Research Article

Quadrature squeezing in six wave mixing process

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Abstract: Squeezing in multi mixing process was studied by number of authors using perturbation method. In this paper we obtained squeezing in six wave mixing process with a high coherent pump beam. We used a very good approximation and found larger squeezing at large interaction times

Keywords: squeezing, mixing of waves, coherent wave

1. Introduction

Squeezing of electromagnetic wave reflects nonclassical nature and cannot be explained by classical optics [1-2]. Squeezing has very good applications in field of detection of gravitational wave [3], optical communication and quantum information theory [4-10], in resonance fluorescence [11], quantum teleportation [12-15], in quantum cryptography [16] and study of dense coding [17].

Process of multi wave mixing has been used for theoretical and experimental study of squeezed state generation [18-20]. Quantum mechanical treatment of mixing of waves have also been studied [21-22]. Mixing of wave have been used in different form to study squeezing such as difference squeezing, sum squeezing, squeezing in four wave mixing process.

Some authors studied amplitude squeezing in six wave mixing using perturbation theory [26]. We reexamine squeezing in six wave mixing in the present paper by an intense coherent pump mode and under a much better approximation, which provides the validity of results for larger interaction times. We found larger squeezing at large times.

2. Definition of Ordinary and Amplitude Squared Squeezing

Generally, quantum fluctuations in both the quadratures are not always equal. For anyone quadrature phase may have reduced quantum fluctuations at the rate of increased quantum fluctuations in another quadrature phase so that the product of both the fluctuations still follows Heisenberg's uncertainty principal relation. This phenomenon is called squeezing of the electromagnetic field.

We define operators X_θ and Y_θ by

$$X_\theta \equiv \frac{1}{2}\left[a e^{-i\theta} + a^\dagger e^{i\theta}\right],\; Y_\theta \equiv \frac{1}{2}\left[a^2 e^{-i\theta} + a^{\dagger 2} e^{i\theta}\right] \tag{1}$$

For these operators, if $\Delta X_\theta \equiv X_\theta - \langle X_\theta \rangle, \Delta Y_\theta \equiv Y_\theta - \langle Y_\theta \rangle$, the minimum variances (minimum against variation of θ) are seen to be

$$\left\langle (\Delta X_\theta)^2 \right\rangle_{\min} = \frac{1}{4} + \frac{1}{4}\left(\langle a^\dagger a\rangle - |\langle a\rangle|^2\right) - \frac{1}{4}\left|\langle a^2\rangle - \langle a\rangle^2\right|. \tag{2}$$

where a and $a^\dagger$ are annihilation and creation operators, and N(= $a^\dagger a$) is number operator.

If $\left\langle (\Delta X_\theta)^2 \right\rangle_{\min} < 1/4$, X_θ is said to be ordinary squeezed.

Conditions for this to occur is

$$\langle a^\dagger a\rangle - |\langle a\rangle|^2 < \left|\langle a\rangle^2 - \langle a\rangle^2\right| \tag{3}$$

respectively.

3. Interaction Hamiltonian for six wave mixing process and the Time -evolution operator

Consider the six wave mixing process which involves the absorption of two pump photons of frequency ω_1 and the emission of 3 probe photons of frequency ω_2 and one single photon of frequency ω_3 with $2\omega_1 = 3\omega_2 + \omega_3$. The interaction Hamiltonian for this process is

$$H_I = \omega_1 a^\dagger a + \omega_2 b^\dagger b + \omega_3 c^\dagger c + g(aab^\dagger b^\dagger b^\dagger c^\dagger + a^\dagger a^\dagger bbbc) \quad (4)$$

where $(a, a^\dagger)$ are operators for pump mode, $(b, b^\dagger)$ and $(c, c^\dagger)$ are operators for the other modes in interaction picture and g is coupling constant. For an intense pump mode initially in the coherent state $\left| x e^{i\theta_\alpha} \right\rangle$ with $x >> 1$, we may write

$$a = Ae^{i\theta_\alpha}, A = \left(x + \overline{A}\right), b = \overline{B}e^{i\theta_\alpha}, c = \overline{C}e^{i\theta_\alpha} \quad (5)$$

and therefore the interaction Hamiltonian will be in the form,

$$H_I = [H_I^{(0)} + H_I^{(1)} + H_I^{(2)}], \quad (6)$$

$$H_I^{(0)} = G(\bar{B}^{\dagger 3}\bar{C} + \bar{B}^3\bar{C}), \quad (7)$$

$$H_I^{(1)} = \frac{2G}{x}(\bar{A}\bar{B}^{\dagger 3}\bar{C}^\dagger + \bar{A}^\dagger\bar{B}^3\bar{C}), \quad (8)$$

$$H_I^{(2)} = \frac{G}{x^2}(\bar{A}^2\bar{B}^{\dagger 3}\bar{C}^\dagger + \bar{A}^{\dagger 2}\bar{B}^3\bar{C}), G = g\,x^2, \quad (9)$$

Equation of motion for the time-evolution operator in interaction picture U_I is $i\dot{U}_I = H_I U_I$. This can be written as $U_I = U_{I0} V$, where

$$U_{I0} = e^{-iH_I^{(0)}t} = e^{-iGt(\bar{B}^3\bar{C} + \overline{B}^{\dagger 3}\overline{C}^\dagger)},$$

and V is solution of

$$i\dot{V} = U_{I0}^\dagger(H_I^{(1)} + H_I^{(2)})\,U_{I0}V = (\tilde{H}_I^{(1)} + \tilde{H}_I^{(2)})V. \quad (10)$$

using Equation (10), which gives

$$U_{I0}^\dagger \begin{pmatrix} \overline{A} \\ \overline{B} \\ \overline{C} \end{pmatrix} U_{I0} = \begin{pmatrix} \overline{A} \\ \overline{B}^3 \cosh Gt - i\overline{C}^\dagger \sinh Gt \\ \overline{C}^\dagger \cosh Gt - i\overline{B}^{\dagger 3} \sinh Gt \end{pmatrix}, \quad (11)$$

expressions for $\tilde{H}_I^{(1)}$ and $\tilde{H}_I^{(2)}$ are obtained as given below-

$$\tilde{H}_I^{(1)} = \frac{2G}{x}[J_1 \cosh^2 Gt - J_2 \sinh^2 Gt - J_3 \sinh Gt \cosh Gt]$$

$$\tilde{H}_I^{(1)} = \frac{2G}{x}[\; J_1 cosh^2 Gt - J_2 sinh^2 Gt - J_3 sinhGt\, coshGt \;], \quad (12)$$

$$\tilde{H}_I^{(2)} = \frac{G}{x^2}[\; K_1 cosh^2 Gt - K_2 sinh^2 Gt - K_3 sinhGt\, coshGt \;] \quad (13)$$

Where $J_1 = (\overline{A}\,\overline{B}^{\dagger 3}\overline{C}^\dagger + \overline{A}^\dagger\overline{B}^3\overline{C})$, $J_2 = \left(\overline{A}\,\overline{B}^3\overline{C} + \overline{A}^\dagger\overline{B}^{\dagger 3}\overline{C}^\dagger\right)$, $J_3 = i(\overline{A}^\dagger - \overline{A})(\overline{B}^{\dagger 3}\overline{B}^3 + \overline{C}^\dagger\overline{C} + 1)$,

$K_1 = (\overline{A}^{\dagger 2}\overline{B}^3\overline{C} + \overline{A}^2\overline{B}^{\dagger 3}\overline{C}^\dagger)$, $K_2 = (\overline{A}^{\dagger 2}\overline{B}^{\dagger 3}\overline{C}^\dagger + \overline{A}^2\overline{B}^3\overline{C})$, and $K_3 = i(\overline{A}^{\dagger 2} - \overline{A}^2)(\overline{B}^{\dagger 3}\overline{B}^3 + \overline{C}^\dagger\overline{C} + 1)$.

4. Ordinary squeezing in four wave mixing process

Using solution of V, for correction up to second order in $1/x$, we get,

$$< A> = x - \frac{2\sinh^2 Gt}{x} - \frac{1}{x} \tag{14}$$

$$< A^2 > = x^2 - 2\sinh^2 Gt - 2 - \sinh^2 2Gt + \frac{1}{x^2} + \frac{\sinh^2 2Gt}{x^2} \tag{15}$$

$$< A^{\dagger}A> = x^2 - 4\sinh^2 Gt - 2 + \frac{1}{x^2} + \frac{\sinh^2 2Gt}{x^2} + \frac{1}{4x^4} + \frac{\sinh^2 2Gt}{4x^4} \tag{16}$$

These lead to

$$\langle(\Delta X_\theta)^2\rangle - \frac{1}{4} = \frac{1}{4}\left(-4 + \frac{sinh^2\, 2Gt}{x^2} - \frac{4\, sinh^4\, Gt}{x^2} + \frac{4\, sinh^2\, Gt}{x^2}\right) - \frac{1}{2}\left(2\, sinh^2\, G\, t - 4 - 2\, sinh^2\, 2\, Gt + \frac{sinh^2\, 2Gt}{x^2} - \frac{4\, sinh^4\, Gt}{x^2} + \frac{4\, sinh^2\, Gt}{x^2}\right) cos\, 2\, (\theta_\alpha - \theta) \tag{17}$$

In above expression the coefficient of $cos\, 2\, (\theta_\alpha - \theta)$ is negative and all other term is positive and so squeezing can be obtained for positive value of $cos\, 2\, (\theta_\alpha - \theta)$ i.e. if $|\theta_\alpha - \theta|$ exist in between 0 to $\frac{\pi}{4}$ or in between $\frac{3\pi}{4}$ to π.

5. Conclusion and Discussion

In this paper we consider $sinh 2Gt << \frac{n_a}{\sqrt{n_b n_c}}$, where n_a, n_b, n_c are the number of photons in given modes and number of photons in pump mode is much greater than one. Our results are valid for much larger times of interaction.

In present work radiation squeezing by mixing of six waves has been examined and result showed that squeezing is dependent on value of "gt". Here we get large radiation squeezing in fundamental mode for small interaction time. In equation (17), coefficient of cos2 ($\theta_\alpha - \theta$) is positive and other terms are small negative values and hence squeezing can be obtained only when cos2($\theta_\alpha - \theta$) is negative, i.e. if $\theta_\alpha - \theta$ lies between $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$.... .We investigated for the case cos2 ($\theta_\alpha - \theta$) = -1 and results are plotted in Figures 1 and 2. We show our results for x^2=100, in figure 1, Gt lies between 0 to 2 and we get larger squeezing which is increasing with time. We show our result for gt=10^{-2} , x^2 lies between 0 to 100 in figure 2 and we get larger squeezing for small interaction time. One may obtain desired degree of squeezing for larger interaction time by using different kind of higher order nonlinear process

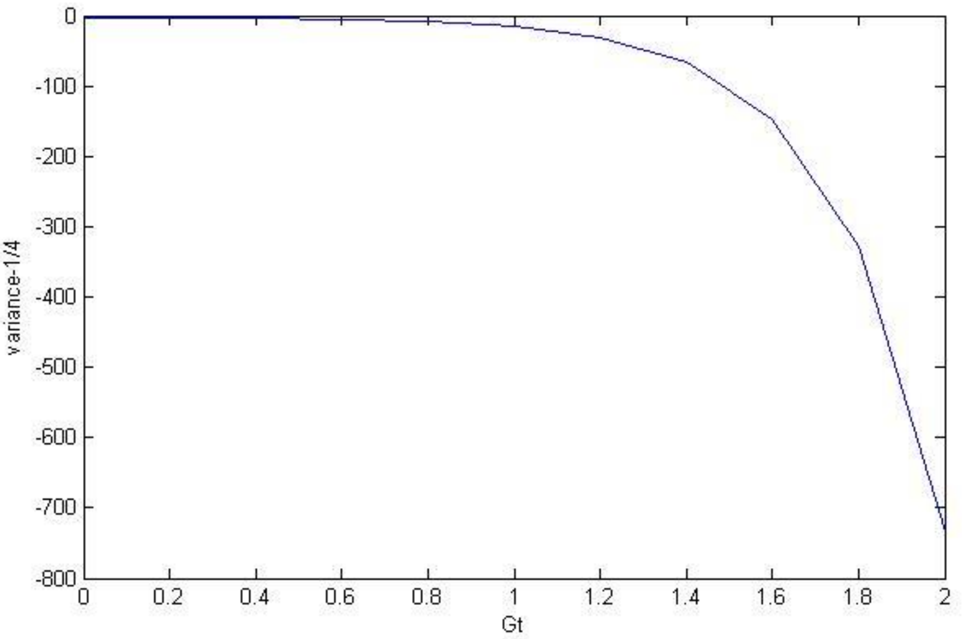

Figure 1: Graph showing variation of $\Delta X_\theta^2(t) - \frac{1}{4}$ Gt for x^2=100 and $\cos 2(\theta_\alpha - \theta)$ = -1

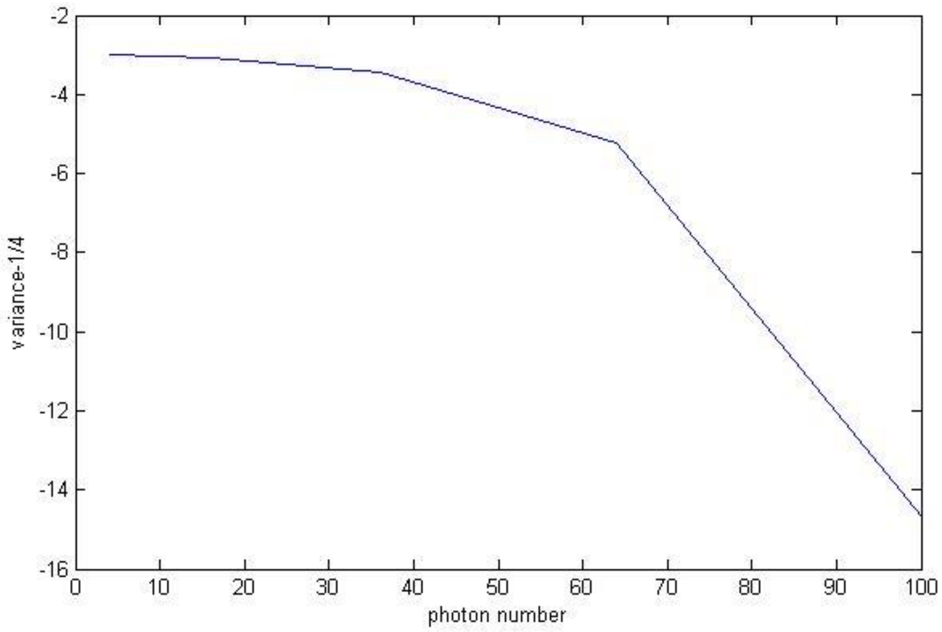

Figure 2: Graph showing variation of $\Delta X_\theta^2(t) - \frac{1}{4}$ with x^2=100 and gt =10^{-2}

6. Acknowledgement

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