Research Article

Δ −Synchronization Of Interval Nutrosophic Automata

N. Mohanarao [1] V. Karthikeyan [2]

[1]Department of Mathematics, Government College of Engineering,Bodinayakkanur, Tamilnadu, India.
[2]Department of Mathematics, Government College of Engineering,Dharmapuri Tamilnadu, India.

Article History: Received: 11 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 23 May 2021

Abstract
The purpose of this paper is to study Δ −Synchronization of interval neutrosophic automata and their characterizations.

Key words: Interval neutrosophic automaton (INA), Δ −Synchronization.

AMS Mathematics subject classification: 03D05, 20M35, 18 B20, 68Q45, 68Q70, 94 A45

1 Introduction

The neutrosophic set was introduced by Florentin Smarandache in 1999 [6]. Fuzzy sets was introduced by Zadeh in 1965 [8]. Bipolar fuzzy sets, YinYang, bipolar fuzzy sets, NPN fuzzy set were introduced by W. R. Zhang in [9, 10, 11]. A NS N is classified by a Truth T_N, Indeterminacy I_N, and Falsity membership F_N where T_N, I_N, and F_N are real standard and non-standard subsets of $]0^-, 1^+[$. Fuzzy automaton was introduced by Wee [7]. The INA was introduced by Tahir Mahmood [4]. Retrievability, subsystem, and strong subsystems of INA are studied in the papers [1, 2, 3]. Here, We study the characterizations of Δ −synchronization of INA.

2 Preliminaries

2.1 Definition [5]

A FA is triple F = (T, I, S) where T, I are set of states, set of input symbols and
S is transition function in $T \times I \times T \rightarrow [0, 1]$.

2.2 Definition [4]

Let U be universal set. A NS S in U is classified as truth K_s, an indeterminacy L_s and a falsity values M_S where K_s, L_s, and M_S are real standard or non- standard subsets of $]0^- \ 1^+[$. $S = \{\langle z, (K_s(z), L_s(z), M_S(z)\rangle, z \in U, K_s, L_s\ M_S \in\ ]0^-\ 1^+[\ \}$ and
$0^- \leq \sup K_s(z) + \sup L_s(z) + \sup M_S(z) \leq 3^+$. We take values [0, 1] instead of
$]0^-, 1^+[$.

2.3 Definition [4]

Let $F = (T, I\ S)$ be INA. T and I are set of states and input symbols respectively, and S =
$\{\langle K_s(z), L_s(z), M_S(z)\rangle\}$ is an INS in $T \times I \times T$. The set of all strings I is denote by I^*.
The empty string is denoted by ϵ and the length of $z \in I^*$ is denoted by |z|.

2.4 Definition [4]

Let $F = (T, I\ S)$ be INA. Define an INS $s^* = \{\langle K_{s*}(z), L_{s*}(z), M_{S*}(z)\rangle\}$ in $T^* \times I^* \times T$ by

$K_s(z)(t_a, \epsilon, t_b) = \begin{cases} [1,1]\ if\ t_a = t_b \\ [0,0]\ if\ t_a \neq t_b \end{cases}$, $L_s(z)(t_a, \epsilon, t_b) = \begin{cases} [0,0]\ if\ t_a = t_b \\ [1,1]\ if\ t_a \neq t_b \end{cases}$, and

$M_s(z)(t_a, \epsilon, t_b) = \begin{cases} [0,0]\ if\ t_a = t_b \\ [1,1]\ if\ t_a \neq t_b \end{cases}$

$K_{s*}(t_a, zz', t_b) = \vee_{t_r \in T} [K_{s*}(t_a, z, t_r) \wedge K_{s*}(t_r, z', t_b)] > [0, 0]$

$L_{s*}(t_a, zz', t_b) = \wedge_{t_r \in T} [L_{s*}(t_a, z, t_r) \vee L_{s*}(t_r, z', t_b)] < [1, 1]$

$M_{s*}(t_a, zz', t_b) = \wedge_{t_r \in T} [M_{s*}(t_a, z, t_r) \vee M_{s*}(t_r, z', t_b)] < [1, 1]\ \ \forall t_a, t_b \in T, z \in I^*$ and $z' \in I$.

3 Δ −Synchronization of Interval Neutrosophic Automata

3.1 Definition

Let F = (T, I, S) be an IVNA. F is called deterministic IVNA, $\forall\ t_a \in T$ and z ∈ I
∃ unique state t_b such that $K_{s*}(t_a$, z, $t_b)$ > [0, 0], $L_{s*}\_(t_a$, z, $t_b)$ < [1, 1], $M_{s*}\_(t_a$, z, $t_b)$ < [1, 1].

3.2 Definition

Let F = (T, I, S) be an IVNA and $\Theta = T_1, T_2, \ldots\ldots T_z$ be a partition of T. If $K_{s*}(t_a$, z, $t_b)$ > [0, 0], $L_{s*}\_(t_a$, z, $t_b)$ < [1, 1], $M_{s*}\_(t_a$, z, $t_b)$ < [1, 1] for some z ∈ I then $t_a \in T_S$ and $t_b \in T_{S+1}$. Then Θ is periodic partition of order $z \geq 2$. An INA F is periodic of period $z \geq 2$ iff $z = Maxcard(\Theta)$, maximum is consider all periodic partitions Θ of F. F has no periodic partition, then F is called aperiodic.

Note.
Throughout this paper we consider aperiodic INA.

3.3 Definition

Let F = (T, I, S) be an IVNA. Two states t_a, t_b interval neutrosophic stability related (INSR) denoted by $t_a\ \Omega\ t_b$, for any string z ∈ I^*, $t_k \in T$ such that

$K_{s*}(t_a, zz', t_k)$ > [0, 0] ⇔ $K_{s*}(t_b, zz', t_k)$ > [0, 0]

$L_{s*}(t_a, zz', t_k)$ < [1, 1] ⇔ $L_{s*}(t_b, zz', t_k)$ < [1, 1]

$M_{s*}(t_a, zz', t_k)$ > [0, 0] ⇔ $M_{s*}(t_b, zz', t_k)$ < [1, 1]

3.4 Example

Let F = (T, I, S) be an IVNA, where $\{T = T_1, T_2, T_3, T_4\}$ I = $\{z, z'\}$ and S are defned as below.

$$(K_s, L_s, M_s)(t_1, z, t_4) = \{[0.3,0.4], [0.4,0.5], [0.6,0.8]\}$$
$$(K_s, L_s, M_s)(t_1, z', t_2) = \{[0.1,0.2], [0.3,0.4], [0.7,0.8]\}$$
$$(K_s, L_s, M_s)(t_2, z, t_3) = \{[0.2,0.3], [0.5,0.6], [0.8,0.9]\}$$
$$(K_s, L_s, M_s)(t_2, z', t_4) = \{[0.7,0.8], [0.3,0.4], [0.2,0.3]\}$$
$$(K_s, L_s, M_s)(t_3, z, t_2) = \{[0.6,0.7], [0.4,0.5], [0.3,0.4]\}$$
$$(K_s, L_s, M_s)(t_3, z', t_4) = \{[0.5,0.6], [0.4,0.5], [0.2,0.3]\}$$
$$(K_s, L_s, M_s)(t_4, z, t_1) = \{[0.8,0.9], [0.2,0.3], [0.1,0.2]\}$$
$$(K_s, L_s, M_s)(t_4, z', t_3) = \{[0.3,0.4], [0.4,0.5], [0.6,0.8]\}$$

For any string $v \in I^*$, there exists a string $zz'z' \in I^*$ such that
$K_{s*}(t_1, vzz'z', t_k)$ > [0, 0] ⇔ $K_{s*}(t_4, vzz'z', t_k)$ >[0,0]
$L_{s*}(t_1, vzz'z', t_k)$ < [1, 1] ⇔ $L_{s*}(t_4, vzz'z', t_k)$ <[1,1]
$M_{s*}(t_1, vzz'z', t_k)$ < [1, 1] ⇔ $M_{s*}(t_4, vzz'z', t_k)$ <[1,1] and
$K_{s*}(t_2, vzz'z', t_l)$ > [0, 0] ⇔ $K_{s*}(t_3, vzz'z', t_l)$ >[0,0]
$L_{s*}(t_2, vzz'z', t_l)$ < [1, 1] ⇔ $L_{s*}(t_3, vzz'z', t_l)$ <[1,1]
$M_{s*}(t_2, vzz'z', t_l)$ < [1, 1] ⇔ $K_{s*}(t_3, v\ zz'z', t_l)$ <[1,1].
The states t_1, t_4 and t_2, t_3 are interval neutrosophic stability related.

3.5 Definition

Let F = (T, I, S) be an IVNA. F is called Δ −Synchronization if ∃ a string $z \in I^*$, $t_b \in T$ and a real number Δ with Δ ∈ (0,1] such that $K_{s*}(t_a, z, t_b)$ ≥ Δ >[0,0], $L_{s*}(t_a, z, t_b)$ ≤ Δ < [1,1], $M_{s*}(t_a, z, t_b)$ ≤ Δ < [1,1] ∀ $t_a \in T$.

4 Algorithm

Let F = (T, I, S) be an IVNA.
1) Find the equivalence classes of the states T using INSR.
2) Construct the quotient $INA\ G$ by considering each equivalence class as a state.
3) Relabel the quotient INA along with neutrosophic values G into G' keeping the stability class.
4) Construct New $INA\ F'$ from G'.
5) $INA\ G'$ gives the synchronized string.

4.1 Example

From Example 3.4 and the quotient $INA\ G$ is as follows.

$$(K_{s*}, L_{s*}, M_{s*})(t_1t_4, z, t_1t_4) = \{[0.3,0.4], [0.4,0.5], [0.6,0.8]\}$$
$$(K_{s*}, L_{s*}, M_{s*})(t_1t_4, z', t_2t_3) = \{[0.1,0.2], [0.4,0.5], [0.7,0.8]\}$$
$$(K_{s*}, L_{s*}, M_{s*})(t_2t_3, z, t_2t_3) = \{[0.2,0.3], [0.5,0.6], [0.8,0.9]\}$$
$$(K_{s*}, L_{s*}, M_{s*})(t_2t_3, z', t_1t_4) = \{[0.5,0.6], [0.4,0.5], [0.2,0.3]\}$$

Relabled quotient $INA\ G'$ is as follows

$$(K_{s*}, L_{s*}, M_{s*})(t_1t_4, z', t_1t_4) = \{[0.1,0.2], [0.4,0.5], [0.7,0.8]\}$$
$$(K_{s*}, L_{s*}, M_{s*})(t_1t_4, z, t_2t_3) = \{[0.3,0.4], [0.4,0.5], [0.6,0.8]\}$$
$$(K_{s*}, L_{s*}, M_{s*})(t_2t_3, z, t_2t_3) = \{[0.2,0.3], [0.5,0.6], [0.8,0.9]\}$$
$$(K_{s*}, L_{s*}, M_{s*})(t_2t_3, z', t_1t_4) = \{[0.5,0.6], [0.4,0.5], [0.2,0.3]\}$$

Relabled $INA\ F'$ from $G'$is as follows

$$(K_s, L_s, M_s)(t_1, z', t_4) = \{[0.5,0.6], [0.4,0.5], [0.2,0.3]\}$$
$$(K_s, L_s, M_s)(t_1, z, t_2) = \{[0.3,0.4], [0.4,0.5], [0.6,0.8]\}$$
$$(K_s, L_s, M_s)(t_2, z, t_3) = \{[0.2,0.3], [0.5,0.6], [0.8,0.9]\}$$
$$(K_s, L_s, M_s)(t_2, z', t_4) = \{[0.7,0.8], [0.3,0.4], [0.2,0.3]\}$$
$$(K_s, L_s, M_s)(t_3, z, t_2) = \{[0.6,0.7], [0.4,0.5], [0.3,0.4]\}$$
$$(K_s, L_s, M_s)(t_3, z', t_4) = \{[0.8,0.9], [0.2,0.3], [0.1,0.2]\}$$
$$(K_s, L_s, M_s)(t_4, z, t_3) = \{[0.3,0.4], [0.4,0.5], [0.6,0.8]\}$$
$$(K_s, L_s, M_s)(t_4, z', t_1) = \{[0.3,0.4], [0.4,0.5], [0.6,0.8]\}$$

In the relabeled INA there exists a string $zz' \in I^*$ in F' such that

$K_{s*}(t_i, zz', t_4) > [0, 0]$, $L_{s*}(t_i, zz', t_4) < [1, 1]$ and $M_{s*}(t_i, zz', t_4) < [1, 1]\ \forall\ t_i \in T$.

5. Procedure for finding Δ −Synchronized String of Interval Neutrosophic Automata

Let F = (T, I, S) be an INA. We define another INA as follows:

$F_S = (2^T, I, M_S, T, D \subseteq T)$ where

T- Starting state on F_S, D- set of all final states on F_S, $M_S -$ Interval neutrosophic transition function and is defined by

$$K_{M_S}(T_a, z, T_b) = \wedge \{(K_s(t_a, z, t_b)\} > [0, 0]$$

$$L_{M_S}(T_a, z, T_b) = \vee \{(L_s(t_a, z, t_b)\} < [1, 1]$$

$M_{M_S}(T_a, z, T_b) = \vee \{(M_s(t_a, z, t_b)\} < [1, 1]$, $t_a \in T_a$, $t_b \in T_b$, T_a, $T_b \in 2^T$ for $z \in I$.

M_S is a deterministic INA and a string $z \in I$ is $\Delta -$ synchronized in F iff $\exists$ a singleton subsets $T_t \in 2^T$ such that

$K_{M_{S*}}(T_a, z, T_t) > [0, 0]$, $L_{M_{S*}}(T_a, z, T_t) < [1, 1]$ and $M_{M_{S*}}(T_a, z, T_b) < [1, 1]$.

6 Conclusion

Δ −Synchronization of INA are introduce, algorithm is given for finding Synchronized string using interval neutrosophic stability relation. Finally procedure is given for finding synchronized string.

References

[1] V. Karthikeyan, and R. Karuppaiaya, Retrievability in interval neutrosophic automata, Advances in Mathematics: Scienti_c Journal, 9(4) (2020), 1637-1644.

[2] V. Karthikeyan, and R. Karuppaiaya, Subsystems of interval neutrosophic automata, Advances in Mathematics: Scienti_c Journal, 9(4) (2020), 1653-1659.

[3] V. Karthikeyan, and R. Karuppaiaya, Strong subsystems of interval neutrosophic automata, Advances in Mathematics: Scienti_c Journal, 9(4) (2020), 1645-1651.

[4] T. Mahmood, and Q. Khan, Interval neutrosophic finite switchboard state machine, Afr. Mat. 20(2)(2016),191-210.

[5]J. N. Mordeson, D. S. Malik, *Fuzzy automata and Languages*, CRC Press, Chapman & Hall, (2002).

[6] F. Smarandache, A Unifying Field in Logics, Neutrosophy: Neutrosophic Probability, set and Logic, Rehoboth: American Research Press, (1999).

[7] W. G. Wee, On generalizations of adaptive algorithms and application of the fuzzy sets concepts to pattern classi_cation Ph.D. Thesis, Purdue University, (1967).

[8] L. A. Zadeh, Fuzzy sets, Information and Control, 8(3) (1965), 338-353.

[9] W. R. Zhang, Bipolar fuzzy sets and relations: A computational framework for cognitive modeling and multiagent decision analysis, Proc. 1st Int. Joint Conf. North American Fuzzy Information Processing Society Biannual Conf., San Antonio, TX, USA, 1994, 305-309.

[10] W. R. Zhang, YinYang bipolar fuzzy sets, Proc. IEEE World Congr. Computational Intelligence, Anchorage, Alaska, 1998, 835-840.

[11] W. R. Zhang, NPN Fuzzy Sets and NPN Qualitative-Algebra: A Computational Framework for Bipolar Cognitive Modeling and Multiagent Decision Analysis, IEEE Trans. on Sys., Man, and Cybern. 26(8), 1996, 561-575.