Research Article

Fuzzy Transportation Problem By Using TriangularFuzzy Numbers With Ranking Using Area Of Trapezium, Rectangle And Centroid At Different Level Of α-Cut

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Abstract:

The objective of this article is to introduce a new ranking method to order any two fuzzy triangular numbers. This new ranking method is used to find the best approximate solution to the fuzzy transportation problem. The uncertainty plays a vital role in any branch of science and technology, engineering, medical and management. In Artificial Intelligence Fuzzy Mathematics and fuzzy logic is used to process Natural language and widely used in decision making. The objective of the transportation problem is to control the amount to be transported from several origins to several destinations such that the total transportation cost is minimized. But in real-life situations, the available supply and forecast demand are often fuzzy because some information is incomplete orunavailable.

In this article, the crisp transportation problem is transformed into a fuzzy transportation problem by using triangular fuzzy numbers. To order any two triangular fuzzy numbers the new and simple method is invented which are based on the area of a trapezium, rectangle, and centroid at prominent places using α-cut method at α=0.2, α= 0.5 and α=0.8 respectively. A computer program was written in MATLAB which is given in this article to make calculation easier and simple. This article is organized as follows. In section first we introduced new ranking method to compare any two triangular fuzzy numbers. I third section we extend this method to solve fuzzy transportation problem using numerical example, while paper is concluded in the last section.

Keywords: Ranking, Trapezium, Centroid, Triangular Fuzzy Number, Fuzzy Transportation

Introduction

Theoretically, fuzzy numbers cannot be compared but merely partially ordered. But, when fuzzy numbers are used in real-world applications, e.g., when we want to determine optimal solution of objective function or want to make the decision among alternatives, then order of fuzzy numbers becomes important. However, there are many methods of fuzzy numbers say qualitative, quantitative and based on α–cuts to the ordering relation between any two fuzzy numbers. However in fuzzy set theory there is no universally acceptable method to compare any two fuzzy numbers. The ranking of fuzzy number using α-weighted valuations was started by*Marcin Detyniecki and Ronald R Yager* [9] in year 2001.Later P.*PhaniBushan Rao and N. Ravi Shankar*[10] obtainedranking fuzzy numbers with a distancemethod usingcircumcenter of centroids and an index ofmodality. *D. Stephen Dinagar, K. Latha*[5] studied some types of type-2 triangular fuzzy matrices. Representation and ranking of fuzzy numberswith heptagonal membership function usingvalue and ambiguity index developed by *K. Rathi and S. Balamohan*[7]. The solution offuzzy game problem using triangular fuzzy numberstudied by R. *Senthil Kumar, S. Kumaraghuru*[11]. *A. Rahmani, F. HosseinzadehLotfi, M. Rostamy-Malkhalifeh, and T. Allahviranloo* [1] applied New Method for Defuzzification and ranking of fuzzy numbers based on the statistical beta distribution.A new approach for ranking fuzzy numbers developed by *M. Lotfi, S. Salahshour, F. Nasr Esfahani, A. Jafarnejad*[8]. In year 2017 *S. Yahya Mohamed and N. Karthikeyan*[12] used A new distance and ranking method for triangularfuzzy numbers. Sub interval average method forranking of linear fuzzy developed by *Stephen Dinagar, Kamalanathan, Rameshan* [13]. Ordering of generalised trapezoidal fuzzy numbers based on area method using euler line of centroids introduced by *A. Hari Ganesh and M. Suresh* [10].

Basic Definitions

Triangular Fuzzy Number:

A triangular fuzzy number $\bar{A}$ or simply triangular number represented with three points as follows (a_1, a_2, a_3) holds the following conditions.

i) a_1 to a_2 is increasing function

ii) a_2 to a_3 is decreasing function

iii) $a_1 \leq a_2 \leq a_3$

Its membership function is defined as follows

$$\mu_A(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a1)} & a_1 \leq x < a_2 \\ 1 & x = a_2 \\ \dfrac{(a_3-x)}{(a_3-a_2)} & a_2 < x \leq a_3 \end{cases}$$

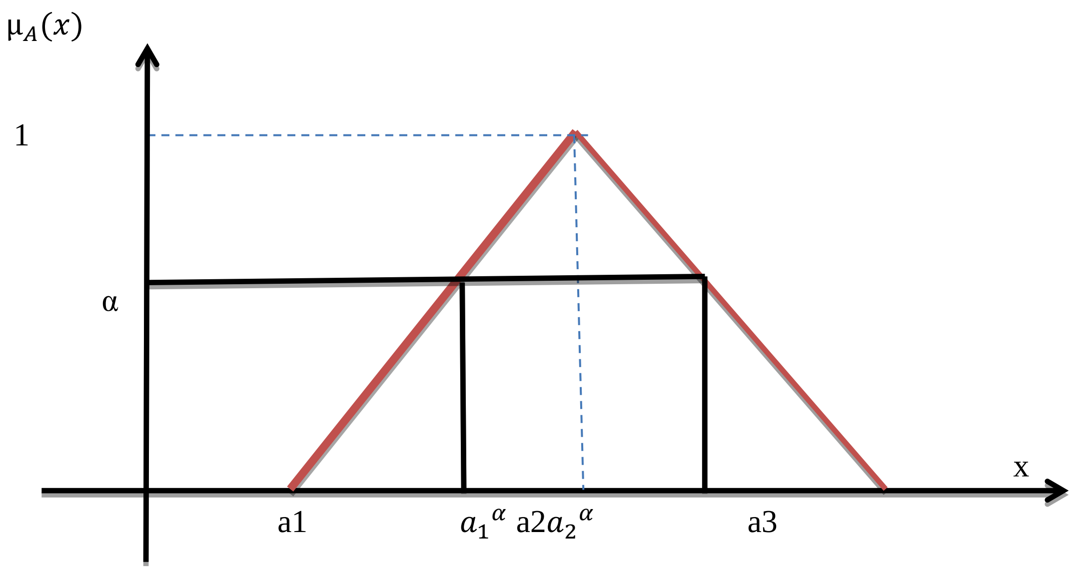

Figure1: Triangular fuzzy number [a_1, a_2, a_3]

α- Cut for triangular fuzzy number:

For any α ∈ [0, 1] from

$$\frac{a_1^\alpha - a_1}{a_2 - a_1} = \alpha, \frac{a_3 - a_3^\alpha}{a_3 - a_2} = \alpha$$

$$a_1^\alpha = (a_2 - a_1)\alpha + a_1,\ a_3^\alpha = = -(a_3 - a_2)\alpha + a_3$$

Thus $\bar{A}_\alpha = [a_1^\alpha, a_3^\alpha] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3]$

Operations on triangular fuzzy number:

Addition, Subtraction and Multiplication of any two triangular fuzzy numbers are alsotriangular fuzzy number. Suppose triangular fuzzy numbers $\bar{A}$ and $\bar{B}$ are defined as,

$$\bar{A} = (a_1, a_2, a_3) \text{ and } \bar{B} = (b_1, b_2, b_3)$$

i) Addition $\bar{A}\ (+)\bar{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3)$

$= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

ii) Subtraction $\bar{A}\ (-)\bar{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3)$

$= (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

iii) Symmetric image: $(-\bar{A}) = (-a_3, -a_2, -a_1)$

iv) Multiplication:

$\bar{A} \times \bar{B} = (\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$.

Proposed Ordering (Ranking) of twoTriangular fuzzy numbers:

The proposed method is based on the area of Trapezium, area of rectangle and centroid of triangle formed due to α-cut taken at values of α = 0.2, 0.5, 0.8 respectively of triangular fuzzy number. The calculation of the proposed area and value of centroid is presented from the point of view of analytical geometry. Given triangular fuzzy number [G H I], is divided into three plane figures. The three plane figures are trapezium (EFIG), Rectangle (CDKJ) and triangle (LAB) respectively. The area of trapezium EFIG,Area= $\frac{1}{2} \times h \times (EF + GI)$. Where the value of h can be calculated by using α-cut. For the Trapezium (EFIG) the value of h=0.2. To determine the value of EF, from Fig. (4) we can observe that side EF is parallel to LM. We assume the value of side LM=x then the value of side EF is also x. Now to determine the value of x, by using α-cut method the value of L= (H– $G)\alpha + H$ and value of M = - (I –H) $\alpha + I$. The value of GI is the core oftriangular fuzzy number [G H I].

Next, we want to determine the area of Rectangle (CDKJ). Suppose $\bar{X} = (x_1, x_2, x_3)$and $\bar{Y} = (y_1, y_2, y_3)$be given any two triangular fuzzy number. Area= *Length (JK)* × *width (CJ)*. The width CJ is equal to value of $\alpha = 0.5$. i.e., CJ =0.5. To determine the value of JK, from Fig. (3) we can observe that length JK is α-cut value at 0.5. Therefore by using α-cut method the value of J= (H– $G)\alpha + H$ and value of K = - (I –$H)\alpha + I$.

Next, we want to determine the centroidvalue of the x-axis of triangle (LAB) formed by α- cut at α = 0.8. Here we are taking only x co-ordinate because the centroid value of the y-axis is always same.Suppose ΔABC a triangle having vertices A(x1, y1), B(x2, y2), C(x3, y3), then the centroid of a triangle can be calculated by using the following formula. Centroid of a triangle for x coordinate = ((x1+x2+x3)/3). In the given triangular fuzzy number[G H I] centroid of Δ LAB from Fig. (6) is = ((H– *G)α*+ *H* + (- (I –*H) α* + *I) + H)/3*where α = 0.8

Suppose the area of trapezium, Rectangle and centroid of Triangle for triangular fuzzy number $\bar{X}$and $\bar{Y}$due to α- cut at α = 0.2, 0.5, 0.8 are A1, A2 A3 and B1, B2, B3 Respectively. If$\frac{(A1+A2+A3)}{3} \leq \frac{(B1+B2+B3)}{3}$then we can say that fuzzy numbers$\bar{X} \leq \bar{Y}$.

The aim for selecting the proposed area of reference is that the three plane figures Trapezium, Rectangle and Triangle are balancing points of each plane figure, of triangular fuzzy number.

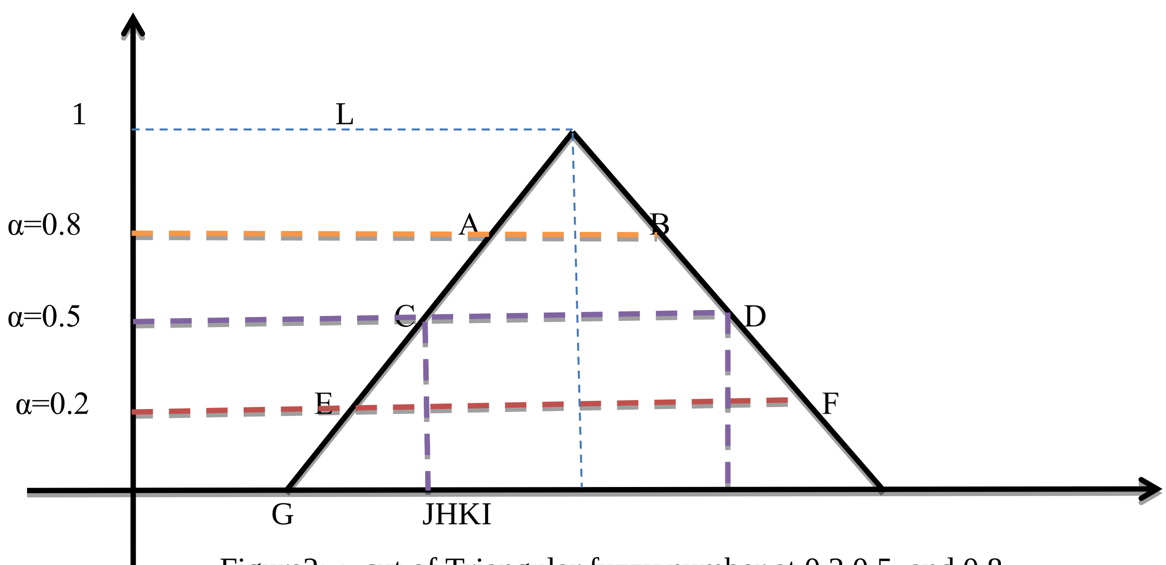

Figure2: α-cut of Triangular fuzzy number at 0.2,0.5, and 0.8.

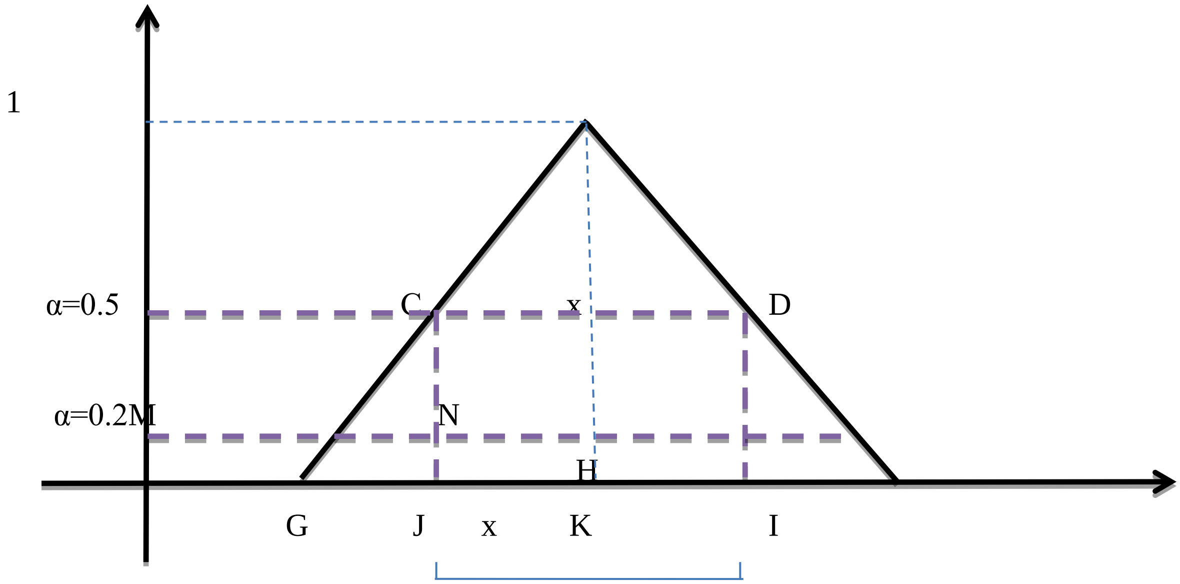

Figure 3: Area of Rectangle CDMN

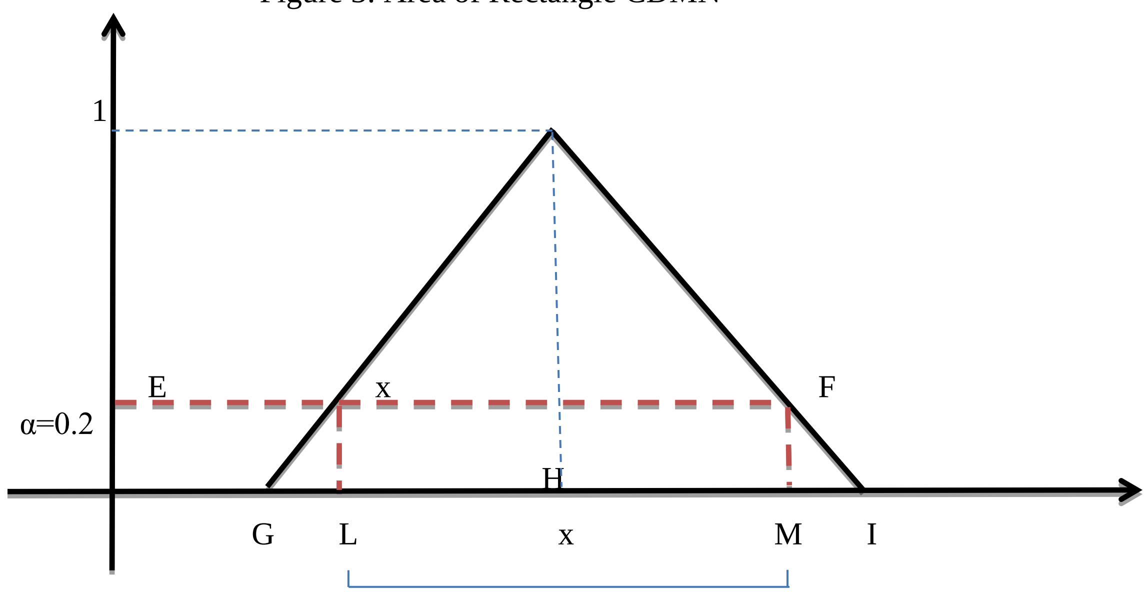

Figure 4: Area of Trapezium EFIG

GI

Figure 5: Trapezium EFIG

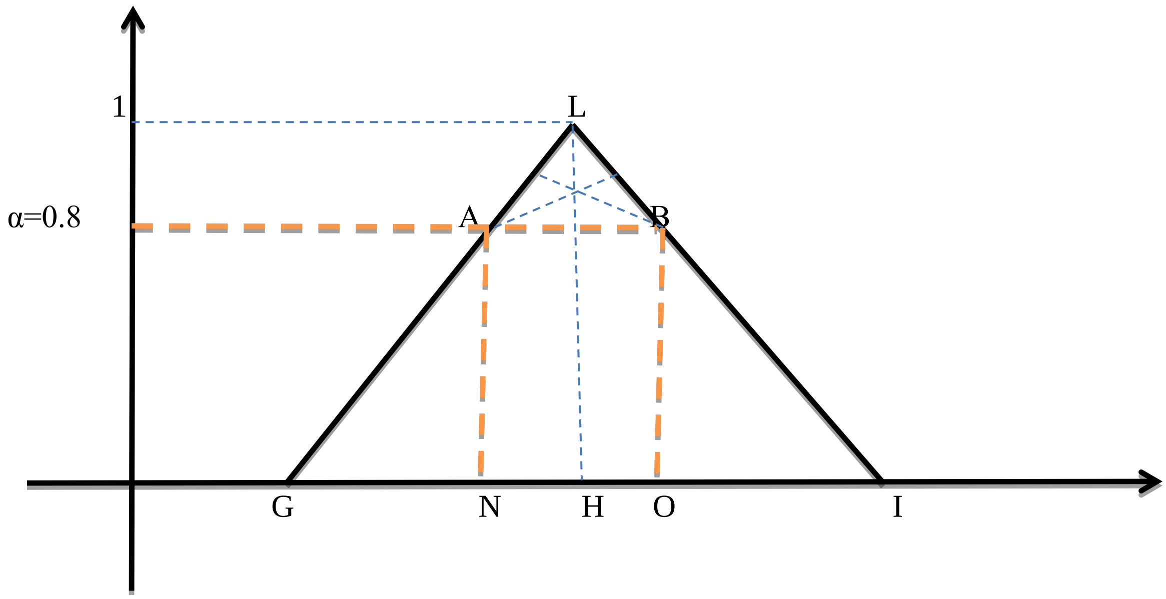

Figure 6: Centroid of triangle LAB

MATLAB Code:

```
clc     %clear the previous output
format short
fprintf('Enter first triangular Fuzzy Number');
a1=input('');
a2=input('');
a3=input('');
A=[a1 a2 a3]   %display the first triangular number
alp=0.5;
fprintf('Enter second triangular Fuzzy Number');
b1=input('');
b2=input('');
b3=input('');
B=[b1 b2 b3]  %display the second triangular number
A1_alpha=[(a2-a1)*alp+a1, -(a3-a2)*alp+a3]
B1_alpha=[(b2-b1)*alp+b1, -(b3-b2)*alp+b3]
A1_area=(alp-0.2)*(A1_alpha(2)-A1_alpha(1)) %area of rectangle for first fuzzy number
B1_area=(alp-0.2)*(B1_alpha(2)-B1_alpha(1)) %area of rectangle for second fuzzy number
alp=0.8;
A2_alpha=[(a2-a1)*alp+a1, -(a3-a2)*alp+a3]
```

```
B2_alpha=[(b2-b1)*alp+b1, -(b3-b2)*alp+b3]
A2_area=(A2_alpha(2)+A2_alpha(1)+a2)/3;    %centroid for first fuzzy number
B2_area=(B2_alpha(2)+B2_alpha(1)+b2)/3;     %centroid for second fuzzy number
alp=0.2;
A3_alpha=[(a2-a1)*alp+a1, -(a3-a2)*alp+a3]
B3_alpha=[(b2-b1)*alp+b1, -(b3-b2)*alp+b3]
A3_area=(alp/2)*(a3-a1)*(A3_alpha(2)-A3_alpha(1)) % area of trapezium for first fuzzy number
B3_area=(alp/2)*(b3-b1)*(B3_alpha(2)-B3_alpha(1)) % area of trapezium for second fuzzy number
area1=A1_area+A2_area+A3_area       %Summation of three value obtained at 0.2,0.5,0.8  for first fuzzy number
area2=B1_area+B2_area+B3_area       %Summation of three value obtained at 0.2,0.5,0.8 for second fuzzy number
if(abs(area1-area2)<0.000001)    % if condition to check order of two fuzzy number
fprintf('A=B\n');
else if(area1-area2>0)
fprintf('B<A\n');
 else
fprintf('A<B\n');
 end
end
```

Comparison of Various Ranking Method:

Here we compare proposed method of order triangular fuzzy number with others by using example. Suppose $\bar{A}$ = (0.1, 0.2, 0.3) and $\bar{B}$ = (0.3, 0.4, 0.9) be any given triangular fuzzy numbers that we want to order

Table 1: Comparison of existing method of ranking two fuzzy numbers with proposed method

Name Ranking Method		$\bar{A}$	$\bar{B}$	Ranking Order
New Method	α = 0.2, 0.5, 0.8	0.2332	0.5455	
Yager	-	0.200	0.500	$\bar{A} \leq \bar{B}$
Chen	β = 0.5	-0.200	0.000	$\bar{A} \leq \bar{B}$
	β = 1	-0.200	0.000	$\bar{A} \leq \bar{B}$

Defuzzification:

If $\bar{x}$ = (a, b, c) beany given triangular fuzzy number then we use mean and center method to Defuzzifyi.e. $x = \frac{a+b+c}{3}$ and method x = b respectively.

If (-a, 0, a) is given fuzzy triangular number then its crisp value is zero.

Method for solving fuzzy transportation problem:

In this section, the crisp transportation problem is converted into fuzzy transportation problem by using Triangular, Pentagonal, and Heptagonal Fuzzy numbers.Three methods are used to obtain the fuzzy transportation costFuzzy North West Corner Method (FNWCM), Fuzzy Matrix Minima Method (FMMM) and Fuzzy Vogel's Approximation Method (FVAM).

Fuzzy Matrix Minima Method (FLCM):

Suppose there are m factories and n warehouses then transportation problem is usually represented in tabular form

Table 2: Fuzzy transportation problem

Destination / Origin	$\bar{\boldsymbol{D}}_1$	$\bar{\boldsymbol{D}}_2$	$\bar{\boldsymbol{D}}_3$	----------	$\bar{\boldsymbol{D}}_n$	Supply
$\bar{\boldsymbol{O}}_1$	$\bar{C}_{11}$	$\bar{C}_{12}$	$\bar{C}_{13}$	----------	$\bar{C}_{1n}$	$\bar{A}_1$
$\bar{\boldsymbol{O}}_2$	$\bar{C}_{21}$	$\bar{C}_{22}$	$\bar{C}_{23}$	----------	$\bar{C}_{2n}$	$\bar{A}_2$
⋮	⋮	⋮	⋮		⋮	⋮
$\bar{\boldsymbol{O}}_m$	$\bar{C}_{m1}$	$\bar{C}_{m2}$	$\bar{C}_{m3}$	----------	$\bar{C}_{mn}$	$\bar{A}_m$
Demand	$\bar{B}_1$	$\bar{B}_2$	$\bar{B}_3$	----------	$\bar{B}_n$	$\sum_{i=1}^{n} \bar{B}_i = \sum_{j=1}^{m} \bar{A}_j$

Step 1: Decide the smallest fuzzy cost in fuzzy transportation table. Let it be $\bar{c}_{ij}$. Find $\bar{x}_{ij}$ = minimum $(\bar{A}_i, \bar{B}_j)$. The following three conditions may arise:

Condition (i): If minimum $(\bar{A}_i, \bar{B}_j) = \bar{A}_i$ then allocate $\bar{x}_{ij} = \bar{A}_i$ in the NWC of $m \times n$ fuzzy transportation table.Ignore the i^{th}row to obtain a new fuzzy transportation table of order (m-1) × n. Replace $\bar{B}_j$ by $\bar{B}_j - \bar{A}_i$ in the obtained fuzzy transportation table. Go to Step 2.

Condition (ii): If minimum $(\bar{A}_i, \bar{B}_j) = \bar{B}_j$ then allocate $\bar{x}_{ij} = \bar{B}_j$ in the NWC of $m \times n$ fuzzy transportation table.Ignore the j^{th} row to obtain a new fuzzy transportation table of order m × (n-1). Replace $\bar{A}_i$ by $\bar{A}_i - \bar{B}_j$ in the obtained fuzzy transportation table. Go to Step2.

Condition (*iii*): If $\overline{A_i} = \overline{B_j}$ then either follow condition(*i*) or condition (*ii*) but not both simultaneously. Go to Step 2.

Step2: RepeatStep 1 for the obtained fuzzy transportation table, until the fuzzy transportation table is reduced into a fuzzy transportation table of order 1 × 1**.**

Step3: Assign all $\bar{x}_{ij}$ in the ij^{th} cell of the given fuzzy transportation table.

Step 4: The obtained IFBFS and initial fuzzy transportation cost are $\bar{x}_{ij}$and $\sum_{i=1}^{m}\sum_{j=1}^{n}\bar{x}_{ij}\,\bar{c}_{ij}$respectively.

Numerical example:

Consider the following crisp transportation problem

Table 3: Crisp transportation problem

Destination / Origin	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$	$\bar{D}_4$	Supply
$\bar{O}_1$	6	3	5	4	22
$\bar{O}_2$	5	9	2	7	15
$\bar{O}_3$	5	7	8	6	8
Demand	7	12	17	9	

Minimum Transportation Cost = 150

Table 4: FLCMfor triangular fuzzy number

Destination / Origin	$\bar{D}_1$	$\bar{D}_2$	$\bar{D}_3$	$\bar{D}_4$	Supply
$\bar{O}_1$	[5,6,7]	[2,3,4] **(11,12,13)**	[4,5,6] **(-2,1,4)**	[3,4,5] **(8,9,10)**	[21,22,23]
$\bar{O}_2$	[4,5,6]	[8,9,10]	[1,2,3] **(14,15,16)**	[6,7,8]	[14,15,16]
$\bar{O}_3$	[4,5,6] **(6,7,8)**	[6,7,8]	[7,8,9] **(-2,1,4)**	[5,6,7]	[6,8,10]
Demand	[6,7,8]	[11,12,13]	[16,17,18]	[8,9,10]	

Minimum Transportation Cost = [22,36,52] + [-8, 5, 24] + [24, 36, 50] + [14, 30, 48]+ [24, 35, 48]+[-14 8 36] = [62, 150, 258] = **156.66**

Final computational value of fuzzy triangular number is**[-9, 0, 9]** = 0

Conclusion and Future Work

This article proposes a simple and concrete method that ranks triangular fuzzy numbers. The MATLAB code is used to get exact ordering of triangular fuzzy number. This method can order normal as well as non-normal fuzzy triangular numbers. This method gives a correct ranking order to the problems for decision making problem under uncertainty since it is easy to calculation and gives acceptable result. In this article the proposed method is applied to fuzzy transportation problem. The crisp transportation problem is convertedinto a fuzzy transportation problem by using various triangular fuzzy numbers and solved by using FLCM (Fuzzy Least Cost Method) methods. Through the numerical example, we can conclude that minimum fuzzy transportation cost obtained from this method is optimum. In Future we want to extend our work by applying this ranking technique to fuzzy optimization techniques such as Network Analysis, Assignment Problem, queuing theory etc.

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