FuzzyInventory Model Under Two Storage Systemfor Deteriorating Items with Selling Price Dependent Demand and Shortages

Paramjit Kaur[a], Vinod Kumar[b]

[a] Guru Kashi University, Talwandi Sabo, Bathinda, Punjab, India.
[b]Department of Mathematics, NIET, NIMS University, Jaipur, Rajasthan, India

Article History: Received: 10 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 4 June 2021

Abstract: A fuzzy inventory model for a single deteriorating item, selling price dependent demand rate,constant deterioration rate and fully backlogged shortages has been developed. In the present market scenario,the management of inventory is of most important to reduce the total inventory cost.Uncertainty in the different prices and factors affecting inventory cost can't dealt with crisp nature of components affecting inventory system. Therefore, to deal with uncertain situation of the market, a fuzzy based two warehouse system of inventory has been discussed in the present paper considering vague nature of holding cost, selling price and other components as well, in both ware-houses. The objective of this paper is to derive the optimal replenishment policy with defuzzification of fuzzy numbers using signed distance and centroid method to minimize the present worth of total relevant inventory cost per unit of time. First a crisp inventory model is developed and corresponding fuzzy inventory model has been derived considering triangular fuzzy numbers for components affecting inventory cost. With the help of numerical example effects of parameters are studied for both crisp a fuzzy models and sensitivity analysis is performed for selected parameters in both the cases.

Keywords: Fuzzy numbers,Centroid and Signed distance methods,selling price dependent demand,Shortages.

1. Introduction

Most of research papers in the field of inventory model considered constant rate of demand, various form of time dependent demands such as linear, non-linear, stock dependent or exponential and this still continues but it is not always fruitful in optimizing inventory cost. IN last decade many researchers considers two ware house inventory system over a single storage management due to space constrains but are maximum of crisp nature. Previously researchers such as K.V.S. Sarma,[1] has developed, "A deterministic inventory model for deteriorating items with two level of storage and an optimum release rule". T.A.Murdeshwar, Y, S. Sathe,[2] developed, "Some aspects of lot size model with two level of storage", U.Dave, [3] has contributed in paper, "On the EOQ models with two level of storage", This papers are of crisp nature developing inventory model for optimizing total relevant inventory cost. Donaldson [5]was the first to consider inventory model with time dependent demand and thereafter many researchers such as Goswami and Chaudhuri [6], Bhunia and Maiti[7]Banerjee and Agrawal [8] etc. were considered the time dependent demands for two-warehouse inventory systems.

Deteriorationis the key factor affecting the inventory costduring the storage period and has drawn much attention of various researchers since past many decades. The problem of deteriorating inventory involved by researchers in developing model to deal real situation of inventories while stored. In the business scenario there are many products which gets deteriorated during their life period and are not in good condition for a long time if not stored proper storage facilities and even the become unusable before its life span. There are some products which falls under the category of deterioration if not stored properly namely these aremedicine, blood, fish, alcohol, gasoline,vegetables and radioactive chemicals. Many researchers, have taken care of deteriorating items in their models and developed the models accordingly.

Storage is a big issue in the business entity particularly in metro cities where limited storage space are available in the heart of markets causing practical problem of real situation arising during business planning and thus need more attention. Various inventory models were developed with single storage space considering unlimited storage capacity which is not reality and problem of storage exists on excess purchasing. A space available will always turned off a limited storage capacity in real life situation until inventory are not supplied continuously during the business. The storage constraints in the market brings the concept of two warehouse storage system one storage having known capacity and other considering of unlimited capacity due to continuous supply from the other storage space and most of the vendor prefers to hire storage space on rent for the period they require.Since specially equipped storage facility is required to reduce the amount of deterioration therefore during rent holding cost of deteriorating items are assumed to be more than the cost incurred in normal storing due to better preservation facilities provided in the rented house which are specially built so for.

In the present market scenario, many products such as fashionable items, garments, electronic items mobile are being produced rapidly and companies are launching new products regular causing uncertainty in the market and increase competition level. The exact cost of components or items available in the market cannot be pre-determined exactly in advance until we arrived the exact situation. Forexample,hike in the prices, affecting the total inventory cost or hike in demand, shortages etc. In the present situation, it is not easy to assess that how much? and /or when an increase/decrease in the components affecting inventory system willoccur inthe future? One of the most concerns of the management is to decide when and how much to be ordered so that the costs associated with the inventory system should minimized. This is more important when some or more products in inventory are deteriorating. This type of uncertainty which need to be forecast for future trend be handled initially by the L.A. Zadeh [9] in seventy centuries considering interval-based membership function describing a graded situation.Thereafter many papers have been developed using fuzzy set theory in fuzzy environment.L.A.Zadeh and R.E.Bellman [10] considered an inventory model on decision making in fuzzy environment.R.Jain[11] developed a fuzzy inventory model on decision making in the presence of fuzzy variables. D.Dubois and H.Prade[12] defined some operations on fuzzy numbers. In general, the demand is to be considered either constant or increasing with time.Sujit D.Kumar,P.K.Kund and A. Goswami[13] developed an economic production quantity modelwith fuzzy demand and deterioration rate.J.K.Syed and L.A.Aziz[14]consider a signed distance method for a fuzzy inventory model without shortages.P.K.De and A.Rawat[15]developed a fuzzy inventory model without shortages using triangular fuzzy number.C.K.Jaggi,S.Pareek,A. Sharma and Nidhi[16] developed a fuzzy inventory model for deteriorating items with time varying demand and shortages.D.Datta and Pawan Kumar[17] considered an optimal replenishment policy for an inventory model without shortages assuming fuzziness in demand.Halim et al.[18] developed a fuzzy inventory model for perishable items with stochastic demand,partial backlogging and fuzzy deterioration rate.Goni and Maheshwari [19] discussed the retailer's ordered policy under the two level of delay in payments considering the demand and selling price as triangular fuzzy numbers. They used graded mean integration representation method for defuzzification.Halim et al. [20] addressed a lot sizing problem in an unreliable production system with stochastic machine breakdown and fuzzy repair time using the signed distance method. Singh,S.R. and Singh, C. [21] developed a fuzzy inventory model for finite rate of replenishment using signed distance method. In recent Palani, R. and Maragatham, M. [22]developed "Fuzzy inventory model for time dependent deteriorating items with lead time stock dependent demand rate and shortages" in which components affecting inventory cost are considered of fuzzy nature and shortages are completely backlogged. Shabnam Fathalizadeh et al.[23] has developed, "Fuzzy inventory models with partial backordering for deteriorating items under stochastic inflationary conditions: Comparative comparison of the modelling methods" in which demand rate is considered to be constant per unit time and purchasing cost is of fuzzy nature and shortages are partially backlogged. Sujit Kumar De and Gour Chandra Mahata[24] developed, "A cloudy fuzzy economic order quantity model for imperfect-quality items with allowable proportionate discounts" in which constant demand rate and triangular fuzzy numbers are used for parameters affecting inventory cost and defuzzyfied with cloudy fuzzy system methodology. Swagatika Sahoo et al. [25] developed, "A three rates of EOQ/EPQ Model for Instantaneous Deteriorating Items Involving Fuzzy Parameter Under Shortages" incorporating selling price and advertisement dependent demand rate and triangular fuzzy numbers are used to deal uncertainty of the parameters involved in the model system. The signed distance method is used to defuzzied the fuzzy numbers.

On deep study of research papers and motivated by, a two ware-house inventory model is being proposed and developed considering demand rate as selling price dependent under fuzzy parameters with fully backlogged shortages and effect of fuzziness is studiedwith fuzzy triangular number. Model is solved using signed distance method and centroid method and results are compared. Parameters involved in modelling are considered as fuzzy triangular number. This study includes only single item. Sensitivity is also performed for the crisp and fuzzy modelson selected parameters and change are observed. In section-1 introduction and literature review are presented followed with section -2 representing preliminary definition followed by assumptions and notations in secion-3. In section-4 Mathematical model is developed considering crisp nature and followed by section-5 with corresponding fuzzy models. Section-6 presents validation of models with the help of numerical example and sensitivity analysis on the models are performed in section-7 followed by concluding remarks and future scope in section-8.

2.0 Definition and Preliminaries

For the development of fuzzy inventory model, we need the following definitions:

(1) A fuzzy set $\tilde{E}$ on a given universal set X is denoted and defined by

$$\tilde{E} = \{(x, \lambda_{\tilde{A}}(x)) : x \,\varepsilon\, X\}$$

Where $\lambda_{\tilde{E}}$: X→ [0,1], is the membership function and $\lambda_{\tilde{E}}$(x) describes degree of x in $\tilde{E}$.

(2)A fuzzy number is specified by the triplet $(x_1 x_2 x_3)$ is known as triangular fuzzy if$x_1 < x_2 < x_3$ and defined by its continuous membership function$\lambda_{E^{\sim}}$: X→ [0,1]as follows:

$$\lambda_{A^{\sim}}(x) = \begin{cases} \dfrac{x - x_1}{x_3 - x_1} & if\ x_1 \leq x \leq x_2 \\ \dfrac{x_3 - x}{x_3 - x_2} & if\ x_2 \leq x \leq x_3 \\ 0 & otherwise \end{cases}$$

(3) Let E˜ be the fuzzy set defined on the R (set of real numbers), then the signed distance of E˜ is defined as

d(E˜,0) = $\frac{1}{2}\int_0^1 [E_L(\alpha) + E_R(\beta)] d_\alpha$

where $E_\alpha = [E_L(\alpha) + E_R(\beta)]$=[a+(b-a) α, d-(d-c) , α] , α ε [0,1] is a α-cut of a fuzzy set E˜.

(4) If E˜= $(x_1 x_2 x_3)$ is a triangular fuzzy number then the signed distance of E˜ is defined as

d (E˜, 0) =$\frac{1}{4}(x_1 + 2\ x_2 + x_3)$

(5) If E˜= $(x_1 x_2 x_3)$ is a triangular fuzzy number then the centroid method on E˜ is defined as

C(E˜) =$\frac{1}{3}(x_1 + x_2 + x_3)$

3.0 Assumptionand Notations

The mathematical model of the two-warehouse inventory problem is based on the following

Assumption and notations.

3.1 Assumptions

I. Demand rate is selling price dependent.
II. The lead time is negligible.
III. The replenishment rate is infinite and instantaneous.
IV. Shortages are allowed and fully backlogged.
V. Deterioration rate assumed to be constant in both ware houses.
VI. The holding cost is constant and higher in RW than OW.
VII. The deteriorated units cannot be repaired or replaced during the period under review.

- Deterioration occurs as soon as items are received into inventory.
- Inventory start without shortages and end with shortages.
- Parameters are considered to be triangular fuzzy number in case of fuzzy model.

3.2 Notation

The following notation is used throughout the paper:

Demand rate (units/unit time) which is ramp type given as

$$D(s)=\begin{cases} a\, s^{-b} & whrer\ a > 0\ and\ b > 0 \\ a & if\ \ b = 0 \end{cases}$$

W_0 Capacity of Own ware house (OW)

α_r Deterioration rate in rented warehouse (RW) such that $0 < \alpha_0 < 1$

α_0 Deterioration rate in OW and $\alpha_0 > 1$.

C_o Ordering cost per order

d_r Deterioration cost per unit of deteriorated item in RW

d_0 Deterioration cost per unit of deteriorated item in OW

h_r Holding cost per unit per unit time in OW

h_0 Holding cost per unit per unit time in RW such that$(h_r - h_0) > 0$

S_c Backlogging cost per unit per unit time

P_c Unit purchasing cost

s Selling price per unit of an item

a Scale parameter of sales price

b Elasticity of sales price

λ Point of Time when inventory vanishes in RW

γ Point of Time when inventory vanishes in OW

M Maximum order quantity at the end of cycle length

T Cycle length

$I_r(t)$ Inventory level in RW inthe system at time t

$I_0(t)$Inventory level in OW at time t in the system

$I_s(t)$Inventory level of shortages quantity in OW at time t in the system

Π (γ, T) Worth average inventory cost for crisp model

$\tilde{\alpha_r}$ Fuzzy deterioration parameter in RW

$\tilde{\alpha_0}$ Fuzzy deterioration parameter in OW

$\tilde{d_r}$ Fuzzy deterioration cost parameter in RW

$\tilde{d_0}$ Fuzzy deterioration cost parameter in OW

$\tilde{h_r}$ Fuzzy holding cost parameter in RW

$\tilde{h_0}$ Fuzzy holding cost parameter in OW

$\tilde{S_c}$ Fuzzy shortages cost parameter

$\tilde{P_c}$ Fuzzy purchasing cost parameter

$\tilde{s}$ Fuzzy selling cost parameter

$\tilde{a}$ Fuzzy scale parameter of selling price

$\tilde{b}$ Fuzzy elasticity parameter of selling price

$\tilde{\Pi}$(γ, T) Worth average inventory cost for fuzzy model

{~ Sign represent the fuzziness of the parameters}

4.0 Mathematical Model considering crisp nature of parameters

In the beginning of the business an order of quantity M is placed. After receiving order quantity,an amount equal to W_0,the capacity of own warehouse is stored in OW and the remaining stock of $M - W_0$ is placed in RW and items are supplied from the RW to decrease the rent being charged. System of depletion of inventory during storages is depicted in Figure-1. Since inventory decreases during time interval [0 λ] in RW due to continuous demand and deterioration therefore, present situation governing by the following differential equations is

$$\frac{dI_r(t)}{dt} = -\alpha_r I_r(t) - D(s) \quad ; \quad 0 \le t \le \lambda \tag{1}$$

During the supply from RW, on hand inventory stocked in OW decreases due to deterioration and after vanishing level of inventory in RW, the demand is being fulfilled from inventory remaining in the OW. Since inventory decreases during time interval [0 λ] in OW due to deterioration only and due to continuous demand after vanishing inventory in RW, the level of inventory depletes due to demand and deterioration both in the time interval [λ γ] and therefore, present situation governing by the following differential equations is

$$\frac{dI_0(t)}{dt} = -\alpha_0 I_r(t) \quad ; \quad 0 \le t \le \lambda \tag{2}$$

$\frac{dI_0(t)}{dt} = -\alpha_0 I_0(t) - D(s)$; $\lambda \leq t \leq \gamma$ (3)

Since there is continuous demand in the market and customers are ready to wait to receive the inventory, they needed due to effective price management and thus it reveals a shortage in the inventory system which are supplied in the beginning of next cycle and shortages quantity is ordered during the next replenishment. The quantity shortages during the time interval [γ T] is governed by the following differential equation

$\frac{dI_s(t)}{dt} = -D(s)$; $\gamma \leq t \leq T$ (4)

Solution of equation (1) with boundary condition $I_r(\lambda) = 0$ is found to be

$I_r(t) = \frac{D(s)}{\alpha_r}\left(e^{\alpha_r(\lambda - t)} - 1\right);$ $0 \leq t \leq \lambda$ (5)

Solution of equation (2) with boundary condition $I_0(0) = W_0$ is found to be

$I_0(t) = W_0 e^{-\alpha_0 t};$ $0 \leq t \leq \lambda$ (6)

Solution of equation (3) with boundary condition $I_0(\gamma) = 0$ is found to be

$I_0(t) = \frac{D(s)}{\alpha_0}\left(e^{\alpha_0(\gamma - t)} - 1\right);$ $\lambda \leq t \leq \gamma$ (7)

Solution of equation (4) with boundary condition $I_0(\gamma) = 0$ is found to be

$I_s(t) = D(s)(\gamma - t);$ $\gamma \leq t \leq T$ (8)

Since at the beginning of inventory level is M and therefore maximum inventory to be purchased in the beginning is

$M = I_r(0) + W_0$

$$= W_0 + \frac{D(s)}{\alpha_r}\left(e^{\alpha_r \lambda} - 1\right)$$

And in the next replenishment after shortages occurred quantity of inventory ordered with backlogged quantity will be

$M_{max} = W_0 + \frac{D(s)}{\alpha_r}\left(e^{\alpha_r(\lambda)} - 1\right) + \frac{D(s)}{2}\{(T - \gamma)^2\}$ (9)

Also, during the supply of inventory continuity of demand reveals that inventory level at $t = \lambda$ are same, therefore from equations (6) and (7) it is obtained thatγ is the function of λ that is

$\gamma = \lambda + \frac{1}{\alpha_0}\log[1 + \frac{\alpha_0 W_0}{D(s)} e^{-\alpha_0 \lambda}]$ (10)

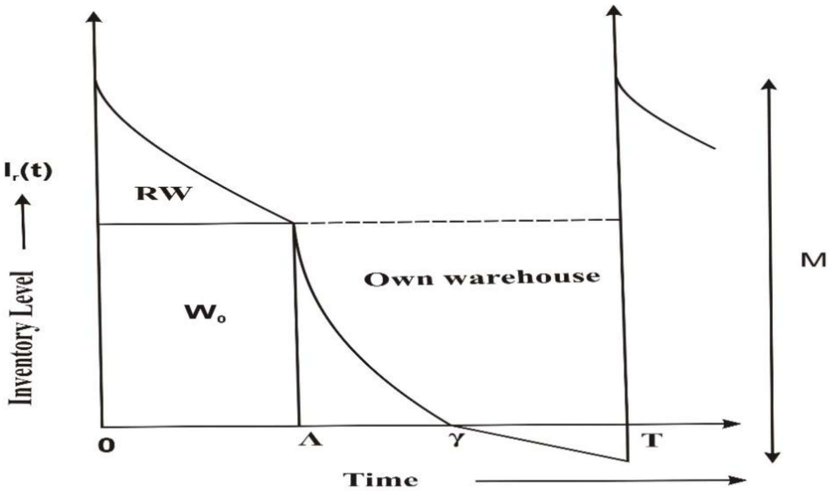

Figure-1: Graph representing depletion of inventory level in warehouses

The inventory cost consists of the following cost

- Ordering cost
- Inventory holding cost in RW
- Inventory holding cost in OW
- Deterioration cost in RW
- Deterioration cost in OW
- Shortages cost in OW
- Purchasing cost

Now above costs are calculated as follows:

Ordering cost C_0

Inventory holding cost in RW is

$$IH_{crw} = h_r \left\{\int_0^{\lambda} I_r(\mathrm{t})\, dt\right\}$$

$$= \frac{h_r\, D(s)}{\alpha_r}\left\{\frac{1}{\alpha_r}\left(e^{\alpha_r(\lambda-t)} - \lambda\right)\right\}$$

Inventory holding cost in OW

$$IH_{cow} = h_0 \left\{\int_0^{\lambda} I_0(\mathrm{t})\, dt + \int_{\lambda}^{\gamma} I_0(\mathrm{t})\, dt\right\}$$

$$= \left\{\frac{h_r\, W_0}{\alpha_0}\left(1 - e^{-\alpha_0\lambda}\right) + \frac{h_r\, W_0}{\alpha_0^2}\left[\left(e^{\alpha_0(\gamma-\lambda)}\right) - \alpha_0\,(\gamma - \lambda)\right]\right\}$$

Inventory deterioration cost in RW

$$ID_{crw} = d_r \left\{I_r(0) - \int_0^{\lambda} D(s)\, dt\right\}$$

$$= \left\{\frac{d_r\, D(s)}{\alpha_r}\left(1 - e^{-\alpha_r\lambda}\right) - d_r\,(D(s)\lambda)\right\}$$

Inventory deterioration cost in OW

$$ID_{cow} = d_0 \left\{W_0 - \int_{\lambda}^{\gamma} D(s)\, dt\right\}$$

$$= \{d_0\,(W_0 - D(s)((\gamma - \lambda))\}$$

Inventory shortages cost in OW is

$$IS_{crw} = S_c \left\{\int_{\gamma}^{T} -I_s(\mathrm{t})\, dt\right\}$$

$$= \frac{S_c\, D(s)}{2}\{(T - \gamma)^2\}$$

Inventory purchase cost at the first cycle and onward is given by

$$IP_c = P_c\,(M_{max})$$

$$= P_c\,(W_0 + \frac{D(s)}{\alpha_r}\left(e^{\alpha_r(\lambda)} - 1\right) + \frac{D(s)}{2}\{(T - \gamma)^2\})$$

Hence the total relevant inventory cost per unit of time during cycle length is given b

$$\Pi(\lambda, \mathrm{T}) = \frac{1}{T}[\mathrm{C}_o + IH_{crw} + IH_{cow} + ID_{crw} + ID_{cow} + IS_{crw} + IP_c] = \frac{1}{T}\Big[\mathrm{C}_o + h_r\left\{\int_0^{\lambda} I_r(\mathrm{t})\, dt\right\} + h_0\left\{\int_0^{\lambda} I_0(\mathrm{t})\, dt + \int_{\lambda}^{\gamma} I_0(\mathrm{t})\, dt\right\} + d_r\left\{I_r(0) - \int_0^{\lambda} D(s)\, dt\right\} + d_0\left\{W_0 - \int_{\lambda}^{\gamma} D(s)\, dt\right\} + S_c\left\{\int_{\gamma}^{T} -I_s(\mathrm{t})\, dt\right\} + P_c\,(M_{max})\Big] \qquad (11)$$

The objective is to Minimize:$\Pi(\lambda, T)$

Subject to: $(\gamma > 0, T > 0)$

The necessary condition for $\Pi(\gamma, T)$ to be minimum is that $\frac{\partial \Pi(\lambda,T)}{\partial \lambda} = 0$

and $\frac{\partial \Pi(\lambda,T)}{\partial T} = 0$, and solving equation (11) we find the optimum values of γ and T say λ^* and T^* for which average inventory cost is minimum and the sufficient condition is

$$\left(\frac{\partial^2 \Pi(\lambda,T)}{\partial \lambda^2}\right)\left(\frac{\partial^2 \Pi(\lambda,T)}{\partial T^2}\right) - \left(\frac{\partial^2 \Pi(\lambda,T)}{\partial \lambda \partial T}\right)^2 < 0$$

5.0 Fuzzy Model considering fuzzy nature of some parameters

In case of uncertain situation crisp nature of parameters are not so meaningful in describing the business specifically. In the present global market, the value of parameters like cost, demand may fluctuate due to the several reasons and uncertainty like low production, natural hazards etc. and it may fluctuate. The fluctuation at any time cannot be pre-determined until we reach the situation of that time. Therefore, the only possibility is to consider the possible range of fluctuation. To deal with such type of uncertain situation, a corresponding fuzzy model is developed, considering vagueness of some parameter affecting the total inventory cost. Parameters affecting inventory cost is considered as triangular fuzzy numbers. The model is solved using signed distance and centroid method to minimize the total inventory costand the results are analysed. Also, sensitivity is performed with some combination of parameters of fuzzy nature and deviation is noticed

Using equation (11) and fuzzy parameters we have,

$\tilde{\alpha_r} = (\tilde{\alpha}_{r1}, \tilde{\alpha}_{r2}, \tilde{\alpha}_{r3})$,$\tilde{\alpha_0} = (\tilde{\alpha}_{01}, \tilde{\alpha}_{r2}, \tilde{\alpha}_{r3})$,$\tilde{h_r} = (\tilde{h}_{r1}, \tilde{h}_{r2}, \tilde{h}_{r3})$, $\tilde{h_0} = (\tilde{h}_{01}, \tilde{h}_{02}, \tilde{h}_{03})$,

$\tilde{d_r} = (\tilde{d}_{r1}, \tilde{d}_{r_{r2}}, \tilde{d}_{r_3})$, $\tilde{d_0} = (\tilde{d}_{01}, \tilde{d}_{0_2}, \tilde{d}_{0_3})$, $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3)$, $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3)$, $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$, $\tilde{S_c} = (\tilde{S}_{c1}, \tilde{S}_{c2}, \tilde{S}_{c3})$, and $\tilde{P_c} = (\tilde{P}_{c1}, \tilde{P}_{c2}, \tilde{P}_{c3})$

Therefore, fuzzy model is given by

$$\tilde{\Pi}\ (\gamma, T) = (\tilde{\Pi}_1\ (\gamma, T), \tilde{\Pi}_2\ (\gamma, T), \tilde{\Pi}_3\ (\gamma, T))$$

$$= \frac{1}{T}\Big[C_o + \tilde{h_r}\left\{\int_0^\lambda I_r(t)\,dt\right\} + \tilde{h_0}\left\{\int_0^\lambda I_0(t)\,dt + \int_\lambda^\gamma I_0(t)\,dt\right\} + \tilde{d_r}\left\{I_r(0) - \int_0^\lambda D(\tilde{s})\,dt\right\} + \tilde{d}_{01}\left\{W_0 - \int_\lambda^\gamma D(\tilde{s})\,dt\right\} + \tilde{S_c}\left\{\int_\gamma^T -I_s(t)\,dt\right\} + \tilde{P_c}\ (M_{max})\Big] \quad (12)$$

Where,

$$\tilde{\Pi}_1(\gamma, T) = \frac{1}{T}\Big[C_o + \tilde{h}_{r1}\left\{\int_0^\lambda I_r(t)\,dt\right\} + \tilde{h}_{01}\left\{\int_0^\lambda I_0(t)\,dt + \int_\lambda^\gamma I_0(t)\,dt\right\} + \tilde{d}_{r1}\left\{I_r(0) - \int_0^\lambda D(\tilde{s_1})\,dt\right\} + \tilde{d}_{01}\left\{W_0 - \int_\lambda^\gamma D(\tilde{s_1})\,dt\right\} + \tilde{S}_{c1}\left\{\int_\gamma^T -I_s(t)\,dt\right\} + \tilde{P}_{c1}\ (M_{max})\Big]$$

$$\tilde{\Pi}_2(\gamma, T) = \frac{1}{T}\Big[C_o + \tilde{h}_{r2}\left\{\int_0^\lambda I_r(t)\,dt\right\} + \tilde{h}_{02}\left\{\int_0^\lambda I_0(t)\,dt + \int_\lambda^\gamma I_0(t)\,dt\right\} + \tilde{d}_{r2}\left\{I_r(0) - \int_0^\lambda D(\tilde{s_2})\,dt\right\} + \tilde{d}_{02}\left\{W_0 - \int_\lambda^\gamma D(\tilde{s_2})\,dt\right\} + \tilde{S}_{c2}\left\{\int_\gamma^T -I_s(t)\,dt\right\} + \tilde{P}_{c2}\ (M_{max})\Big]$$

$$\tilde{\Pi}_3(\gamma, T) = \frac{1}{T}\Big[C_o + \tilde{h}_{r3}\left\{\int_0^\lambda I_r(t)\,dt\right\} + \tilde{h}_{03}\left\{\int_0^\lambda I_0(t)\,dt + \int_\lambda^\gamma I_0(t)\,dt\right\} + \tilde{d}_{r3}\left\{I_r(0) - \int_0^\lambda D(\tilde{s_3})\,dt\right\} + \tilde{d}_{03}\left\{W_0 - \int_\lambda^\gamma D(\tilde{s_3})\,dt\right\} + \tilde{S}_{c3}\left\{\int_\gamma^T -I_s(t)\,dt\right\} + \tilde{P}_{c3}\ (M_{max})\Big]$$

and$M_{max} = W_0 + \frac{D(\tilde{s_i})}{\alpha_{ri}}\left(e^{\alpha_r(\lambda)} - 1\right) + \frac{D(\tilde{s_i})}{2}\{(T-\gamma)^2\}$ such that $i = 1,2,3$

By signed distance method total average inventory cost is given by

$$\tilde{\Pi}\ (\gamma, T) = \frac{1}{4T}\left[\tilde{\Pi}_1(\gamma, T) + 2\tilde{\Pi}_2(\gamma, T) + \tilde{\Pi}_3\ (\gamma, T)\right] \quad (13)$$

and by centroid method total average inventory cost is given by

$$\tilde{\Pi}\ (\gamma, T) = \frac{1}{3T}\left[\tilde{\Pi}_1(\gamma, T) + \tilde{\Pi}_2(\gamma, T) + \tilde{\Pi}_3\ (\gamma, T)\right] \quad (14)$$

The objective is to

To minimize: $\widetilde{\Pi}\,(\lambda, T)$

Subject to: $(\lambda > 0, T > 0)$

The necessary condition for $\widetilde{\Pi}\,(\lambda, T)$ to be minimum is that $\frac{\partial \widetilde{\Pi}\,(\gamma,T)}{\partial \lambda} = 0$

and $\frac{\partial \widetilde{\Pi}\,(\gamma,T)}{\partial T} = 0$, and solving equations (13) and (14) we find the optimum values of γ and T say $\lambda^*$and T^* applying two methods respectively under the condition of optimality and average inventory cost is minimum and the sufficient condition is

$$\left(\frac{\partial^2 \widetilde{\Pi}\,(\gamma,T)}{\partial \lambda^2}\right)\left(\frac{\partial^2 \widetilde{\Pi}\,(\gamma,T)}{\partial T^2}\right) - \left(\frac{\partial^2 \widetilde{\Pi}\,(\gamma,T)}{\partial \lambda \partial T}\right)^2 < 0$$

6.0 Numerical example:

To analyse the model, following example is taken under the random selection of parameters the demand rate function is to be D(s)= a s^{-b}, where a scale factor of sales price and is the initial demand rate at b=0 and b is the price elasticity of salesparameter. If b=o, demand remain constant. The exponential function has to be solved up to first approximation. The values of parameters are not collected from any real-life case study but these values are realistic and chosen randomly to illustrate and validate the model. Considering the value of parameters in an appropriate unit (displayed in Table-A) and using suitable mathematical software, the optimal average inventory cost has been obtained which are displayed in Table-1& Table-2 for two models. Sensitivity analysis is performed for crisp model and fuzzy model on some selected parameters only.

Table-1

Parameter	C_o	s	a	b	d_r	d_o	S_c	h_r	h_o	α_r	α_o	P_c	W
Example	5000	100	50	0.5	48	50	50	50	35	0.1	0.2	95	100

Table-2: Crisp Model

λ^*	γ^*	T^*	$\Pi\,(\gamma, T)$
0.0094	2.1213	4.0436	3885.55

Table-3: Fuzzy Model:

Method	λ^*	γ^*	T^*	$\Pi\,(\gamma, T)$
Signed distance	0.0684	2.1836	3.9222	3903.21
Centroid	0.0932	2.2112	3.8849	3913.94

7. Sensitivity Performance

7.1 Crisp Model

Table-4

Parameter	values	T^*	$\Pi\,(\gamma, T)$
α_0			
	0.30	5.9093	4451.63
	0.40	7.3597	5165.63
	0.50	8.5630	5822.13
α_r			
	0.12	11.6426	5546.96
	0.13	26.0123	7842.01
	0.14	32.7635	9644.04

Parameter	values	T^*	$\Pi\,(\gamma, T)$
h_r			
	55	4.2435	3989.05

	60	4.4468	3930.96
	65	4.6477	3979.16
h_0			
	40	3.8448	4092.73
	45	3.6807	4279.05
	50	3.5427	4450.55
s			
	125	4.0459	3885.55
	150	4.0480	3858.29
	175	4.0500	3848.22
C_p			
	100	3.9443	3893.00
	125	3.6450	4108.98
	150	4.9258	4894.09
S_c			
	55	3.5345	4541.19
	60	3.5270	4633.22
	75	3.5077	4918.75

7.2 Fuzzy Model

Table-5

Variation in total inventory cost with respect to $D(\tilde{s}) = [\tilde{a}(49,50,51), \tilde{b}(0.04, 0.05, 0.06), \tilde{s}(99,100,101)]$

Parameter→ Method ↓	λ^*	γ^*	T^*	$\tilde{\Pi}(\gamma, T)$
Signed distance	0.0094	2.1213	4.0436	3885.59
Centroid	0.0094	2.1213	4.0436	3885.60

Variation in total inventory cost with respect to $D(\tilde{s}) = [\tilde{a}(49,50,51), \tilde{b}(0.04, 0.05, 0.06), \tilde{s}(99,100,101)] \& \tilde{\alpha}_0(0.1,0.2,0.3)$

Parameter→ Method ↓	λ^*	γ^*	T^*	$\tilde{\Pi}(\gamma, T)$
Signed distance	0.0117	1.9974	4.5236	4006.42
Centroid	0.0236	2.1360	4.0094	3887.03

Variation in total inventory cost with respect to $D(\tilde{s}) = [\tilde{a}(49,50,51), \tilde{b}(0.04, 0.05, 0.06), \tilde{s}(99,100,101)] \& \tilde{\alpha}_r(0.09, 0.2, 0.11)$

Parameter→ Method ↓	λ^*	γ^*	T^*	$\tilde{\Pi}(\gamma, T)$
Signed distance	0.0803	2.0427	4.4370	3915.64
Centroid	0.0182	2.0939	4.1213	3886.53

Variation in total inventory cost with respect to $D(\tilde{s}) = [\tilde{a}(49,50,51), \tilde{b}(0.04, 0.05, 0.06), \tilde{s}(99,100,101)]$ & $\tilde{h}_r(49, 50, 51)$				
Parameter→ Method ↓	λ^*	γ^*	T^*	$\tilde{\Pi}(\gamma, T)$
Signed distance	0.0098	2.1219	4.0431	3885.26
Centroid	0.0097	2.1215	4.0429	3885.75

Variation in total inventory cost with respect to $D(\tilde{s}) = [\tilde{a}(49,50,51), \tilde{b}(0.04, 0.05, 0.06), \tilde{s}(99,100,101)]$ & $\tilde{h}_0(34, 35, 36)$				
Parameter→ Method ↓	λ^*	γ^*	T^*	$\tilde{\Pi}(\gamma, T)$
Signed distance	0.0168	2.1242	4.0343	3886.72
Centroid	0.0093	2.1217	4.0442	3884.83

Section-8: Graphical representation of Models

Convexity of Crisp Model presented through 3-D graphs

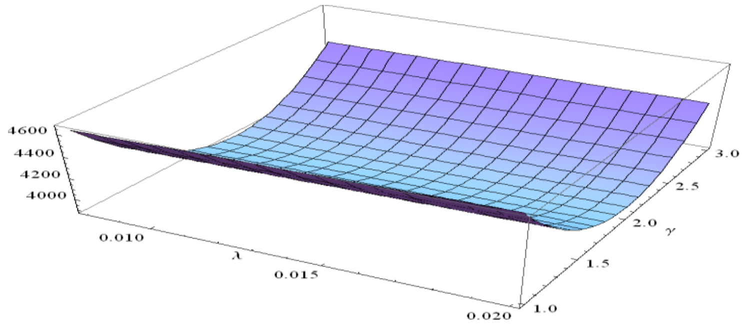

Figure-2: Graph representing convex nature ofcrisp model (When T is constant)

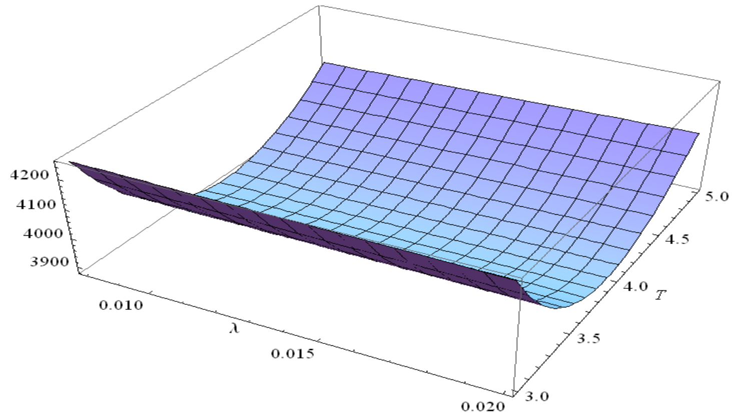

Figure-3: Graph representing convex nature of crisp model (When γ is constant)

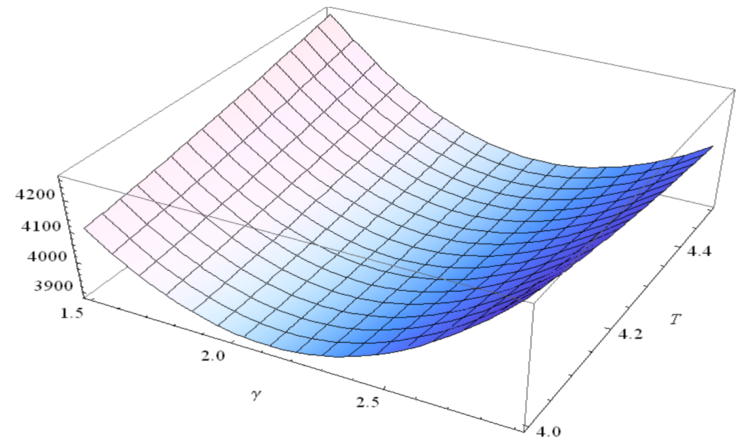

Figure-4: Graph representing convex nature of crisp model (When λ is constant)

Convexity of Fuzzy Model presented through 3-D graphs

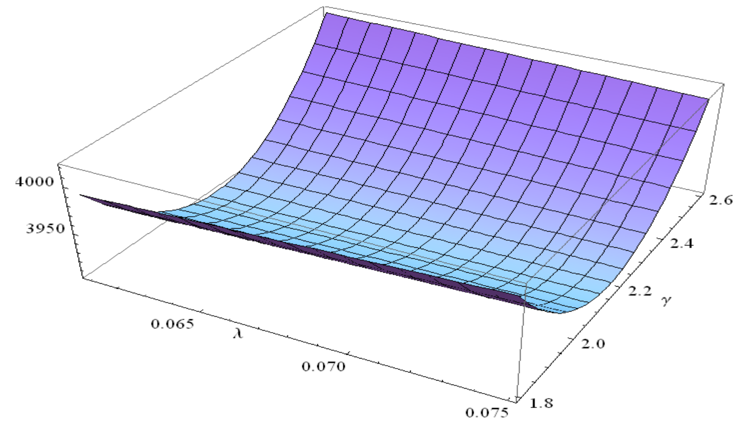

Figure-5: Graph representing convex nature of crisp model (When T is constant)

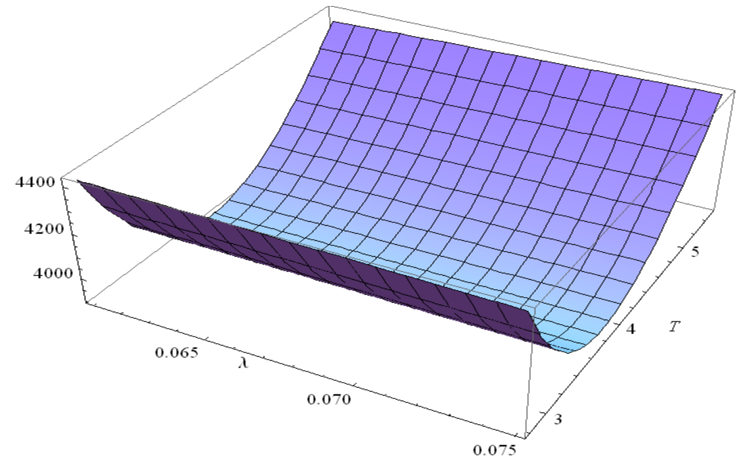

Figure-6: Graph representing convex nature of crisp model (When γ is constant)

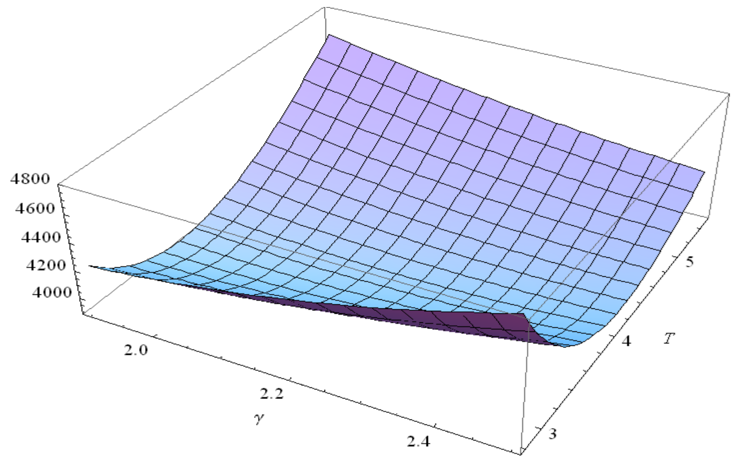

Figure-7: Graph representing convex nature of crisp model (When λ is constant)

8.Sensitivity analysis and observance:

On some selected and important parameters of model sensitivity performance reveals the following points on two models respectively: -

Crisp model and Fuzzy model

- ✓ When all the given conditions and constraints are satisfied, theoptimal solution is obtained. From Table-2 & Table-3, it is observed that the averageminimal present value of total relevant inventory cost in an appropriate unitisminimum as in case of crisp model but fuzzy model has more flexibility on choosing a range of values of parameters in respect of future planning and there is increment in the average inventory cost as compared to crisp model.
- ✓ From Table-3, it is observed that in two methods of defuzzification of model the total inventory costs and cycle lengths are moderately differ and signed distance method yielding low value of inventory cost as compared to centroid method.
- ✓ From Table-4, it is observed that in the case of crisp model when there is slightly increment in the value of deterioration rate in own warehouse then the change in the total inventory cost increase and correspondingly there is also increase in cycle length while very low increment in the deterioration rate in RW shows rapid change in the total inventory cost and ordering cycle length.
- ✓ From Table-4, it is observed that in the case of crisp model when the value of selling price increase (keeping scale and elasticity factors constant) total inventory cost decreases and cycle length increase. Increment in the purchasing cost yields increment in the total average inventory cost in proportion and moderately cycle length also increases.
- ✓ From Table-4, it is observed that in the case of crisp model when the value of holding cost in RW increase, total inventory cost increases and cycle length also increases. Increment in the holding cost in OW yields high increment in the total average inventory cost as compared to RW and moderately cycle length also increases. Change in the shortages cost also affect the total average inventory cost.
- ✓ From Table-5, it is observed that variation in the sales parameters of the fuzzy model alone has effect on the total inventory cost and is decreases in the both method of solutions.
- ✓ From Table-5, it is observed that variation in the deterioration rate in OW in combination with sales factors yield changes in average total inventory cost and decreases in centroid method while increases in the signed distance method also cycle length increases in both the method of solution.
- ✓ From Table-5, it is observed that variation in the deterioration rate in RW in combination with sales factors yield changes in average total inventory cost and decreases in centroid method while increases in the signed distance method also cycle length increases in both the method of solution.
- ✓ From Table-5, it is observed that variation in the holding cost rate in RW in combination with sales factors yield changes in average total inventory cost and decreases in both methods of solutions, centroid and signed distance. Yet cycle length increases in both the method of solution.
- ✓ From Table-5, it is observed that variation in the holding cost rate in OW in combination with sales factors yield changes in average total inventory cost and decreases in both methods of solutions, centroid and signed distance. Yet cycle length increases in both the method of solution.
- ✓ The convexity of graphs shown in Section-8 for crisp modeland Fuzzy model for shows that there is apoint where inventory system has minimal cost with the condition of optimality.

9.0 Conclusion:

This paper presents a deterministic inventory model for two warehouse inventory system withselling price dependent demand, constant deterioration rates in both warehousesand fully backlogging assumption. The two modelsnamely a crisp model and a fuzzy model is developed and numerical example is given to illustrate and validate themodels. Mathematica 9.0 software is used to find solution of models using different solution methods and results are compared. Sensitivity analysis is performed on some selected parameters. Fuzzy model is solved withdefuzzificationof triangular fuzzy numbers with the help of signed distance method and centroid method. Further this model can be generalised by considering different combination of demand rates with inflation and trade credit payment options other realistic combinations.

References

1. K.V.S.Sarma, A deterministic inventory model for deteriorating items with two level of storage and an optimum release rule,Opsearch 20 (1983) 175-180.
2. T.A.Murdeshwar, Y, S. Sathe,Some aspects of lot size model with two level of storage, Opsearch 22 (1985) 255-262.
3. U.Dave,On the EOQ models with two level of storage. Opsearch25 (1988) 190-196.
4. K.V.S.Sarma, A deterministic order level inventory model for deteriorating items with two storage facilities,Eur.J. Oper.Res.29 (1987) 70-73.
5. T.P.M. Pakala,K.K. Archary,Discrete time inventory model for deteriorating items with two warehouses,Opsearch 29 (1992a) 90-103.
6. T.P.M. Pakala,K.K. Archary,A deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate, Eur.J. Oper.Res.57 (1992b) 71-76.
7. H.L. Yang, Two warehouse inventory models for deteriorating items with shortages under inflation, Eur.J. Oper.Res.157 (2004) 344-356.
8. C.Y. Dye,L.Y.Ouyang,T.P. Hsieh,deterministic inventory model for deteriorating items with capacity constraints and time proportional backlogging rate, Eur.J. Oper.Res.178 (3) (2007) 789-807.
9. L.A.Zadeh, Fuzzy Set, Information Control,8(1965),338-353.
10. L.A.Zadeh andR.E.Bellman,Decision making in a fuzzy environment,Management Science,17(1970),140-164.
11. R. Jain, Decision making in the presence of fuzzy variables,IIIE Transaction on Systems,Man and Cybernetics,17(1976)698-703.
12. D.Dubois and H. Prade,Operations on fuzzy numbers,International Journals of System Science,9(1978) 613-626.
13. S.Kumar, D.Kundu and A.Goswami,An Economic Production Inventory Model Involving Fuzzy Demand Rate and Fuzzy Deterioration Rate,Journal of Applied Mathematics and Compuing,12(2007) 251-260.
14. J.K. Syed and L.A.Aziz,Fuzzy Inventory Model without shortages by Using Signed Distance method.Applied Mathematics and Information Science,1(2007) 203-209.
15. P.K.De and A.Rawat,Fuzzy Inventory Model Without Shortage Using Triangular Fuzzy Number,Fuzzy Information andEngineering ,3(2001) 59-61.
16. C.K.Jaggi,S.Pareek,A.Sharma and Nidhi,Fuzzy Inventory Model for Deteriorating Items wiyh Time Varying Demand and Shortages.American Journal of Operational Research,2(2012)81-92.
17. D.Datta and P. Kumar, Fuzzy Inventory Model without shortages by Using Trapezoidal Fuzzy Number with Sensitivity Analysis.IOSR Journal of Mathematics,4(2012) 32-37.
18. K.A.Halem,B.C.Giri and K.S.Chaudhari, Fuzzy economic order quantity model for perishable items with stochastic demand, partial backlogging and fuzzy deteriorating rate.International Journal of Operational Research,3(2008) 77-96.
19. A.Goni and S.Maheshwari,Supply chain model for retailer’s ordering policy under two level of delay payments in fuzzy environment.Applied Mathematical Sciences,4(2010) 1155-1164.
20. K.A. Halem,B.C.Giri and K.S.Chaudhari, Lot sizing in an unreliable manufacturing system with fuzzy demand and repair time.International Journal of industrial and System Engineering,5(2010),485-500.
21. S.R.Singh and C.Singh,Fuzzy Inventory Model for finite rate of Replenishment using Signed Distance Method.International Transactions in Mathematical Sciences and Computer,1(1) (2008) 27-34.

22. Palani, R. and Maragatham, M., Fuzzy inventory model for time dependent deteriorating items with lead time stock dependent demand rate and shortages,International Journal of Development Research, Vol. 07, Issue, 10 (2017)15988-15995.
23. Shabnam Fathalizadeh, AbolfazlMirzazadehand Ali Ghodratnama, Fuzzy inventory models with partial backordering for deteriorating items under stochastic inflationary conditions: Comparative comparison of the modelling methods, Cogent Engineering (2019), 6: 1648630.
24. Sujit Kumar De andGour Chandra Mahata, A cloudy fuzzy economic order quantity model for imperfect-quality items with allowable proportionate discounts, Journal of Industrial Engineering International 15 (2019),571–583.
25. Swagatika Sahoo, Milu Acharya and MitaliMadhusmita Nayak,A three rates of EOQ/EPQ Model for Instantaneous Deteriorating Items Involving Fuzzy Parameter Under Shortages,International Journal of Innovative Technology and Exploring Engineering (IJITEE), Volume-8(2019) Issue-8S2,405-418.