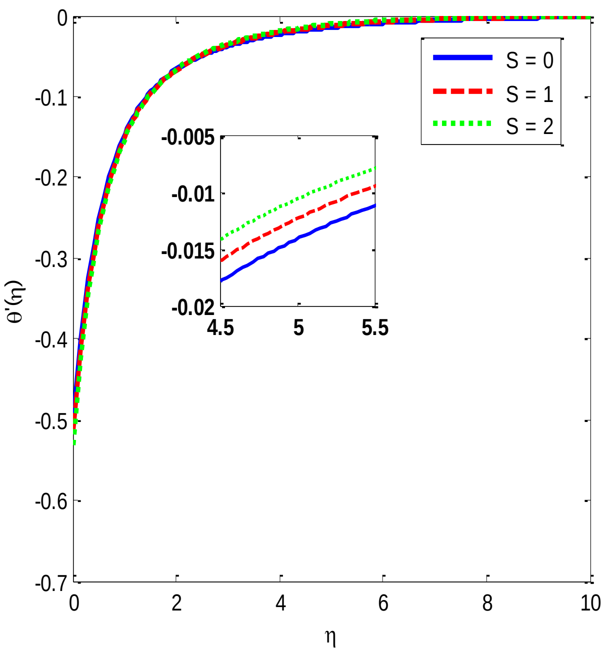

Fig. 19. $\theta'(\eta)$ **with** η **for distinct values of** S **.**

Table 1: A numerical analogy for friction factor coefficient $-f''(0)$ at the sheet $\eta = 0$ for diverse values of λ_1, S & M .

M	λ_1	S	$-f''(0)$
1.0	0.3	0.5	2.198442
2.0	0.3	0.5	2.520513
3.0	0.3	0.5	2.800320
1.0	**0.2**	0.5	2.098458
1.0	**0.4**	0.5	2.295839
1.0	**0.6**	0.5	2.483935
1.0	0.3	**0.5**	2.198442
1.0	0.3	**1.0**	2.596169
1.0	0.3	**1.5**	3.045347

Table 2: A numerical analogy for Nusselt number $-\theta'(0)$ at the sheet $\eta = 0$ for various values of $\Pr$, λ_1, S, St & M .

M	λ_1	S	St	$\Pr$	$-\theta'(0)$
1.0	0.3	0.5	0.5	0.7	0.596085
2.0	0.3	0.5	0.5	0.7	0.564501
3.0	0.3	0.5	0.5	0.7	0.541406
1.0	**0.2**	0.5	0.5	0.7	0.606136
1.0	**0.4**	0.5	0.5	0.7	0.586773
1.0	**0.6**	0.5	0.5	0.7	0.570026
1.0	0.3	**0.5**	0.5	0.7	0.596085
1.0	0.3	**1.0**	0.5	0.7	0.694789
1.0	0.3	**1.5**	0.5	0.7	0.810096
1.0	0.3	0.5	**0.1**	0.7	0.119217

1.0	0.3	0.5	**0.2**	0.7	0.238433
1.0	0.3	0.5	**0.3**	0.7	0.357650
1.0	0.3	0.5	0.5	**0.2**	0.195929
1.0	0.3	0.5	0.5	**0.4**	0.349138
1.0	0.3	0.5	0.5	**0.6**	0.497642

Table 3: A numerical analogy for $-\theta'(0)$ contrary those of Bidin & Nazar (2009) and Swati Mukhopadhyay (2013)
for diverse values of $\Pr$.

$\Pr$	Bidin & Nazar (2009) with $K = E = 0$	Swati Mukhopadhyay (2013) with $S = St = M = 0$	Present results with $S = St = M = \lambda_1 = 0$
1.0	0.9547	0.9547	0.9547
2.0	1.4714	1.4714	1.4714
3.0	1.8961	1.8961	1.8961

5. Conclusion

In current investigation to furnish the numerical results of the boundary layer flow of conducting Jeffrey liquid through an exponentially enlarge sheet entrenched in a stratified thermal medium in occurrence of suction. The outcomes of parameters S, M and λ_1 on invisible liquid repress the momentum of the fluid which in turn causes the enrichment of the coefficient of skin-friction. The rate of heat transport is sinking with rising Jeffrey parameter and magnetic parameter. The temperature diminishes with growing stratification parameter and Prandtl number.

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