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Research Article

Odd And Even Edge Magic Total Labeling Of Union Of Cycles

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ABSTRACT
A graph G is said to be edge magic if a 1-1 onto function $\delta: V(G) \cup E(G) \to \{1,2,3, \ldots |V(G)| + |E(G)|\}$ exists such that $\delta(x) + \delta(xy) + \delta(y)$ is a constant h for each edge $xy \in E(G)$, where h is defined as the magic constant of δ. If a graph G is said to have even edge magic if $\delta(V(G)) = \{2,4,6, \ldots 2|V(G)|\}$. If $\delta(V(G)) = \{1,3,5, \ldots, 2|V(G)| - 1\}$ therefore a graph G is odd edge magic. In this article, we present odd and even EMTL of $C_4 \cup C_{4\theta-3}, \theta > 1$.

Keywords: Edge Magic, Labelling, Union of cycles.

1. INTRODUCTION

In this article, we only look at finite, simple, and undirected graphs. The vertex and edge sets of a graph G are denoted by V(G) and E(G) respectively, where |V(G)| = n and |E(G)| = m. Sedlacek [5] pioneered the magic labeling of graphs in 1968, and since then, they have achieved numerous results in magic labeling, especially edge magic labeling. For new findings in graph labeling, see [1] preliminary results in consecutive EMTL can be found in [6]. C.Y. Ponnappan et. al [4] introduced the concept of odd EMTL and even EMTL. They call an EMTL is odd iff $\delta(V) = \{1,3,5, \ldots 2n - 1\}$. Similarly, the EMTL is called even if $\delta(V) = \{2,4,6, \ldots, 2n\}$. In [2], CT.Nagaraj, C.Y.Ponnappan and G.Prabakaran prove that the graph $C_3 \cup C_{4\theta-2}, \theta > 1, C_3 \cup C_{4\theta}, \theta > 1$ and $C_4 \cup C_{4\theta-1}, \theta > 1$ are even edge magic graphs. In [3], C.Y.Ponnappan and CT. Nagaraj, prove that the graph $C_3 \cup C_{4\theta-2}, \theta > 1, C_3 \cup C_{4\theta}, \theta \geq 1$ and $C_4 \cup C_{4\theta-1}, \theta > 1$ are odd edge magic graphs.

2. EVEN EDGE MAGIC TOTAL LABELING OF $C_4 \cup C_{4\theta-3}, \theta > 1$

Theorem 2.1
The graph $C_4 \cup C_{4\theta-3}$ for $\theta > 1$ is an even edge magic graph.
Proof.
We label the vertex and edges of C_4 consecutively as $[4\theta + 2, 3, 8\theta, 7, 4\theta - 2, 5, 8\theta + 2, 1]$.

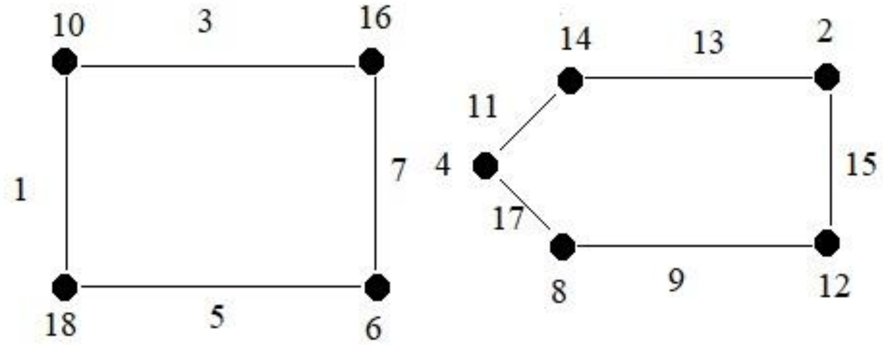
The vertex label of $C_{4\theta-3}$ is as follows

$$f(v_r) = \begin{cases} r + 3 & if\ r = 1\ mod\ 4 \\ r + 4\theta + 4 & if\ r = 2\ mod\ 4 \\ r - 1 & if\ r = 3\ mod\ 4 \\ r + 4\theta & if\ r = 0\ mod\ 4 \end{cases}$$

The edges labels of $C_{4\theta-3}$ is as follows:

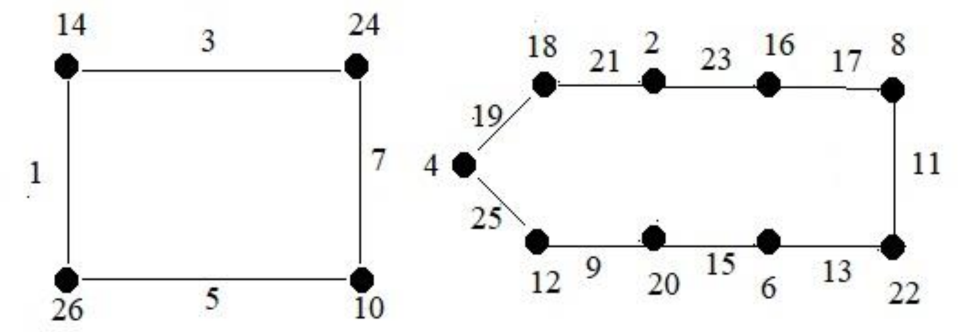
$$f(e_r) = \begin{cases} 8\theta - 2r - 3 & if\ r = 1\ mod\ 4, \quad r \neq 4\theta - 3 \\ 8\theta - 2r + 1 & if\ r = 0,2\ mod\ 4 \\ 8\theta - 2r + 5 & if\ r = 3\ mod\ 4 \\ 8\theta + 1 & if\ r = 4\theta - 3 \end{cases}$$

It is easy to verify that $C_4 \cup C_{4\theta-3}, \theta > 1$ is an even edge magic graph with magic constant $h = 12\theta + 5$.
Example 2.2



$C_4 \cup C_5$, Magic constant $h = 29$

Example 2.3



$C_4 \cup C_9$, **Magic constant** $h = 41$

3. ODD EDGE MAGIC TOTAL LABELING OF $C_4 \cup C_{4\theta-3}, \theta > 1$

Theorem 3.1
The graph $C_4 \cup C_{4\theta-3}, \theta > 1$ is an odd edge magic graph.
Proof.
We label the vertex and edges of C_4 consecutively as $[4\theta+1, 4, 8\theta-1, 8, 4\theta-3, 6, 8\theta+1, 2]$.
$C_{4\theta-3}$ has the following vertex labels:

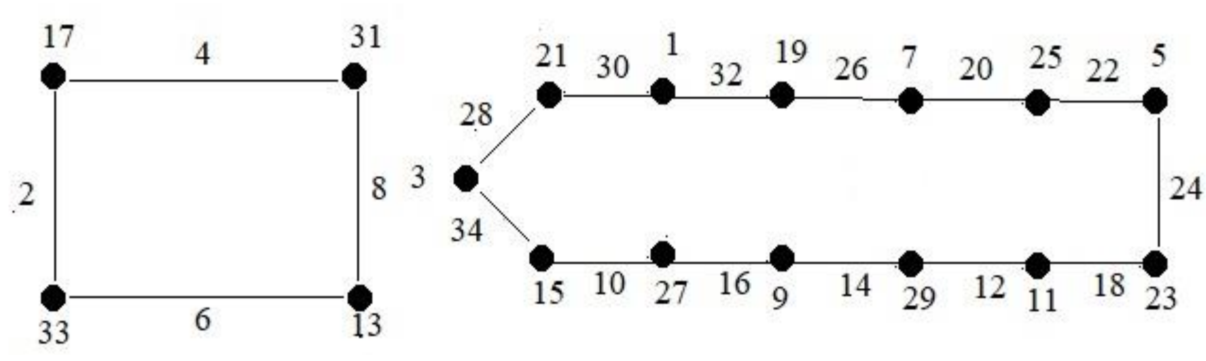
$$f(v_r) = \begin{cases} r+2 \ if \ r = 1 \ mod \ 4 \\ r+4\theta+3 \ if \ r = 2 \ mod \ 4 \\ r-2 \ if \ r = 3 \ mod \ 4 \\ r+4\theta-1 \ if \ r = 0 \ mod \ 4 \end{cases}$$

The edges labels of $C_{4\theta-3}$ is as follows:

$$f(e_r) = \begin{cases} 8\theta-2r-2 \ if \ r = 1 \ mod \ 4, \ r \neq 4\theta-3 \\ 8\theta-2r+2 \ if \ r = 0,2 \ mod \ 4 \\ 8\theta-2r+6 \ if \ r = 3 \ mod \ 4 \\ 8\theta+2 \ if \ r = 4\theta-3 \end{cases}$$

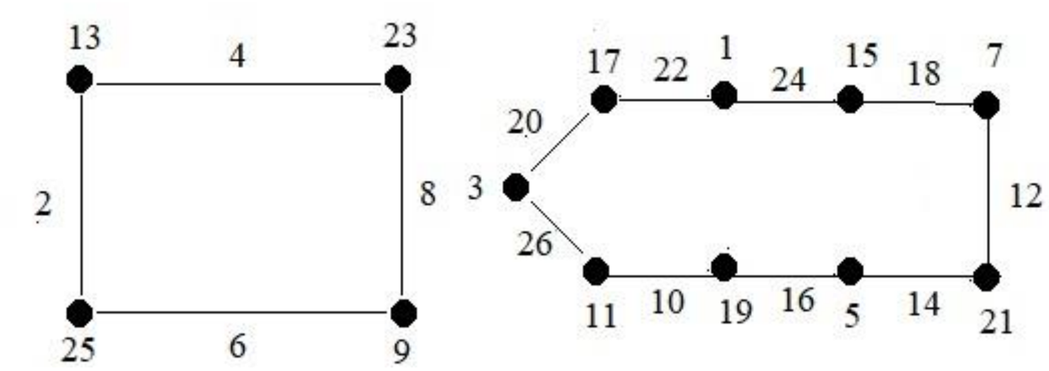
It is easy to verify that $C_4 \cup C_{4\theta-3}, \theta > 1$ is an odd edge magic graph with magic constant $h = 12\theta + 4$.

Example 3.2



$C_4 \cup C_{13}$, **Magic constant** $h = 52$

Example 3.3



$C_4 \cup C_9$, **Magic constant** $h = 40$

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