Research Article

The Minimum Edge Dominating Energy of a Triangular Book and A Globe Graph

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Abstract

Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{e_1, e_2, \ldots, e_m\}$. A subset D' of E is called an edge dominating set of G if every edge of $E - D'$ is adjacent to some edge in D'. Any edge dominating set with minimum cardinality is called a Minimum Edge Dominating set [1]. Let D' be a minimum edge dominating set of a graph G. The Minimum Edge Dominating matrix of G is the m x m matrix defined by

$$D'(G) = [d_{ij}'] \text{ ,where } d_{ij}' = \begin{cases} 1 & if\ e_i\ and\ e_j\ are\ incident \\ 1 & if\ i = j\ e_i \in D' \\ 0 & otherwise \end{cases}$$

The characteristic polynomial of $D'(G)$ is denoted by

$$f_m(G, \rho) = det\,(\rho I - D'\,(G)).$$

The Minimum Edge Dominating Eigen values of a graph G are the eigen values of$D'(G)$. Minimum Edge Dominating Energy of G [13] is defined as the sum of the absolute values of the Minimum Edge Dominating Eigen values. i.e.,

$$E_{D'}(G) = \sum_{i=1}^{m} |\rho_i|$$

In this paper we have computed the Minimum Edge Dominating Energy of a Triangular Book B(3,n) [11] and a Globe graph Gl(n) [12]. In this paper we have considered simple, finite and undirected graphs.

Key Words:

Edge adjacency matrix, Edge energy, Edge dominating set, Minimum Edge Dominating matrix, Minimum Edge Dominating Eigen values, Minimum Edge Dominating Energy.
AMS Subject Classification: 05C50, 05C69

1. INTRODUCTION

Many real-life situations can conveniently be described by means of a diagram consisting of a set of points together with lines joining certain pairs of these points. Since they are represented graphically graphs got this name. It is easy to understand because of the graphical representation. Graphs are used to model many types of relations and processes in physical, biological, social and information systems. Many practical problems can be represented using graphs. Graph Theory began with Leonhard Euler in his study of the Bridges of Konigsberg problem. The paper written by Leonhard Euler on the seven Bridges of Konigsberg and published in 1736 is regarded as the first paper in the history of graph theory. Graph energy was defined during the year 1978 by Ivan Gutman from theoretical chemistry. Many results were found later. For instance, during the year 2004 Bapat and Pati [3] proved that if the energy of a graph is rational then it must be an even integer, while Pirzada and Gutman [9] established that the energy of a graph is never the square root of an odd integer. Due to the interest in graph energy many energies like Laplacian energy [6], Seidel energy [8], Distance energy [4], Randic energy [7], Minimum Dominating energy [10] etc., were defined and their properties were discussed. Inspired by all these energies we have defined a new energy called the Minimum Edge Dominating energy [13] and the energy for various graphs were found.

In this paper we have computed the Minimum Edge Dominating energy of a Triangular Book B(3,n) [11] and a Globe graph Gl(n) [12]. In this paper we have considered simple, finite and undirected graphs.

2. PRELIMINARIES

In this section, we give the basic definitions and notations relevant to this paper.

Defintion 2.1: - The **adjacency matrix A(G)** of a graph G (V, E) with a vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and an edge set E = { e_1, e_2 ,….,e_m } is an n x n matrix

$$A = (a_{ij}) = \begin{cases} 1 \ if \ v_i is \ adjacent \ to \ v_j \\ 0 \ , otherwise \end{cases}$$

A is a real symmetric matrix [2].

Definition 2.2: - The eigen values $\lambda_1, \lambda_2, \cdots, \lambda_n$ of A, assumed in non increasing order, are the eigen values of the graph G. As A is real symmetric, the eigen values of G are real with sum equal to zero. The **Energy E(G) of G** is defined to be the sum of the absolute values of the eigen values of G. i.e., E (G) $= \sum_{i=1}^{n} |\lambda_i|$ [5].

Definition 2.3:- Let G be a simple graph of order n with vertex set V= {$v_1, v_2, \ldots, v_n$} and edge set E = {$e_1, e_2, \ldots, e_m$}. A subset D' of E is called an edge dominating set of G if every edge of E - D' is incident to some edge in D'. Any edge dominating set with minimum cardinality is called a Minimum Edge Dominating set [ ]. Let D' be a Minimum Edge Dominating set of a graph G. The **Minimum Edge Dominating matrix of G** is the m x m matrix defined by

$$D'(G) = (d'_{ij}), \text{ where } (d'_{ij}) = \begin{cases} 1 \ \ if \ e_i \ and \ e_j \ are \ incident \\ 1 \ \ \ \ if \ i = j \ and \ e_i \in D' \\ 0 \ \ \ \ \ \ otherwise \end{cases}$$

The characteristic polynomial of $D'(G)$ is denoted by

$$f_m(G, \rho) = \det(\rho I - D'(G)).$$

The Minimum Edge Dominating Eigen values of a graph G are the eigen values $\rho_1, \rho_2, \ldots, \rho_m$ of D'(G). The Minimum Edge Dominating Energy of G is defined as $E_{D'}$ (G) = $\sum_{i=1}^{m} |\rho_i|$ [13].

Definition 2.4:- The **Triangular Book** with n-pages is defined as n copies of cycle C_3 sharing a common edge. The common edge is called the spine or base of the book. This graph is denoted by B(3, n). In other words it is the complete tripartite graph $K_{1,1,n}$ [11].

Definition 2.5:-A **Globe graph Gl(n)** is a graph obtained from two isolated vertex are joined by n paths of length two [12].

Example: 1

Consider the Wheel graph (W_6) in Figure 1.

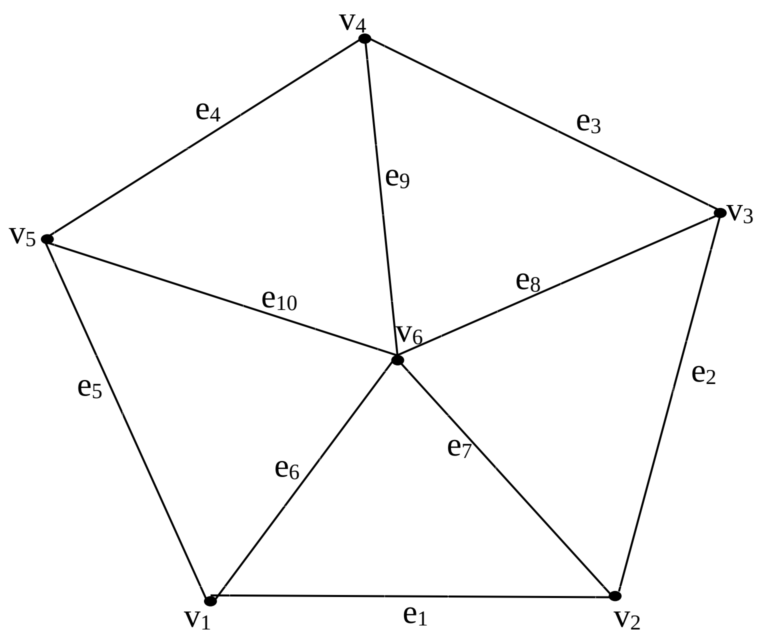

Figure 1

1. Let the vertex set be V = {$v_1, v_2, v_3, v_4, v_5, v_6$} and the Edge set be

E = {e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}}.

(i) Let the Minimum Edge Dominating set be D_1'= {e_1, e_3, e_{10}}.

Then the Minimum Edge Dominating adjacency matrix be

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The characteristic equation is

$$\rho^{10} - 3\rho^9 - 22\rho^8 + 20\rho^7 + 148\rho^6 + 4\rho^5 - 364\rho^4 - 188\rho^3 + 240\rho^2 + 192\rho + 32 = 0$$

The Minimum Edge Dominating Eigen values are

$\rho_1 \approx -2$, $\rho_2 \approx -1.9209$, $\rho_3 \approx -1.6751$, $\rho_4 \approx -1.3656$, $\rho_5 \approx -0.5392$, $\rho_6 \approx -0.2597$, $\rho_7 \approx 1.1112$, $\rho_8 \approx 1.9143$, $\rho_9 \approx 2.2143$, $\rho_{10} \approx 5.5208$.

The Minimum Edge Dominating Energy, $E_{D'}(G) \approx 18.5211$.

(ii) Let the Minimum Edge Dominating set be D_1'= {e_6, e_8, e_{10}}.

Then the Minimum Edge Dominating adjacency matrix be

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The characteristic equation is

$$\rho^{10} - 3\rho^9 - 22\rho^8 + 16\rho^7 + 129\rho^6 - \rho^5 - 272\rho^4 - 76\rho^3 + 168\rho^2 + 48\rho + 0 = 0$$

The Minimum Edge Dominating Eigen values are

$\rho_1 \approx -2$, $\rho_2 \approx -1.9133$, $\rho_3 \approx -1.7321$, $\rho_4 \approx -1.3989$, $\rho_5 \approx -0.2850$, $\rho_6 \approx 0$, $\rho_7 \approx 0.9494$, $\rho_8 \approx 1.7321$, $\rho_9 \approx 1.9326$, $\rho_{10} \approx 5.7153$.

The Minimum Edge Dominating Energy, $E_{D'}(G) \approx 17.6587$.

This example illustrates the fact that the Minimum edge dominating energy of a graph G depends on the choice the Minimum edge dominating set.
i.e. the Minimum edge dominating energy is not a graph invariant.

MAIN RESULTS

Theorem 1

For $n \geq 2$, the Minimum Edge Dominating Energy of a Triangular Book B(3, n) is

$3n - 4 + \sqrt{n^2 + 6n + 1}$.

Proof:
Consider a Triangular Book B(3, n) with vertex set $V = \{v_1, v_2, v_3, \ldots, v_{n+2}\}$ and edge set

$E = \{e_1, e_2, e_3, \ldots, e_{2n+1}\}$.

Let the minimum dominating set be $D' = \{e_1\}$.

$$D'(B(3, n)) = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & \cdots & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & \cdots & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ \vdots & \cdots & \cdots & \vdots & \ddots & \ddots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots & \ddots & \ddots & \cdots & \vdots \\ 1 & 0 & 1 & 0 & \ddots & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & \cdots & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & \cdots & 1 & 1 & 0 \end{bmatrix}_{(2n+1)X\,(2n+1)}$$

$$\text{The characteristic polynomial of } D'(B(3, n)) = \begin{vmatrix} \rho - 1 & -1 & -1 & -1 & \cdots & -1 & -1 & -1 \\ -1 & \rho & -1 & -1 & \cdots & 0 & -1 & 0 \\ -1 & -1 & \rho & 0 & \cdots & -1 & 0 & -1 \\ -1 & -1 & 0 & \rho & \cdots & 0 & -1 & 0 \\ \vdots & \cdots & \cdots & \vdots & \ddots & \ddots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots & \ddots & \ddots & \cdots & \vdots \\ -1 & 0 & -1 & 0 & \ddots & \rho & 0 & -1 \\ -1 & -1 & 0 & -1 & \cdots & 0 & \rho & -1 \\ -1 & 0 & -1 & 0 & \cdots & -1 & -1 & \rho \end{vmatrix}$$

The characteristic equation is $(\rho+2)^{n-1} \rho^{n-1}(\rho-(n-2)) (\rho^2 - (n+1)\rho - n) = 0$.

The Minimum Edge Dominating Eigen values are

$\rho = -2$ (n-1) times, $\rho = 0$ (n-1) times, $\rho = n-2$, $\rho = \frac{(n+1)+\sqrt{n^2+6n+1}}{2}$, $\rho = \frac{(n+1)-\sqrt{n^2+6n+1}}{2}$

The Minimum Edge Dominating Energy is

$$E_{D'}(B(3, n)) = |-2|(n-1) + 0(n-1) + \left| \frac{(n+1)+\sqrt{n^2+6n+1}}{2} \right| + \left| \frac{(n+1)-\sqrt{n^2+6n+1}}{2} \right|$$

$$= 3n - 4 + \sqrt{n^2 + 6n + 1}$$

Theorem 2

For $n \geq 2$, the Minimum Edge Dominating Energy of a Globe graph Gl(n) is $3n - 3 + \sqrt{n^2 - 2n + 5}$.

Proof:
Consider a Globe graph Gl(n) with vertex set $V = \{v_1, v_2, v_3, \ldots, v_{n+2}\}$ and edge set

$E = \{e_1, e_2, e_3, \ldots, e_{2n}\}$.

Let the minimum dominating set be $D' = \{e_1, e_{n+1}\}$.

$$D^{'}(Gl(n)) = \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 & \dots & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & \dots & 0 & \dots & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & \dots & 0 & \dots & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & \dots & 0 & \dots & 0 & 0 & 1 \\ \vdots & \dots & \dots & \vdots & \ddots & \ddots & \ddots & \ddots & \dots & \vdots \\ 1 & 0 & 0 & 0 & \ddots & 1 & \ddots & 0 & 1 & 1 \\ \vdots & \dots & \dots & \vdots & \ddots & \ddots & \ddots & \ddots & \dots & \vdots \\ 0 & 0 & 1 & 0 & \ddots & 0 & \dots & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & \dots & 0 & \dots & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & \dots & 1 & \dots & 1 & 1 & 0 \end{bmatrix}_{2n \, X \, 2n}$$

The characteristic polynomial of $D^{'}$(Gl(n))

$$= \begin{vmatrix} \rho-1 & -1 & -1 & -1 & \dots & -1 & \dots & 0 & 0 & 0 \\ -1 & \rho & -1 & -1 & \dots & 0 & \dots & 0 & 0 & 0 \\ -1 & -1 & \rho & -1 & \dots & 0 & \dots & -1 & -1 & 0 \\ -1 & -1 & -1 & \rho & \dots & 0 & \dots & 0 & 0 & -1 \\ \vdots & \dots & \dots & \vdots & \ddots & 0 & \ddots & \ddots & \dots & \vdots \\ -1 & 0 & 0 & 0 & \ddots & \rho-1 & \ddots & 0 & -1 & -1 \\ \vdots & \dots & \dots & \vdots & \ddots & \ddots & \ddots & \ddots & \dots & \vdots \\ 0 & 0 & -1 & 0 & \ddots & 0 & \dots & \rho & 0 & -1 \\ 0 & -1 & 0 & -1 & \dots & 0 & \dots & 0 & \rho & -1 \\ 0 & 0 & -1 & 0 & \dots & -1 & \dots & -1 & -1 & \rho \end{vmatrix}$$

The characteristic equation is

$$\rho^{n-2} (\rho + 2)^{n-2} (\rho^2 - (n-3) \rho - (n-1)) (\rho^2 - (n+1) \rho + (n-1)) = 0$$

The Minimum Edge Dominating Eigen values are

$\rho = 0$ (n-2) times, $\rho = -2$ (n-2) times, $\rho = \frac{(n-3)+\sqrt{n^2-2n+5}}{2}$, $\rho = \frac{(n-3)-\sqrt{n^2-2n+5}}{2}$,

$\rho = \frac{(n+1)+\sqrt{n^2-2n+5}}{2}$, $\rho = \frac{(n+1)-\sqrt{n^2-2n+5}}{2}$.

The Minimum Edge Dominating Energy is

$$E_{D'}(Gl(n)) = 0(n-2) + |-2|(n-2) + \left|\frac{(n-3)+\sqrt{n^2-2n+5}}{2}\right|$$

$$+ \left|\frac{(n-3)-\sqrt{n^2-2n+5}}{2}\right| + \left|\frac{(n+1)+\sqrt{n^2-2n+5}}{2}\right| + \left|\frac{(n+1)-\sqrt{n^2-2n+5}}{2}\right|$$

$$= 2(n-2) + \sqrt{n^2-2n+5} + (n+1)$$

$$= 3n - 3 + \sqrt{n^2 - 2n+5} .$$

4. CONCLUSION

In this paper we have found the Minimum Edge Dominating Energy of some graphs. Further studies are going on in finding the Minimum Edge Dominating Energy of some special graphs.

REFERENCES

1. Arumugam. S, Velammal. S, Edge domination in graphs, Taiwanese Journal of Mathematics, vol. 2, no. 2, 173–179 (1998).
2. Balakrishnan. R,Ranganathan. K, A Textbook of Graph Theory, Springer, New York, (2000).
3. Bapat.R.B, Pati .S, Energy of a graph is never an odd integer,Bull. Kerala Math. Assoc.1, 129-132, (2004).

4. Gopalapillai Indulal, Ivan Gutman , Ambat Vijayakumar, On Distance Energy Of Graphs, Communications in Mathematical and in Computer Chemistry, ISSN 0340 – 625, Vol. 60, 461-472, (2008).
5. Gutman. I, The energy of a graph, Ber. Math-Satist. Sekt. Forschungsz.Graz103, 1-22 (1978).
6. Ivan Gutman, Bo Zhou, Laplacian energy of a graph, Linear Algebra and its Applications, Vol. 414, Issue 1, 29-36, (2006).
7. Kinkar Ch. Das, Sezer Sorguna, On Randic Energy of Graphs, Communications in Mathematical and in Computer Chemistry, ISSN 0340 – 6253, Vol. 72 , 227-238, (2014).
8. Mohammad Reza Oboudi, Energy and Seidel Energy of Graphs, Communications in Mathematical and in Computer Chemistry, ISSN 0340 – 6253, Vol. 75, 291-303, (2016).
9. Pirzada. S,Gutman. I, Energy of a graph is never the square root of an odd integer, Applicable Analysis and Discrete Mathematics, Vol.2, No.1, 118-121, (2008).
10. Rajesh Kanna. M.R, Dharmendra. B.N,Sridhara. G, The Minimum Dominating energy of a graph, International Journal of Pure and Applied Mathematics, Volume 85 No. 4, 707-718, (2013).
11. Rathod . N.B, Kanani. K.K, k-cordial Labeling of Triangular Book, Triangular Book with Book Mark & Jewel Graph, Global Journal of Pure and Applied Mathematics, Vol. 13, 6979–6989, (2017).
12. Shanti S. Khunti, Mehul A. Chaurasiya and Mehul P. Rupani, Maximum eccentricity energy of globe graph, bistar graph and some graph related to bistar graph, Malaya Journal of Matematik, Vol. 8, No. 4, 1521-1526, (2020).
13. Sharmila. A, Lavanya. S, The minimum edge dominating energy of a graph, Journal of Computer and Mathematical Sciences, vol. 8, 824-828 (2017).