

Strong Reset Fuzzy Automata

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Abstract

Strong necks, strong trap, strong trap-reset fuzzy automata and strong mergeable are introduced. We prove that strong neck is non-empty then it is subautomata and prove that if strong neck exists, then it is kernel.

Key words: Fuzzy automata (FA), Strong Reset Fuzzy Automata

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1 Introduction

Fuzzy sets were introduced by Zadeh in 1965 [5] and it is used in many applications. Fuzzy automaton was introduced by Wee [4]. Directable automata are also called reset automata. Reset automata has important applications in computer science. T. Petkovi et al [3] introduced and studied trap-directable, trapped automata etc. V. Karthikeyan et al [1] introduced and studied μ -necks of fuzzy automata. In this paper, strong necks, strong trap, strong trap-reset, strong mergeable of fuzzy automata are introduced and discuss their properties. We prove that strong neck is non-empty then it is subautomata and if strong neck exists in fuzzy automata, then it is kernel. Also we prove the necessary and sufficient condition for strong reset fuzzy automata.

2 Preliminaries

2.1 Definition [2]

A fuzzy automata is $F = (T, I, \beta)$ where,

T – set of states

I – set of input symbols

β – fuzzy transition function in $T \times I \times T \rightarrow [0,1]$

2.2 Definition

Let $F = (T, I, \beta)$ be FA and $t_i \in Q$. The FA F_1 is generated by t_i is $\langle t_i \rangle$,

$\langle t_i \rangle = \{t_s \mid \beta(t_i, u, t_s) > 0\}$. F_1 is called least subautomata.

2.3 Definition

Let $F = (T, I, \beta)$ be FA and $T_1 \subseteq T, T_1 \neq \emptyset$. Then F_1 generated by T_1 is $\langle T_1 \rangle = \{t_s \mid \beta(t_i, y, t_s) > 0, t_i \in T_1\}$. F_1 is called least subautomata having T_1 .

2.4 Definition

Let $F = (T, I, \beta)$ be FA and $t_i \in T$ is strong neck if $\exists y \in I^*, \forall t_i \in T, \beta^*(t_i, y, t_s) = \eta > 0, \eta = \max$. Weight in $F, \eta \in [0,1]$. Y is called strong reset string of F and F is called strong reset fuzzy automaton. The set of all strong neck is denoted by $SN(F)$. The set of all strong reset strings of F is denoted by $SRS(F)$.

2.5 Definition

Let $F = (T, I, \beta)$ be FA and $t_i \in T$ is called strong trap if $\beta^*(t_i, y, t_i) = \eta > 0, \forall y \in I^*$. The set of all strong traps of F is denoted by $STR(F)$. F is strong trapped fuzzy automata, $\exists y \in I^*$ such that $\beta^*(t_i, y, t_j) = \eta, t_j \in STR(F)$.

2.6 Definition

Let $F = (T, I, \beta)$ be FA. If F has a one strong neck then F is called a strong trap-reset fuzzy automata.

2.7 Definition

Let $F = (T, I, \beta)$ be FA. Two states $t_a, t_b \in T$ are called strong mergeable if $\exists y \in I^*$ and $t_c \in T$ such that $\beta^*(t_a, y, t_c) = \eta > 0 \Leftrightarrow \beta^*(t_b, y, t_c) = \eta > 0$.

3 Properties of Strong Necks of Reset Fuzzy Automata

Theorem 3.1 Let $F = (T, I, \beta)$ be a FA. If $SN(F) \neq \emptyset$ then $SN(F)$ is subautomata.

Proof.

Let $F = (T, I, \beta)$ be a FA. Let $t_j \in SN(F)$, $y \in I^*$ and t_j is strong neck. Then $\forall t_i \in T$ we have $\beta^*(t_i, xy, t_l) = \bigwedge_{t_j \in T} \{\beta^*(t_i, x, t_j), \beta^*(t_j, y, t_l)\} = \eta > 0$. Hence $t_i \in SN(F)$. Therefore, $SN(F)$ is a subautomaton of F .

Theorem 3.2 Let $F = (T, I, \beta)$ be strong reset fuzzy automata. Then $SN(F)$ is kernel of F .

Proof.

Let $F = (T, I, \beta)$ be strong reset fuzzy automata. Let $t_l \in SN(F)$ and $t_k \in T$. Then $\beta^*(t_k, y, t_l) = \eta, \forall y \in RS(F)$. Hence $t_l \in \langle t_k \rangle$. So, $SN(F) \subseteq \langle t_k \rangle, \forall t_k \in T$. Therefore, $SN(F)$ is kernel of F .

Theorem 3.3 Let $F = (T, I, \beta)$ be strong reset fuzzy automata. If F' is subautomata of F then F' is strong reset fuzzy automata and $SN(F') = SN(F)$.

Proof.

Let $F = (T, I, \beta)$ be a FA and $SN(F) \subseteq F'$. Hence F' is strong reset fuzzy automata. $SN(F)$ is kernel of F and $SN(F) \subseteq SN(F')$. Also $SN(F')$ is kernel then $SN(F') \subseteq SN(F)$. Hence $SN(F') = SN(F)$.

Theorem 3.4 Let $F = (T, I, \beta)$ is strong reset fuzzy automata iff all pairs are strong mergeable.

Proof.

Let F is strong reset fuzzy automata. Then $\forall t_l \in T$ we have $\beta^*(t_l, y, t_k) = \eta > 0$.

Let $t_a, t_b \in T$. By strong mergeable, $\beta^*(t_a, y, t_k) = \eta \Leftrightarrow \beta^*(t_b, y, t_k) = \eta$. Hence t_a, t_b are strong mergeable.

Conversely, suppose F is not a strong reset fuzzy automaton. Then assume all states are strong mergeable in two states t_c and t_d in T . Then $\exists y_1 \in I^*$ such that $\beta^*(t_i, y_1, t_c) = \eta > 0$ and $\beta^*(t_j, y_1, t_d) = \eta > 0$, for t_i 's, t_j 's $\in T$.

Now, consider t_c and t_d . Then by hypothesis, t_c and t_d are strong mergeable. Then \exists a string $y_2 \in I^*$ and $t_f \in T$ such that $\beta^*(t_c, y_2, t_f) = \eta > 0 \Leftrightarrow \beta^*(t_d, y_2, t_f) > 0$. Now, $\beta^*(t_i, y_1 y_2, t_f) = \eta > 0, \forall t_i \in T$, which is a contradiction to our assumption. Hence, F is a strong reset fuzzy automaton.

4 Conclusion

Strong necks, strong trap, strong trap-reset fuzzy automata and strong mergeable are introduce and discuss their properties. We prove that strong neck is non-empty then it is subautomata, if strong neck exists then it is kernel. Finally prove that necessary and sufficient condition for strong reset fuzzy automata.

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