N Generalized Conharmonic Curvature Tensor Of The Locally Conformal Kahler Manifold

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ABSTRACTIn this research, the geometrical properties of one of the almost Hermation manifold classes have been studied. In particular, we study generalized conharmonic curvature tensor (GC curvature tensor) in some aspects Hermeation manifold ($AH-$ manifold) in particular Of the Locally Conformal Kahler manifold W_4 (LCK- manifold) .The main results of this study are stated below:-
1) Proved that this tensor possesses the classical symmetry properties of the Riemannian curvature .
2) Computing components of generalized conharmonic curvature tensor (GC curvature tensor) in LCK-manifold W_4 .
3) Got some results and establish relationships between the components of the tensor in this manifold.

KEYWORDSGeneralized conharmonic curvature tensor, Locally Conformal Kahler Manifold.

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1- Introduction

One of the representative work of differential geometry is an almost Hermititian structure , and conformal transformations of Riemannien structures are the very significant object of study differential geometry,which is keeping the property of smooth harmonic function.It is known,that such transformations have tensor in variant so-called generalized conharmonic curvature tensor. In 1957Y. Ishi[7] has studied conharmonic transformation which is a conformal transformation.

In 1975 a great change was made on these studies by The Russian researcher Kirichenko found an interesting method to study the different classes of almost Hermitian manifold , Kirichenko studied the almost Hermitian manifold by adjoined G-structure space in particular, he defined two tensors which were the structure and virtual tensors [8]. These tensors helped him to find the structure group of almost Hermitian manifold. In 1993, Banaru[2] succed in re- classifying the sixteen classes of almost Hermitian manifold by using the structure and Virtual tensors, which were named Kirichenko's tensors [9].

In 2018 [**15**]Ali A.Shihab and Dhabia`a M. Ali were studied conharmonic curvature tensor of nearly Kahler manifold. In 2019 [1]Ali A.Shihab and MaathAbduallah MohammedAbd were studied some Aspects of the geometry for conharmonic curvature tensor of of Locally Conformal Kahler manifold .In 2019 [14]Ali A.Shihab and Abdulhadi Ahmed Abd were studied generalized conharmonic curvature tensor of Vaisman –Gray manifold.

In this paper we have studied the generalized conharmonic curvature tensor of Locally Conformal Kahler manifold.

2- Preliminaries

let M be smooth manifold of dimension 2n, $C^\infty(M)$ is algebra of smooth function on M; X(M) is the module of smooth vector fields on manifold of M; g= <. , > isRiemanian metrics, ∇ is Riemannian connection of the metrics g on M;d is the operator of exterior differentiation. In the further all manifold, Tensor field, etc. objects are assumed smooth a class $C^\infty(M)$. generalized conharmonic curvature tensor was introducerd will be Reminded G.Gan(1978) [5]. as a tensor of type (4, 0) on n-dimensional Riemannian manifold . An AH - manifold is called a locally conformal Kahler manifold , if foreach point $m \in M$ there exist an open neighborhood U of this point and there exists $f \in C^\infty(M)$ such that $\tilde{U}_f$ is Kahler manifold [6] . We willdenoted to the locally conformal Kahler manifold by L.C.K.

Definition 1: [6]

An AH- manifold M is called a locally conformal Kahler manifold, if for each point $m \in M$ there exists an open neighborhood U of this point and there existsf $\in C^\infty(U)$ such that $\tilde{U}_f$is Kahler manifold .

Remark2:

We shall denoted to the locally conformal Kahler manifold by L.c.k-manifold.

Remark 3: [3]

1)From the Banaru's classification of AH-manifold, the class locally conformal Kahler manifold statistics the following conditions :

$$B^{abc} = 0 \ , \quad B^{ab}_{c} = \propto^{[a}\ \delta^{b]}_{c}$$

2)The value of Riemannian metric is g define bythe form:

1) $gab = g\hat{a}\hat{b} = 0$
2) $g\hat{a}b = \delta^{a}_{b}$
3) $ga\hat{b} = \delta^{b}_{a}$

The structure equation of L.c.k manifoldprovide by the following theorem.

Theorem 4 : [12]

The collection of the structure equation of L.c.K-manifold in the adjoint G-structure space has the following forms:

1)$dw^{a} = w^{a}_{b} \Lambda\, w^{b} + B^{ab}_{c} w^{c} \Lambda\, w_{b}$
2) $dw_{a} = -w^{b}_{a} \Lambda\, w_{b} + B^{c}_{ab} w_{c} \Lambda\, w^{b}$
3)$dw^{a}_{b} = w^{a}_{c} \Lambda\, w^{c}_{b} + A^{ad}_{bc} w^{c} \Lambda\, w_{d} + \{\frac{1}{2} \propto^{a[c}\ \delta^{d]}_{b} + \frac{1}{4} \propto^{a} \propto^{[c}\ \delta^{d]}_{b}\} w_{c} \Lambda w_{d}$

Where $\{w^{i}\}$ are the components of mixture form , $\{w^{i}_{j}\}$ are the components of Rimannian connection of metric g and $\{A^{ad}_{bc}\}$ are system function in the adjoint G-structure space .

Theorem 5: [12]

The component of the Riemannian curvature tensor of L.c.K-manifold in the adjoint G-structure space are given as the following forms:

1) $Rabcd = 0$
2) $R\hat{a}\hat{b}\hat{c}\hat{d} = 0$
3) $R\hat{a}bcd = \propto_{a[c}\ \delta^{b}_{d]} + \frac{1}{2} \propto_{a}\ \propto_{[c}\ \delta^{b}_{d]}$
4) $Ra\hat{b}cd = -\propto_{a[c}\ \delta^{b}_{d]} - \frac{1}{2} \propto_{a}\ \propto_{[c}\ \delta^{b}_{d]}$
5) $Rab\hat{c}d = \propto_{[a|d|}\ \delta^{c}_{b]} - \propto_{[a}\ \delta^{h}_{b]} \propto_{[h}\ \delta^{c}_{d]}$
6) $Rabc\hat{d} = \propto_{[a|c|}\ \delta^{d}_{b]} - \propto_{[a}\ \delta^{h}_{b]} \propto_{[h}\ \delta^{d}_{c]}$
7)$R\hat{a}\hat{b}cd = -2 \propto^{[a}_{[c}\ \delta^{b]}_{d]}$
8)$Rab\hat{c}\hat{d} = 2 \propto^{[c}_{[a}\ \delta^{d]}_{b]}$
9)$R\hat{a}b\hat{c}d = A^{ac}_{bd} - \propto^{[a}\ \delta^{h]}_{d} \propto_{[b}\ \delta^{c}_{h]}$
10)$R\hat{a}bc\hat{d} = A^{ad}_{bc} - \propto^{[a}\ \delta^{h]}_{c} \propto_{[h}\ \delta^{d}_{b]}$
11) $Ra\hat{b}\hat{c}d = -A^{bc}_{ad} + \propto^{[h}\ \delta^{b]}_{d} \propto_{[a}\ \delta^{c}_{h]}$
12)$Ra\hat{b}c\hat{d} = -A^{bd}_{ac} + \propto^{[b}\ \delta^{h]}_{c} \propto_{[a}\ \delta^{d}_{h]}$
13)$R\hat{a}\hat{b}\hat{c}d = -\propto^{[a|c|}\ \delta^{b]}_{a} + \propto^{[a}\ \delta^{b]}_{h} \propto^{[h}\ \delta^{c]}_{d}$
14)$R\hat{a}\hat{b}c\hat{d} = -\propto^{[a|d|}\ \delta^{b]}_{c} + \propto^{[a}\ \delta^{b]}_{h} \propto^{[h}\ \delta^{d]}_{c}$
15)$R\hat{a}b\hat{c}\hat{d} = \propto^{a[c}\ \delta^{d]}_{b} + \frac{1}{2} \propto^{a} \propto^{[c}\ \delta^{d]}_{b}$
16)$Ra\hat{b}\hat{c}\hat{d} = -\propto^{a[c}\ \delta^{d]}_{b} - \frac{1}{2} \propto^{a} \propto^{[c}\ \delta^{d]}_{b}$

Definition (4):[10]

A tensor of type (2,0) which is defined as

$r_{ij} = R^{k}_{ijk} = g^{kl} R_{kijl}$ is called

Theorem 6 :

The component of Ricci tensor of L.c.K-manifold in the adjoint G-structure space are given by the following forms:

1) $rab = \propto c\ _{[b}\delta^{a}_{c]} + \frac{1}{2} \propto c\ \propto_{[b}\ \delta^{a}_{c]} + \propto\ _{[c\,|b|}\delta^{c}_{a]}$
$$-\propto\ _{[c}\delta^{h}_{a]} \propto\ _{[h}\delta^{b}_{c]}$$
2) $r\hat{a}b = -2 \propto\ ^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \propto^{[a}\ \delta^{h]}_{b} \propto\ _{[c}\,\delta^{c}_{h]}$
3) $ra\hat{b} = A^{cb}_{ac} - \propto\ ^{[c}\delta^{h]}_{c} \propto\ _{[a}\,\delta^{c}_{h]} + 2 \propto^{[b}_{[c}\ \delta^{c]}_{a]}$
4) $r\hat{a}\hat{b} = -\propto\ ^{[c|b|}\delta^{a]}_{c} + \propto^{[c}\ \delta^{a]}_{h} \propto^{[h}\ \delta^{b]}_{c} - \propto^{c[b}\ \delta^{c]}_{a}$
$$-\frac{1}{2} \propto^{c}\ \propto^{[b}\ \delta^{c]}_{a}$$

Proof :

By Using the theorem (3) and definition (4) we have:

1)put $i = a\ , j = b$ we obtain

$$rab = R^{k}_{abk}$$
$= R^{c}_{abc} + R^{\hat{c}}_{ab\hat{c}} = R\hat{c}abc + Rcab\hat{c}$
$= \propto_{c[b}\delta^{a}_{c]} + \frac{1}{2}\propto_{c}\propto_{[b}\delta^{a}_{c]} + \propto_{[c|b|}\delta^{c}_{a]} - \propto_{[c}\delta^{h}_{a]}\propto_{[c}\delta^{c}_{h]}$
2) put $i = \hat{a}, j = b$ we obtain
$$r\hat{a}b = R^{k}_{\hat{a}bk}$$
$= R^{c}_{\hat{a}bc} + R^{\hat{c}}_{\hat{a}b\hat{c}} = R\hat{c}\hat{a}bc + Rc\hat{a}b\hat{c}$
$= -2\propto_{c}{}^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \propto^{[a}\delta^{h]}_{b}\propto_{[c}\delta^{c}_{h]}$
3) put $i = a, j = \hat{b}$ we obtain
$$ra\hat{b} = R^{k}_{a\hat{b}k}$$
$= R^{c}_{a\hat{b}c} + R^{\hat{c}}_{a\hat{b}\hat{c}} = R\hat{c}a\hat{b}c + Rca\hat{b}\hat{c}$
$= A^{cb}_{ac} - \propto^{[c}\delta^{h]}_{c}\propto_{[c}\delta^{c}_{h]}\propto_{[a}\delta^{b}_{h]} + 2\propto_{c}{}^{[b}_{[c}\delta^{c]}_{a]}$
4) put $i = \hat{a}, j = \hat{b}$ we obtain
$$r\hat{a}\hat{b} = R^{k}_{\hat{a}\hat{b}k}$$
$= R^{c}_{\hat{a}\hat{b}c} + R^{\hat{c}}_{\hat{a}\hat{b}\hat{c}} = R\hat{c}\hat{a}\hat{b}c + Rc\hat{a}\hat{b}\hat{c}$
$= -\propto^{[c|b|}\delta^{a]}_{c} + \propto^{[c}\delta^{a]}_{h}\propto^{[h}\delta^{b]}_{c} - 2\propto^{c[b}\delta^{c]}_{a} - \frac{1}{2}\propto^{c}\propto^{[b}\delta^{c]}_{a}$

Definition 7:
A generalized Riemannian Curvature tensor on AH- manifold M is a tensor of type (4.0) which is defined as following form:

$$(GR)(X,Y,Z,W) = \frac{1}{16}\{3\,[R(X,Y,Z,W) + R(JX,JY,Z,W)$$
$$+R(X,Y,JZ,JW) + R(JX,JY,JZ,JW)] - R(X,Z,JW,JY) - R(JX,JZ,W,Y) - R(X,W,JY,JZ)$$
$$- R(JX,JW,Y,Z) + R(JX,Z,JW,Y) + R(X,JZ,W,JY) + R(JX,W,Y,JZ) + R(X,JW,JY,Z)]$$

where $R(X,Y,Z,W)$ is Riemannian curvature tensor.
$(X,Y,Z,W) \in Tp\,(M)$ and satisfies the following properties :
1) $(GR)(X,Y,Z,W) = -(GR)(Y,X,Z,W)$
2) $(GR)(X,Y,Z,W) = -(GR)(X,Y,W,Z)$
3) $(GR)(X,Y,Z,W) = (GR)(Z,W,X,Y)$
4) $(GR)(X,Y,Z,W) + (GR)(X,Z,W,Y) + (GR)(X,W,Y,Z) = 0$
Consider this equation in the adjoined G-structure space we obtain:

$$(GR)\,abcd = \frac{1}{16}\{3\,[Rabcd + R\hat{a}\hat{b}cd + Rab\hat{c}\hat{d} + R\hat{a}\hat{b}\hat{e}\hat{d}] - Rac\hat{d}\hat{b} - R\hat{a}\hat{c}db - Rad\hat{b}\hat{c} - R\hat{a}\hat{d}bc$$
$$+ R\hat{a}c\hat{d}b + Ra\hat{c}d\hat{b} + R\hat{a}db\hat{c} + Ra\hat{d}\hat{b}c\}$$

Theorem8:
The components of the generalized Riemannian curvature tensor of L.c.k-manifold in the adjoint G-structure space are given at the following forms:
1) $(GR)abcd = (GR)\hat{a}bcd = (GR)a\hat{b}cd = (GR)ab\hat{c}d =$
$$(GR)abc\hat{d} = (GR)\hat{a}\hat{b}cd = 0$$
2) $(GR)\hat{a}b\hat{c}d = A^{a\,c}_{b\,d} - \propto^{[a}\delta^{h]}_{b}\propto_{[h}\delta^{c}_{d]} - \frac{1}{2}\propto^{[a}_{[d}\delta^{c]}_{b]}$
3) $(GR)\hat{a}bc\hat{d} = \frac{1}{2}A^{a\,d}_{b\,c} - \frac{1}{2}\propto^{[a}\delta^{h]}_{c}\propto_{[h}\delta^{d}_{b]} - \frac{1}{2}\propto^{[a}_{[b}\delta^{d]}_{c]}$
Proof:
1)For $i = a,\ j = b, k = c$ and $l = d$ we obtain

$$(GR)abcd = \frac{1}{16}\{3[\,Rabcd - Rabcd - Rabcd + Rabcd] +$$
$$Racdb + Racdb + Radbc + Radbc - Racdb - Racdb - Radbc - Radbc\,\}$$

$(GR)abcd = 0$
2) For $i = \hat{a},\ j = b, k = c$ and $l = d$ we obtain

$$(GR)\hat{a}bcd = \frac{1}{16}\{3[\,R\hat{a}bcd + R\hat{a}bcd - R\hat{a}bcd - R\hat{a}bcd] +$$
$$R\hat{a}cdb - R\hat{a}cdb + R\hat{a}dbc - R\hat{a}dbc + R\hat{a}cdb - R\hat{a}cdb + R\hat{a}dbc - R\hat{a}dbc\,\}$$

$(GR)\hat{a}bcd = 0$
3) For $i = a,\ j = \hat{b}, k = c$ and $l = d$ we obtain

$$(GR)a\hat{b}cd = \frac{1}{16}\{3\big[\,Ra\hat{b}cd + Ra\hat{b}cd - Ra\hat{b}cd - Ra\hat{b}cd\big] -$$
$$Racd\hat{b} + Racd\hat{b} - Rad\hat{b}c + Rad\hat{b}c + Racd\hat{b} - Rad\hat{b}c + Rad\hat{b}c\,\}$$

$(GR)a\hat{b}cd = 0$

4) For $i = a,\ j = b,\ k = \hat{c}$ and $l = d$ we obtain

$$(GR)ab\hat{c}d = \frac{1}{16}\{3[R ab\hat{c}d - R ab\hat{c}d + R ab\hat{c}d - R ab\hat{c}d] + R a\hat{c}db - R a\hat{c}db - R adb\hat{c} + R adb\hat{c} - R a\hat{c}db + R a\hat{c}db + R adb\hat{c} - R adb\hat{c}\}$$

$(GR)ab\hat{c}d = 0$

5) For $i = a,\ j = b,\ k = c$ and $l = \hat{d}$ we obtain

$$(GR)abc\hat{d} = \frac{1}{16}\{3[R abc\hat{d} - R abc\hat{d} + R abc\hat{d} - R abc\hat{d}] - R ac\hat{d}b + R ac\hat{d}b + R a\hat{d}bc - R a\hat{d}bc + R ac\hat{d}b - R ac\hat{d}b - R a\hat{d}bc + R a\hat{d}bc\}$$

$(GR)abc\hat{d} = 0$

6) For $i = \hat{a},\ j = \hat{b},\ k = c$ and $l = d$ we obtain

$$(GR)\hat{a}\hat{b}cd = \frac{1}{16}\{3[R\hat{a}\hat{b}cd - R\hat{a}\hat{b}cd - R\hat{a}\hat{b}cd + R\hat{a}\hat{b}cd] - R\hat{a}cd\hat{b} - R\hat{a}cd\hat{b} - R\hat{a}d\hat{b}c - R\hat{a}d\hat{b}c + R\hat{a}cd\hat{b} + R\hat{a}cd\hat{b} + R\hat{a}d\hat{b}c + R\hat{a}d\hat{b}c\}$$

$(GR)\hat{a}\hat{b}cd = 0$

7) For $i = \hat{a},\ j = b,\ k = \hat{c}$ and $l = d$ we obtain

$$(GR)\hat{a}b\hat{c}d = \frac{1}{16}\{3[R\hat{a}b\hat{c}d + R\hat{a}b\hat{c}d + R\hat{a}b\hat{c}d + R\hat{a}b\hat{c}d] + R\hat{a}\hat{c}db + R\hat{a}\hat{c}db - R\hat{a}db\hat{c} + R\hat{a}db\hat{c} + R\hat{a}\hat{c}db + R\hat{a}\hat{c}db - R\hat{a}db\hat{c} - R\hat{a}db\hat{c}\}$$

$$(GR)\hat{a}b\hat{c}d = \frac{1}{16}[12\, R\hat{a}b\hat{c}d + 4\, R\hat{a}\hat{c}db - 4R\hat{a}db\hat{c}]$$

$$= \frac{1}{4}[3\, R\hat{a}b\hat{c}d + R\hat{a}\hat{c}db - R\hat{a}db\hat{c}]$$

$$= \frac{1}{4}\{3A^{a\,c}_{b\,d} - 3\propto^{[a}\delta^{h]}_{d}\propto_{[b}\delta^{c}_{h]} - 2\propto^{[a}_{[d}\delta^{c]}_{b]} - A^{a\,c}_{b\,d} - \propto^{[a}\delta^{h]}_{b}\propto_{[h}\delta^{c}_{d]}\}$$

$$= \frac{1}{4}\{3A^{a\,c}_{b\,d} - 3\propto^{[a}\delta^{h]}_{d}\propto_{[b}\delta^{c}_{h]} - 2\propto^{[a}_{[d}\delta^{c]}_{b]} - A^{a\,c}_{b\,d} - \propto^{[a}\delta^{h]}_{d}\propto_{[h}\delta^{c}_{b]}\}$$

$$= \frac{1}{4}\{4A^{a\,c}_{b\,d} - 4\propto^{[a}\delta^{h]}_{b}\propto_{[h}\delta^{c}_{d]} - 2\propto^{[a}_{[d}\delta^{c]}_{b]}\}$$

$$= A^{a\,c}_{b\,d} - \propto^{[a}\delta^{h]}_{b}\propto_{[h}\delta^{c}_{d]} - \frac{1}{2}\propto^{[a}_{[d}\delta^{c]}_{b]}$$

8) For $i = \hat{a},\ j = b,\ k = c$ and $l = \hat{d}$ we obtain

$$(GR)\hat{a}bc\hat{d} = \frac{1}{16}\{3[R\hat{a}bc\hat{d} + R\hat{a}bc\hat{d} + R\hat{a}bc\hat{d} + R\hat{a}bc\hat{d}] - R\hat{a}c\hat{d}b - R\hat{a}cdb + R\hat{a}\hat{d}bc + R\hat{a}\hat{d}bc - R\hat{a}c\hat{d}b + R\hat{a}\hat{d}bc + R\hat{a}\hat{d}bc\}$$

$$(GR)\hat{a}bc\hat{d} = \frac{1}{16}[12R\hat{a}bc\hat{d} + 4R\hat{a}\hat{d}bc - 4R\hat{a}c\hat{d}b] = \frac{1}{4}[3R\hat{a}bc\hat{d} + R\hat{a}\hat{d}bc - R\hat{a}c\hat{d}b]$$

$$= \frac{1}{4}\{3A^{a\,d}_{b\,c} - 3\propto^{[a}\delta^{h]}_{c}\propto_{[h}\delta^{d}_{b]} - 2\propto^{[a}_{[d}\delta^{c]}_{b]} - A^{a\,c}_{b\,d} - \propto^{[a}\delta^{h]}_{b}\propto_{[h}\delta^{c}_{d]}\}$$

$$= \frac{1}{4}\{3A^{a\,c}_{b\,d} - 3\propto^{[a}\delta^{h]}_{d}\propto_{[b}\delta^{c}_{h]} - 2\propto^{[a}_{[b}\delta^{d]}_{c]} - A^{a\,d}_{c\,b} + \propto^{[a}\delta^{h]}_{b}\propto_{[c}\delta^{d}_{h]}\}$$

$$= \frac{1}{4}\{3A^{a\,d}_{b\,c} - 3\propto^{[a}\delta^{h]}_{c}\propto_{[h}\delta^{d}_{b]} - 2\propto^{[a}_{[b}\delta^{d]}_{c]} + A^{a\,d}_{b\,c} + \propto^{[a}\delta^{h]}_{b}\propto_{[c}\delta^{d}_{h]}\}$$

$$(GR)\hat{a}bc\hat{d} = \frac{1}{4}\{2A^{a\,d}_{b\,c} - 2\propto^{[a}\delta^{h]}_{c}\propto_{[h}\delta^{d}_{b]} - 2\propto^{[a}_{[b}\delta^{d]}_{c]}$$

$$= \frac{1}{2}A^{a\,d}_{b\,c} - \frac{1}{2}\propto^{[a}\delta^{h]}_{c}\propto_{[h}\delta^{d}_{b]} - \frac{1}{2}\propto^{[a}_{[b}\delta^{d]}_{c]}$$

Definition 9: [15]

A tensor of type (2,0) which is defined as $r(GR)_{ij} = (GR)^{k}_{ijk}$ is called a generalized Ricci tensor .

Theorem 10:

The components of the generalized Ricci tensor of L.c.k-manifold in the adjoint G-structure space are given as the following forms:

1) $r(GR)ab = 0$
2) $r(GR)\hat{a}\hat{b} = 0$
3) $r(GR)\hat{a}b = A^{a\,c}_{c\,b} - \propto^{[a}\delta^{h]}_{c}\propto_{[h}\delta^{c}_{b]} + \frac{1}{2}\propto^{[a}_{[b}\delta^{c]}_{c]}$

Proof

By Using theorem (8) we and definition (9) we can get the components of generalized Ricci tensor as follows:

1)put $i = a , j = b$ we obtain

$$r(GR)ab = (GR)_{abk}^{k}$$

$= (GR)_{abc}^{c} + (GR)_{ab\hat{c}}^{\hat{c}}$

$= (GR)\hat{c}abc + (GR)cab\hat{c}$

$$r(GR)ab = 0 + 0 = 0$$

2) put $i = \hat{a} , j = \hat{b}$ we obtain

$$r(GR)\hat{a}\hat{b} = R_{\hat{a}\hat{b}k}^{k}$$

$= R_{\hat{a}\hat{b}c}^{c} + R_{\hat{a}\hat{b}\hat{c}}^{\hat{c}}$

$= R\hat{c}\hat{a}\hat{b}c + Rc\hat{a}\hat{b}\hat{c}$

$r(GR)\hat{a}\hat{b} = 0 + 0 = 0$

3) put $i = \hat{a} , j = b$ we obtain

$$r(GR)\hat{a}b = R_{\hat{a}bk}^{k}$$

$= R_{\hat{a}bc}^{c} + R_{\hat{a}b\hat{c}}^{\hat{c}}$

$= R\hat{c}\hat{a}bc + Rc\hat{a}b\hat{c}$

$= A_{cb}^{ac} - \propto^{[a} \delta_{c}^{h]} \propto_{[h} \delta_{b]}^{c} \propto_{[a} \delta_{h]}^{b} - \frac{1}{2} \propto_{[b}^{[a} \delta_{c]}^{c]}$

Definition11: [1]

Let (M, J, g) is a AH- manifold . The conharmonic curvature of the L.C.K-manifold M of type (4.0) which is defined as following form:

$Cijkl = Rijkl - \frac{1}{2(n-1)}[rilgjk - rjkgil - rikgjl]$

Where r , R and g are respectively Ricci tensor, Riemannian curvature tensor and Riemannian metric .

Theorem 12: [1]

The components of the conharmonic tensor of the L.c.k-manifold is given by the following forms:

1) $Cabcd = Rabcd$
2) $C\hat{a}bcd = R\hat{a}bcd$
3) $Ca\hat{b}cd = Ra\hat{b}cd$
4) $Cab\hat{c}d = Rab\hat{c}d$
5) $Cabc\hat{d} = Rabc\hat{d}$
6) $Cab\hat{c}\hat{d} = Rab\hat{c}\hat{d}$
7) $Ca\hat{b}\hat{c}\hat{d} = Ra\hat{b}\hat{c}\hat{d}$
8) $Ca\hat{b}c\hat{d} = Ra\hat{b}c\hat{d} - \frac{1}{(n-1)} r_{[a}^{[d}\delta_{c]}^{b]}$
9) $Ca\hat{b}\hat{c}d = Ra\hat{b}\hat{c}d + \frac{1}{(n+1)} r_{[d}^{[b}\delta_{a]}^{c]}$

And the other are conjugate of them .

Definition 13:

Let (M, J, g) is a AH- manifold . The generalized conharmonic curvature of the L.C.K-manifold M of type (4.0) which is defined as following form:

$(GC)ijkl = (GR)ijkl$

$- \frac{1}{2(n-1)}[r(GR)ilgjk - r(GR)jlgik + r(GR)jkgil - r(GR)ikgjl]$

Where (GR)is the generalized Riemannian curvature tensor,r(GR) is the generalized Ricci tensor and g is Riemannian metric .

Proposition 14:

The generalized conharmoniccurvature of the L.C.K-manifold satisfies all the properties the algebraic :

1) $(GC)(X, Y, Z, W) = -(GC)(Y, X, Z, W)$
2) $(GC)(X, Y, Z, W) = -(GC)(X, Y, W, Z)$
3) $(GC)(X, Y, Z, W) = (GC)(Z, W, X, Y)$
4) $(GC)(X, Y, Z, W) + (GC)(X, Z, W, Y) + (GC)(X, W, Y, Z) = 0$

$$X, Y, Z, W \in X(M)$$

Proof We shall prove just (1)

$(GC)(X, Y, Z, W) = (GR)(X, Y, Z, W)$

$- \frac{1}{2(n-1)}[r(GR)(X, W)g(Y, Z) - r(GR)(Y, W)g(X, Z)$

$+r(GR)(Y, Z)g(X, W) - r(GR)(X, Z)g(Y, W)]$

$$= -(GC)(X,Y,Z,W) + \frac{1}{2(n-1)}[r(GR)(X,W)g(Y,Z) - r(GR)(Y,W)g(X,Z) + r(GR)(Y,Z)g(X,W) - r(GR)(X,Z)g(Y,W)]$$

$$= -(GC)(Y,X,Z,W)$$

The fallowing properties are similarity proved.

Theorem 15: [1]

The components of the generalized conharmonic curvature tensor of the L.c.k-manifold in the adjoint G-structure are given as the following form:

1)$(GC)abcd = (GC)\hat{a}bcd = (GC)a\hat{b}cd = (GC)ab\hat{c}d = \quad (GC)abc\hat{d} = 0$

2)$(GC)\hat{a}\hat{b}cd = \frac{-1}{(n-1)}[r(GR)\ {}^{[a}_{[d}\delta^{b]}_{c]} + r(GR)\ {}^{[b}_{[c}\delta^{a]}_{d]}]$

3)$(GC)\hat{a}b\hat{c}d = (GC)\hat{a}b\hat{c}d - \frac{1}{(n-1)}[r(GR)\ {}^{[a}_{[d}\delta^{c]}_{b]}$

4)$(GC)\hat{a}bc\hat{d} = (GC)\hat{a}bc\hat{d} + \frac{1}{(n-1)}[r(GR)\ {}^{[a}_{[c}\delta^{d]}_{b]}$

Proof: By the theorem (8) and definition (13)we can get components of generalized conharmonic curvature tensor as follows:

1)put $i = a,\ j = b\ , k = c$ and $l = d$ we obtain

$$(GC)abcd = (GC)abcd - \frac{1}{2(n-1)}[r(GC)adgbc - r(GC)bdgac + r(GC)bcgad - r(GC)acgbd]$$

$$= 0 - \frac{1}{2(n-1)}[(0)(0) - (0)(0) + (0)(0) - (0)(0)]$$

$$(GC)abcd = 0$$

2) put $i = \hat{a},\ j = b\ , k = c$ and $l = d$ we obtain

$$(GC)\hat{a}bcd = (GC)\hat{a}bcd - \frac{1}{2(n-1)}[r(GC)\hat{a}dgbc - r(GC)bdg\hat{a}c + r(GC)bcg\hat{a}d - r(GC)\hat{a}cgbd]$$

$$= 0 - \frac{1}{2(n-1)}[r(GC)\hat{a}d(0) - (0)\delta^a_c + (0)\delta^a_d - r(GC)\hat{a}c(0)]$$

$$(GC)\hat{a}bcd = 0$$

3) put $i = a,\ j = \hat{b}\ , k = c$ and $l = d$ we obtain

$$(GC)a\hat{b}cd = (GC)a\hat{b}cd - \frac{1}{2(n-1)}[r(GC)adg\hat{b}c - r(GC)\hat{b}dgac + r(GC)\hat{b}cgad - r(GC)acg\hat{b}d]$$

$$= 0 - \frac{1}{2(n-1)}\left[(0)\delta^b_c - (0)\delta^a_c + r(GC)\hat{b}c(0) - (0)\delta^b_d\right]$$

$$(GC)a\hat{b}cd = 0$$

4) put $i = a,\ j = b\ , k = \hat{c}$ and $l = d$ we obtain

$$(GC)ab\hat{c}d = (GC)ab\hat{c}d - \frac{1}{2(n-1)}[r(GC)adgb\hat{c} - r(GC)bdga\hat{c} + r(GC)b\hat{c}gad - r(GC)a\hat{c}gbd]$$

$$= 0 - \frac{1}{2(n-1)}[(0)\delta^c_b - (0)\delta^c_a + r(GC)b\hat{c}(0) - r(GC)a\hat{c}(0)]$$

$$(GC)ab\hat{c}d = 0$$

5) put $i = a,\ j = b\ , k = c$ and $l = \hat{d}$we obtain

$$(GC)abc\hat{d} = (GC)abc\hat{d} - \frac{1}{2(n-1)}[r(GC)a\hat{d}gbc - r(GC)b\hat{d}gac + r(GC)bcga\hat{d} - r(GC)acgb\hat{d}]$$

$$= 0 - \frac{1}{2(n-1)}\left[r(GC)a\hat{d}(0) - r(GC)b\hat{d}(0) + (0)\delta^d_a - (0)\delta^d_b\right]$$

$$(GC)abc\hat{d} = 0$$

6) put $i = \hat{a},\ j = \hat{b}\ , k = c$ and $l = d$ we obtain

$$(GC)\hat{a}\hat{b}cd = (GC)\hat{a}\hat{b}cd - \frac{1}{2(n-1)}[r(GC)\hat{a}dg\hat{b}c - r(GC)\hat{b}dg\hat{a}c + r(GC)\hat{b}cg\hat{a}d - r(GC)\hat{a}cg\hat{b}d]$$

$$= 0 - \frac{1}{2(n-1)}\left[r(GR)\ {}^{[a}_{[d}\delta^{b]}_{c]} - r(GR)\ {}^{[b}_{[d}\delta^{a]}_{c]} + r(GR)\ {}^{[b}_{[c}\delta^{a]}_{d]} - r(GR)\ {}^{[a}_{[c}\delta^{b]}_{d]}\right]$$

$$= \frac{-1}{(n-1)}(r(GR)\ {}^{[a}_{[d}\delta^{b]}_{c]} + r(GR)\ {}^{[b}_{[c}\delta^{a]}_{d]})$$

7) put $i = \hat{a},\ j = b\ , k = \hat{c}$ and $l = d$ we obtain

$$(GC)\hat{a}b\hat{c}d = (GC)\hat{a}b\hat{c}d - \frac{1}{2(n-1)}[r(GC)\hat{a}dgb\hat{c} - r(GC)bdg\hat{a}\hat{c} + r(GC)b\hat{c}g\hat{a}d - r(GC)\hat{a}\hat{c}gbd]$$

$$= (GC)\hat{a}b\hat{c}d - \frac{1}{2(n-1)}\left[r(GR)\ {}^{[a}_{[d}\delta^{b]}_{c]} - (0)(0) + r(GR)\ {}^{[b}_{[c}\delta^{a]}_{d]} - (0)(0)\right]$$

$$= (GC)\hat{a}b\hat{c}d - \frac{1}{2(n-1)}[r(GR)\ {}^{[a}_{[d}\delta^{c]}_{b]} + r(GR)\ {}^{[c}_{[b}\delta^{a]}_{d]}]$$

$$= (GC)\hat{a}b\hat{c}d - \frac{1}{(n-1)}[r(GR)\ {}^{[a}_{[d}\delta^{c]}_{b]}]$$

8) put $i = a$, $j = b$, $k = c$ and $l = d$ we obtain

$$(GC)\hat{a}bc\hat{d} = (GC)\hat{a}bc\hat{d} - \frac{1}{2(n-1)}[r(GC)\hat{a}\hat{d}gbc - r(GC)b\hat{d}g\hat{a}c + r(GC)bcg\hat{a}\hat{d} - r(GC)\hat{a}cgb\hat{d}]$$

$$= (GC)\hat{a}bc\hat{d} - \frac{1}{2(n-1)}\left[(0)(0) - r(GR)\ {}^{[d}_{[b}\delta^{a]}_{c]} + (0)(0) - r(GR)\ {}^{[c}_{[c}\delta^{d]}_{b]}\right]$$

$$= (GC)\hat{a}bc\hat{d} + \frac{1}{2(n-1)}\left[r(GR)\ {}^{[d}_{[b}\delta^{a]}_{c]} + r(GR)\ {}^{[c}_{[c}\delta^{d]}_{b]}\right]$$

$$= (GC)\hat{a}bc\hat{d} + \frac{1}{(n-1)}[r(GR)\ {}^{[a}_{[c}\delta^{d]}_{b]}]$$

Definition 16: [8]

The set of tensor of type (r,1), linear or an linear in its argument and the sum of the components in the tensor T is called the spectrum of the tensor T, and the tensors themselves are culled elements of the spectrum.

Properties 17:

By definition of a spectrum tensor

$$(GC)(X,Y)Z = (GC)_0(X,Y)Z + (GC)_1(X,Y)Z + (GC)_2(X,Y)Z + (GC)_3(X,Y)Z + (GC)_4(X,Y)Z + (GC)_5(X,Y)Z + (GC)_6(X,Y)Z + (GC)_7(X,Y)Z$$

Tensor $(GC)_0(X,Y)Z$ as non-Zero the components can have only components of the form

$\{(GC)_{0}{}^{a}_{bcd}, (GC)_{0}{}^{\hat{a}}_{\hat{b}\hat{c}\hat{d}}\} = \{(GC)^{a}_{bcd}, (GC)^{\hat{a}}_{\hat{b}\hat{c}\hat{d}}\}$

Tensor $(GC)_1(X,Y)Z$ components of the form

$\{(GC)_{1}{}^{a}_{bc\hat{d}}, (GC)_{1}{}^{\hat{a}}_{\hat{b}\hat{c}d}\} = \{(GC)^{a}_{bc\hat{d}}, (GC)^{\hat{a}}_{\hat{b}\hat{c}d}\}$

Tensor $(GC)_2(X,Y)Z$ components of the form

$\{(GC)_{2}{}^{a}_{b\hat{c}d}, (GC)_{2}{}^{\hat{a}}_{\hat{b}c\hat{d}}\} = \{(GC)^{a}_{b\hat{c}d}, (GC)^{\hat{a}}_{\hat{b}c\hat{d}}\}$

Tensor $(GC)_3(X,Y)Z$ components of the form

$\{(GC)_{3}{}^{a}_{\hat{b}cd}, (GC)_{3}{}^{\hat{a}}_{b\hat{c}\hat{d}}\} = \{(GC)^{a}_{\hat{b}cd}, (GC)^{\hat{a}}_{b\hat{c}\hat{d}}\}$

Tensor $(GC)_4(X,Y)Z$ components of the form

$\{(GC)_{4}{}^{a}_{b\hat{c}\hat{d}}, (GC)_{4}{}^{\hat{a}}_{\hat{b}cd}\} = \{(GC)^{a}_{b\hat{c}\hat{d}}, (GC)^{\hat{a}}_{\hat{b}cd}\}$

Tensor $(GC)_5(X,Y)Z$components of the form

$\{(GC)_{5}{}^{a}_{\hat{b}c\hat{d}}, (GC)_{5}{}^{\hat{a}}_{b\hat{c}d}\} = \{(GC)^{a}_{\hat{b}c\hat{d}}, (GC)^{\hat{a}}_{b\hat{c}d}\}$

Tensor $(GC)_6(X,Y)Z$components of the form

$\{(GC)_{6}{}^{a}_{\hat{b}\hat{c}d}, (GC)_{6}{}^{\hat{a}}_{bc\hat{d}}\} = \{(GC)^{a}_{\hat{b}\hat{c}d}, (GC)^{\hat{a}}_{bc\hat{d}}\}$

Tensor $(GC)_7(X,Y)Z$components of the form

$\{(GC)_{7}{}^{a}_{\hat{b}\hat{c}\hat{d}}, (GC)_{7}{}^{\hat{a}}_{bcd}\} = \{(GC)^{a}_{\hat{b}\hat{c}\hat{d}}, (GC)^{\hat{a}}_{bcd}\}$

Tensor $(GC)_0 = (GC)_0(X,Y)Z$, $(GC)_1 = (GC)_1(X,Y)Z \ldots\ldots (GC)_7 = (GC)_7(X,Y)Z$

we shall name the basic invariants Con harmonic

L.c.k-manifold.

Definition18:

L.c.k-manifold for which $(GC)_i = 0$ is calld

L.c.k-manifold of class $(GC)i$, $\forall\ i = 0,1,\ldots,7$

Theorem 19:

1)L.c.k-manifold of class $(GC)_0$ characterized by identity

$$(GC)(X,Y)Z - (GC)(X,JY)JZ - (GC)(JX,Y)JZ - (GC)(JX,JY)Z - J(GC)(X,Y)JZ - J(GC)(JX,Y)Z - J(GC)(JX,Y)JZ + J(GC)(JX,JY)JZ = 0$$

$$;\ X,Y,Z \in X(M)$$

2) L.c.k-manifold of class $(GC)_1$ characterized by identity

$$(GC)(X,Y)Z + (GC)(X,JY)JZ - (GC)(JX,Y)JZ + (GC)(JX,JY)Z + J(GC)(X,Y)JZ - J(GC)(X,JY)Z - J(GC)(JX,Y)Z - J(GC)(JX,JY)JZ = 0;\ X,Y,Z \in X(M)$$

3) L.c.k-manifold of class $(GC)_2$ characterized by identity

$$(GC)(X,Y)Z - (GC)(X,JY)JZ + (GC)(JX,Y)JZ + (GC)(JX,JY)Z - J(GC)(X,Y)JZ - J(GC)(X,JY)Z + J(GC)(JX,Y)Z - J(GC)(JX,JY)JZ = 0;\ X,Y,Z \in X(M)$$

4) L.c.k-manifold of class $(GC)_3$ characterized by identity

$$(GC)(X,Y)Z + (GC)(X,JY)JZ + (GC)(JX,Y)JZ - (GC)(JX,JY)Z - J(GC)(X,Y)JZ + J(GC)(X,JY)Z + J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ = 0 \quad ; X,Y,Z \in X(M)$$

5) L.c.k-manifold of class $(GC)_4$ characterized by identity

$$(GC)(X,Y)Z + (GC)(X,JY)JZ + (GC)(JX,Y)JZ - (GC)(JX,JY)Z + J(GC)(X,Y)JZ - J(GC)(X,JY)Z - J(GC)(JX,Y)Z - J(GC)(JX,JY)JZ = 0 \; ; X,Y,Z \in X(M)$$

6) L.c.k-manifold of class $(GC)_5$ characterized by identity

$$(GC)(X,Y)Z - (GC)(X,JY)JZ + (GC)(JX,Y)JZ + (GC)(JX,JY)Z + J(GC)(X,Y)JZ + J(GC)(X,JY)Z - J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ = 0 \; ; X,Y,Z \in X(M)$$

7) L.c.k-manifold of class $(GC)_6$ characterized by identity

$$(GC)(X,Y)Z + (GC)(X,JY)JZ - (GC)(JX,Y)JZ + (GC)(JX,JY)Z + J(GC)(X,Y)JZ - J(GC)(X,JY)Z + J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ = 0 \; ; X,Y,Z \in X(M)$$

8) L.c.k-manifold of class $(GC)_7$ characterized by identity

$$(GC)(X,Y)Z - (GC)(X,JY)JZ - (GC)(JX,Y)JZ - (GC)(JX,JY)Z + J(GC)(X,Y)JZ + J(GC)(X,JY)Z + J(GC)(JX,Y)Z - J(GC)(JX,JY)JZ = 0 \; ; X,Y,Z \in X(M)$$

Proof :

The manifold of class $(GC)_0$ is characterized by a condition

$(GC)_0 abcd = 0$ or $(GC)abcd = 0$,

$$\text{i.e } [(GC)(\varepsilon c, \varepsilon d)\varepsilon b] \quad a\,\varepsilon\, a = 0$$

As σ -aprojector on $D_J^{\sqrt{-1}}$ that

$$\sigma^0\{(GC)(HR)(\sigma X, \sigma Y)\sigma Z\} = 0, \text{ i.e}\left(id - \sqrt{-1}J\right)\left\{(GC)\left(X - \sqrt{-1}JX, Y - \sqrt{-1}JY\right)\left(Z - \sqrt{-1}JZ\right)\right\} = 0$$

Removing the torackets we shall receive

$(GC)(X,Y)Z - (GC)(X,JY)JZ - (GC)(JX,Y)JZ - (GC)(JX,JY)Z - J(GC)(X,Y)JZ - J(GC)(X,JY)Z - J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ - \sqrt{-1}\{(GC)(X,Y)Z + (GC)(X,JY)Z + (GC)(JX,Y)Z - (GC)(JX,JY)JZ\} - \{J(GC)(X,Y)Z - J(GC)(X,JY)JZ - J(GC)(JX,Y)JZ - J(GC)(JX,JY)Z\} = 0$,i.e

1) $(GC)(X,Y)Z - (GC)(X,JY)JZ - (GC)(JX,Y)JZ - (GC)(JX,JY)Z - J(GC)(X,Y)JZ - J(GC)(X,JY)Z - J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ = 0$

Thus, L.c.k-manifold of class $(GC)_0$ are characterized by identity

2) $(GC)(X,Y)JZ + (GC)(X,JY)Z + (GC)(JX,Y)Z - (GC)(JX,JY)JZ + J(GC)(X,Y)Z - J(GC)(X,JY)JZ - J(GC)(JX,Y)JZ - J(GC)(JX,JY)Z = 0 \;, X,Y,Z \in X(M)$

These equality are equivalent, the second equality turns out from the first replacement Z on JZ

$$(GC)(X,Y)Z - (GC)(X,JY)JZ - (GC)(JX,Y)JZ - (GC)(JX,JY)Z - J(GC)(X,Y)JZ - J(GC)(X,JY)Z - J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ = 0 \;, X,Y,Z \in X(M)$$

Similarly, considering L.c.k-manifold of class

$$(GC)_1, \ldots \ldots (GC)_7$$

Theorem20:

The following inclusion relation have been found:

1) $(GC)_0 = (GC)_3$

2) $(GC)_1 = (GC)_2$

3) $(GC)_4 = (GC)_7$

4) $(GC)_5 = (GC)_6$

proof

we shall prove (1)

$$(GC)_0 = (GC)(X,Y)Z - (GC)(X,JY)JZ - (GC)(JX,Y)JZ - (GC)(JX,JY)Z - J(GC)(X,Y)JZ - J(GC)(X,JY)Z - J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ \ldots\ldots\ldots\ldots (1)$$

$$\begin{aligned}(GC)_0 = {} & (GC)(X,Y)Z - (GC)\left(X, -\sqrt{-1}\,JY\right)\left(-\sqrt{-1}\right)JZ - (GC)\left(-\sqrt{-1}JX, Y\right)\left(-\sqrt{-1}\right)JZ \\ & - (GC)\left(-\sqrt{-1}JX, -\sqrt{-1}JY\right)Z - \left(-\sqrt{-1}\right)J(GC)(X,Y)\left(-\sqrt{-1}\right)JZ \\ & - \left(-\sqrt{-1}\right)J(GC)\left(X, -\sqrt{-1}JY\right)Z - \left(-\sqrt{-1}\right)J(GC)\left(-\sqrt{-1}JX, Y\right)Z \\ & + \left(-\sqrt{-1}\right)J(GC)\left(-\sqrt{-1}JX, -\sqrt{-1}JY\right)\left(-\sqrt{-1}\right)JZ\end{aligned}$$

$$(GC)_0 = (GC)(X,Y)Z + (GC)(X,JY)JZ + (GC)(JX,Y)JZ - (GC)(JX,JY)Z - J(GC)(X,Y)JZ + JGC(GC)(X,JY)Z + J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ \ldots\ldots\ldots\ldots (2)$$

From(1) and (2) we get

$$(GC)_0 = (GC)(X,Y)Z - (GC)(JX,JY)Z - J(GC)(X,Y)JZ + J(GC)(JX,JY)JZ \ldots\ldots\ldots\ldots (3)$$

$$(GC)_3 = (GC)(X,Y)Z + (GC)(HR)(X,JY)JZ + (GC)(JX,Y)JZ - (GC)(JX,JY)Z - J(GC)(X,Y)JZ + J(GC)(X,JY)Z + J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ \ldots\ldots\ldots\ldots (4)$$

$$
\begin{aligned}
(GC)_3 = {} & (GC)(X,Y)Z + (GC)\left(X, -\sqrt{-1}\,JY\right)\left(-\sqrt{-1}\right)JZ + (GC)\left(-\sqrt{-1}JX, Y\right)\left(-\sqrt{-1}\right)JZ \\
& - (GC)\left(-\sqrt{-1}JX, -\sqrt{-1}JY\right)Z - \left(-\sqrt{-1}\right)J(GC)(X,Y)\left(-\sqrt{-1}\right)JZ \\
& + \left(-\sqrt{-1}\right)J(GC)\left(X, -\sqrt{-1}JY\right)Z + \left(-\sqrt{-1}\right) J(GC)\left(-\sqrt{-1}JX, Y\right)Z \\
& + \left(-\sqrt{-1}\right)J(GC)\left(-\sqrt{-1}JX, -\sqrt{-1}JY\right)\left(-\sqrt{-1}\right)JZ
\end{aligned}
$$

$$
\begin{aligned}
(GC)_3 = {} & (GC)(X,Y)Z - (GC)(X,JY)JZ - (GC)(JX,Y)JZ - (GC)(JX,JY)Z - J(GC)(X,Y)JZ - J(GC)(X,JY)Z \\
& - J(GC)(JX,Y)Z + J(GC)(JX,JY)JZ \ldots\ldots\ldots\ldots\ldots..(5)
\end{aligned}
$$

from (4) and (5) we get.

$$(GC)_3 = (GC)(X,Y)Z - (GC)(JX,JY)Z - J(GC)(X,Y)JZ + J(GC)(JX,JY)JZ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(6)$$

From (3) and (6) we get $(GC)_0 = (GC)_3$

In the same way we brave:

$(GC)_1 = (GC)_2$

$(GC)_4 = (GC)_7$

$(GC)_5 = (GC)_6$

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