Research Article

Solving a Circuit System Using Fuzzy Aboodh Transform

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Abstract:
This paper proposed a new fuzzy transform that based on Aboodh transform and using this new fuzzy transform to calculate the exact solutions of first order fuzzy differential equations. To explain this approach, a related theorems and properties are proved in detail associated with some examples. In addition, this approach has been applied on physical application of an electrical circuit system.

Keywords: fuzzy number; fuzzy differential equation; strongly generalized differentiable; fuzzy Aboodh transforms; fuzzy first-order differential equation.

Introduction:

In the last decades, fuzzy differential equations have been used in many fields due to their numerous and important applications in a wide range of fields. In order to keep pace with the rapid development and progress in the field of fuzzy differential equations, we presented this paper that contains a new method for solving this type of equations, and our research will be limited to solving fuzzy differential equations of the first order. Fuzzy derivative was first introduced by Chang and Zadeh (1972) [1], then the concept of fuzzy differential equations was introduced by Kandel and Byatt (1978-1980) [2], after a while over time, the method of numerical solution was introduced for solving fuzzy differential equations by Abbasbandy and Allahviranloo (2002) [3]. Seikkala (1987) defined the fuzzy derivative which is a generalization of the Hukuhara derivative [4]. Generalized differential is strongly introduced in Bede and Gal (2005) [5] and studied by Bede et al (2006) [6]. This paper will construct a new fuzzy transform based on Aboodh transform, which for solving this type of equations and a circuit system is showed as an application for this technique.

Definition 1 [7]

A fuzzy number in parametric form is an ordered pair$(\underline{s},\overline{s})$ of functions $\overline{s}(\kappa)$, $\underline{s}(\kappa)$, $\kappa \in [0,1]$,which satisfies the following requirements:

- $\underline{s}(\kappa)$is a bounded non-decreasing, right Continuous at 0 and left continuous function in (0,1].
- $\overline{s}(\kappa)$ is a bounded non- increasing, right Continuous at 0 and left continuous function in (0,1].
- $\underline{s}(\kappa) \leq \overline{s}(\kappa)$, $\kappa \in [0,1]$.

Definition 2 [8]

Let η and ζ are fuzzy numbers, where $\eta = (\underline{\eta}(\kappa), \overline{\eta}(\kappa)), \zeta = (\underline{\zeta}(\kappa), \overline{\zeta}(\kappa)), 0 \leq \kappa \leq 1$ and $\alpha > 0$ we define

- Addition $\eta \oplus \zeta = (\underline{\eta}(\kappa) + \underline{\zeta}(\kappa), \overline{\eta}(\kappa) + \overline{\zeta}(\kappa))$.
- Subtraction $\eta \ominus \zeta = (\underline{\eta}(\kappa) - \overline{\zeta}(\kappa), \overline{\eta}(\kappa) - \underline{\zeta}(\kappa))$.
- Scalar multiplication $\alpha \odot \eta = \begin{Bmatrix} (\alpha\underline{\eta}, \alpha\overline{\eta} & , \alpha \geq 0 \\ \left(\alpha\overline{\eta}, \alpha\underline{\eta}\right), & \alpha < 0 \end{Bmatrix}$.

Definition 3 [9]

Let η and ζ are fuzzy numbers then the distance between fuzzy numbers in the Hausdorff is given by $\Gamma: \Re_f \times \Re_f \to [0, +\infty]$,where $\Re_f$ be the set of all fuzzy numbers on $\Re$:

$\Gamma(\eta,\zeta) = \sup_{\kappa\in[0,1]} \max\left\{\left|\underline{\eta}(\kappa) - \underline{\zeta}(\kappa)\right|, \left|\overline{\eta}(\kappa) - \overline{\zeta}(\kappa)\right|\right\}$,where $\eta = (\underline{\eta}(\kappa), \overline{\eta}(\kappa))$,$\zeta = (\underline{\zeta}(\kappa), \overline{\zeta}(\kappa))$ and $(\Re_f, \Gamma)$ is a complete metric space and following properties are well known:

- $\Gamma(\eta\oplus\omega, \zeta\oplus\omega) = \Gamma(\eta,\zeta), \forall \eta,\zeta,\omega \in \Re_f$.
- $\Gamma(\kappa\odot\eta, \kappa\odot\zeta) = |\kappa|\,\Gamma(\eta,\zeta), \forall \eta,\zeta, \in \Re_f, \kappa \in \Re$.
- $\Gamma(\eta\oplus\omega, \zeta\oplus\nu) \le \Gamma(\eta,\zeta) + \Gamma(\omega,\nu), \forall \eta,\zeta,\omega,\nu \in \Re_f$.

Definition 4 [6]

Suppose that $x, y \in \Re_f$. If there exists $z \in \Re_f$ such that $x = y + z$ then z is called the H-differential of x and y and it is denoted by $x \ominus y$.

Note that in this work, the sign $\ominus$ always meant the H-difference as well as $x \ominus y \ne x + (-1)y$.

Definition 5 [10]

Let continuous fuzzy - valued function $\upsilon: (a,b): \longrightarrow \Re_f$ and $x_0 \in (a,b)$. We say that a mapping υ is strongly generalized differentiable at x_0 if there exists an element $\upsilon\grave{}(x_0) \in \Re_f$, such that :

i. For all $\tau > 0$ sufficiently small, $\exists \upsilon(x_0 + \tau) \ominus \upsilon(x_0), \upsilon(x_0) \ominus \upsilon(x_0 - \tau)$,

where $\lim\limits_{\tau\to0} \frac{\upsilon(x_0+\tau)\ominus\upsilon(x_0)}{\tau} = \lim\limits_{\tau\to0} \frac{\upsilon(x_0)\ominus\upsilon(x_0-\tau)}{\tau} = \upsilon\grave{}(x_0)$,

or

ii. For all $\tau > 0$ sufficiently small, $\exists \upsilon(x_0) \ominus \upsilon(x_0 + \tau), \upsilon(x_0 - \tau) \ominus \upsilon(x_0)$

where $\lim\limits_{\tau\to0} \frac{\upsilon(x_0)\ominus\upsilon(x_0+\tau)}{-\tau} = \lim\limits_{\tau\to0} \frac{\upsilon(x_0-\tau)\ominus\upsilon(x_0)}{-\tau} = \upsilon\grave{}(x_0)$,

or

iii. For all $\tau > 0$ sufficiently small, $\exists \upsilon(x_0 + \tau) \ominus \upsilon(x_0), \upsilon(x_0 - \tau) \ominus \upsilon(x_0)$

where $\lim\limits_{\tau\to0} \frac{\upsilon(x_0+\tau)\ominus\upsilon(x_0)}{\tau} = \lim\limits_{\tau\to0} \frac{\upsilon(x_0-\tau)\ominus\upsilon(x_0)}{-\tau} = \upsilon\grave{}(x_0)$,

or

iv. For all $\tau > 0$ sufficiently small, $\exists \upsilon(x_0) \ominus \upsilon(x_0 + \tau), \upsilon(x_0) \ominus \upsilon(x_0 - \tau)$

where $\lim\limits_{\tau\to0} \frac{\upsilon(x_0)\ominus\upsilon(x_0+\tau)}{-\tau} = \lim\limits_{\tau\to0} \frac{\upsilon(x_0)\ominus\upsilon(x_0-\tau)}{\tau} = \upsilon\grave{}(x_0)$.

Theorem 1 [11]

Assume that $\Gamma: [a,b] \to [0,1]$, be a function such that $[\Gamma(\gamma)]_\omega = [\upsilon_\omega(\gamma), \pi_\omega(\gamma)]$ for each ω∈ [0,1]. Then:

(i) If Γ is differentiable of the first form (i), then υ_ω and π_ω are differentiable functions and $[\Gamma'(\gamma)]_\omega = [\upsilon'(\gamma), \pi'(\pi)]$.

(ii) If F is differentiable of the second form (ii), then f_α and g_α are differentiable functions and $[\Gamma'(\gamma)]_\omega = [\pi'(\pi), \upsilon'(\gamma)]$.

Theorem 2 [12]

Let υ(x) be a fuzzy valued function on [a, ∞) represented by$((\underline{\upsilon}(x,\kappa), \overline{\upsilon}(x,\kappa))$ .For any fixed $\kappa \in [0,1]$, let $(\underline{\upsilon}(x,\kappa), \overline{\upsilon}(x,\kappa)$ are Riemann-integrals on [a, b]. For every b ≥a, if there exists two positive functions $\underline{M}(\kappa)$ and $\overline{M}(\kappa)$ such that$\int_a^b \left|\underline{\upsilon}(x,\kappa)\right| dx \le \underline{M}(\kappa)$ and $\int_a^b \left|\overline{\upsilon}(x,\kappa)\right| dx \le \overline{M}(\kappa)$ for every b ≥a, then υ(x) is said to be improper fuzzy Riemann-Liouville integrals function on [a,∞), i.e.

$\int_a^\infty \upsilon(x)\,dx = [\int_a^\infty (\underline{\upsilon}(x,\kappa)dx, \int_a^\infty \overline{\upsilon}(x,\kappa)dx]$.

Definition 6 [14]

Let υ(x) be a fuzzy valued function on [a, b]. Suppose that $\underline{\upsilon}(x,\kappa)$ and $\overline{\upsilon}(x,\kappa)$ are improper Rieman-integrable on [a, b], then υ(x) is an improper on [a, b] and $\underline{\left(\int_a^b \upsilon(\tau,\kappa)d\tau\right)} = \left(\int_a^b \underline{\upsilon(\tau,\kappa)}d\tau\right)$,

$\overline{(\int_a^b \upsilon(\tau,\kappa)d\tau)} = (\int_a^b \overline{\upsilon(\tau,\kappa)}d\tau)$.

Definition 7 [14]

If υ :(a, b) $\rightarrow \Re_f$ is a continuous fuzzy valued function, then $\pi(x) = \int_a^x \upsilon(\tau)d\tau$ is differentiable with derivative

$\pi'(x) = \upsilon(x)$.

Theorem 3 [15]

Let $\upsilon(x): [a,b] \longrightarrow \Re_f$ be a function and denote $\upsilon(x) = ((\underline{\upsilon}(x,\kappa),\overline{\upsilon}(x,\kappa))$ for each $\kappa \in [0,1]$. Then:

- If υ is differentiable of the first form (i), then $(\underline{\upsilon}(x,\kappa)$ and $\overline{\upsilon}(x,\kappa)$ are differentiable functions and
  $\upsilon\grave{}(x) = (\underline{\upsilon\grave{}}(x,\kappa), \overline{\upsilon\grave{}}(x,\kappa))$.
- If υ is differentiable of the second form (ii), then $\underline{\upsilon}(x,\kappa)$ and $\overline{\upsilon}(x,\kappa)$ are differentiable functions and $\upsilon\grave{}(x) = (\overline{\upsilon\grave{}}(x,\kappa), \underline{\upsilon\grave{}}(x,\kappa))$.

Definition 8 [16]

Consider functions in the set H defined by $H = \{\upsilon(x): \exists M,, \kappa_1, \kappa_2 > 0, |\upsilon(x)| < Me^{-sx}\}$, for a given function in the set H, the constant M must be finite number, κ_1, κ_2 may be finite or infinite. The Aboodh transform denoted by the operator A and defined by the integral equations

$A[\upsilon(x)] = H(s) = \frac{1}{s}\int_0^\infty \upsilon(x)e^{-sx}dx \quad , x \geq 0, \kappa_1 \leq s \leq \kappa_2$, the variable s in this transform is used to factor the variable x in the argument of the function υ .

Theorem 4 [17]

Let $\upsilon(x)$ is a continuous function in $[0,k]$ and $A[\upsilon(x)] = H(s)$,then

- $A[\upsilon(ax)] = \frac{1}{a^2}H\left(\frac{s}{a}\right)$, for any constant a.
- For any functions $\upsilon(x)$ and $\pi(x)$ and any constants a, b then:
  $A[a\,\upsilon(x) + b\,\pi(x)] = a(A[\upsilon(x)]) + b(A[\pi(x)])$
- $A[\upsilon^{(n)}(x)] = s^n H(s) - \sum_{i=1}^{n-1} \frac{\upsilon^{(i)}(0)}{s^{2-n-i}}$
- If $A[\upsilon(x)] = H(s)$ and $L[\upsilon(x)] = F(s)$, then $H(s) = \frac{1}{s}F(s)$
  Where F(s)is Laplace transformation of $\upsilon(x)$.

Definition 9

Let $\upsilon(x)$ be a continuous fuzzy-valued function. Suppose that $\frac{1}{s}\upsilon(x)e^{-sx}$ is an improper fuzzy Rimann-integrable on $[0,\infty)$, then $\frac{1}{s}\int_0^\infty \upsilon(x)e^{-sx}dx$ is called fuzzy Aboodh transform and it is denoted by

$\widehat{A}[\upsilon(x)] = \frac{1}{s}\int_0^\infty \upsilon(x)e^{-sx}dx$, ($s > 0$ and integer). For theorem (2),we have.

$\frac{1}{s}\int_0^\infty \upsilon(x)e^{-sx}dx = \left(\frac{1}{s}\int_0^\infty \underline{\upsilon}(x,\kappa)e^{-sx}dx, \frac{1}{s}\int_0^\infty \overline{\upsilon}(x,\kappa)e^{-sx}dx\right)$.

Using the definition of classical Aboodh transform, we have.

$A[\underline{\upsilon}(x,\kappa)] = \frac{1}{s}\int_0^\infty \underline{\upsilon}(x,\kappa)e^{-sx}dx$ and $A[\overline{\upsilon}(x,\kappa)] = \frac{1}{s}\int_0^\infty \overline{\upsilon}(x,\kappa)e^{-sx}dx$,then

$\widehat{A}[\upsilon(x)] = (A[\underline{\upsilon}(x,\kappa)], A[\overline{\upsilon}(x,\kappa)])$.

Definition 10

The integral transform $\widehat{A}[\upsilon(x)] = \frac{1}{s}\int_0^\infty \upsilon(x)e^{-sx}\,dx$ is said to be absolutely convergent integral if $\frac{1}{s}\lim_{x\to\infty}\int_0^\infty |\upsilon(x)e^{-sx}dx|$ exists, that is mean:
$\frac{1}{s}\lim_{x\to\infty}\int_0^\infty |\underline{\upsilon}(x,\kappa)e^{-sx}dx|$ and $\frac{1}{s}\lim_{x\to\infty}\int_0^\infty |\overline{\upsilon}(x,\kappa)e^{-sx}dx|$ are exist.

Theorem 5

Let $\upsilon(x)$, $\pi(x)$ be continuous fuzzy-valued functions assume that c_1 and c_2 are constants then

(a) $\widehat{A}[c_1\upsilon(x)] = c_1\widehat{A}[\upsilon(x)]$.

(b) $\widehat{A}\big[c_1(\upsilon(x))\oplus c_2(\pi(x))\big] = c_1\widehat{A}[\upsilon(x)]\oplus c_2\widehat{A}[\pi(x)]$.

Proof

(a)

$$\widehat{A}[c_1\upsilon(x)] = (A[c_1\,\underline{\upsilon}(x,\kappa)], A[c_1\overline{\upsilon}(x,\kappa)]) = (\frac{1}{s}\int_0^\infty c_1\,\underline{\upsilon}(x,\kappa)e^{-sx}\,dx, \frac{1}{s}\int_0^\infty c_1\,\overline{\upsilon}(x,\kappa)e^{-sx}\,dx)$$

$$= (\frac{c_1}{s}\int_0^\infty \underline{\upsilon}(x,\kappa)e^{-sx}\,dx, \frac{c_1}{s}\int_0^\infty \overline{\upsilon}(x,\kappa)e^{-sx}\,dx) = c_1(\frac{1}{s}\int_0^\infty \underline{\upsilon}(x,\kappa)e^{-sx}\,dx, \frac{1}{s}\int_0^\infty \overline{\upsilon}(x,\kappa)e^{-sx}\,dx)$$

$$= c_1\widehat{A}[\upsilon(x)]$$

(b)

Suppose $\upsilon(x) = (\underline{\upsilon}(x,\kappa), \overline{\upsilon}(x,\kappa))$, $\pi(x) = (\underline{\pi}(x,\kappa), \overline{\pi}(x,\kappa)$

$$\widehat{A}\big[c_1(\upsilon(x))\oplus c_2(\pi(x))\big] = (A[\,c_1(\underline{\upsilon}(x,\kappa) + c_2(\underline{\pi}(x,\kappa)], A[c_1\,\overline{\upsilon}(x,\kappa) + c_2\overline{\pi}(x,\kappa)])$$

$$= (\frac{1}{s}\int_0^\infty e^{-sx}\left(c_1\underline{\upsilon}(x,\kappa) + c_2\underline{\pi}(x,\kappa)\right), \frac{1}{s}\int_0^\infty e^{-sx}\left(c_1\,\overline{\upsilon}(x,\kappa) + c_2\overline{\pi}(x,\kappa)\right))$$

$$= (\frac{1}{s}\int_0^\infty e^{-sx}\,c_1\underline{\upsilon}(x,\kappa) + \frac{1}{s}\int_0^\infty e^{-sx}\,c_2\underline{\pi}(x,\kappa)), (\frac{1}{s}\int_0^\infty e^{-sx}\,c_1\,\overline{\upsilon}(x,\kappa) + \frac{1}{s}\int_0^\infty e^{-sx}\,c_2\overline{\pi}(x,\kappa))$$

$$= \left(\frac{1}{s}\int_0^\infty e^{-sx}\,c_1\underline{\upsilon}(x,\kappa), \frac{1}{s}\int_0^\infty e^{-sx}\,c_1\,\overline{\upsilon}(x,\kappa)\right) + (\frac{1}{s}\int_0^\infty e^{-sx}\,c_2\underline{\pi}(x,\kappa), \frac{1}{s}\int_0^\infty e^{-sx}c_2\overline{\pi}(x,\kappa))$$

$$= c_1\left(\frac{1}{s}\int_0^\infty e^{-sx}\,\underline{\upsilon}(x,\kappa), \frac{1}{s}\int_0^\infty e^{-sx}\,\overline{\upsilon}(x,\kappa)\right) + c_2(\frac{1}{s}\int_0^\infty e^{-sx}\,\underline{\pi}(x,\kappa), \frac{1}{s}\int_0^\infty e^{-sx}\overline{\pi}(x,\kappa))$$

$$= c_1A\left(\underline{\upsilon}(x,\kappa), \overline{\upsilon}(x,\kappa)\right) + c_2A\left(\underline{\pi}(x,\kappa), \overline{\pi}(x,\kappa)\right)$$

$$= c_1\widehat{A}\,[\upsilon(x)]\oplus c_2\widehat{A}\,[\pi(x)] .$$

Theorem 6

Let $\mu(x)$ is the primitive of $\grave{\mu}(x)$ on $[0,\infty)$ and $\mu(x)$ be an integrable fuzzy-valued function then

a) if μ is (i)-differentiable then $\widehat{A}[\grave{\mu}(x)] = s\widehat{A}[\mu(x)] \ominus \frac{1}{s}\mu(0)$.

b) if μ is (ii)-differentiable then $\widehat{A}[\grave{\mu}(x)] = (-\frac{1}{s}\mu(0)) \ominus (-s\widehat{A}[\mu(x)]$.

Proof (a)

For arbitrary fixed $\kappa \in [0,1]$

$$s\widehat{A}[\mu(x)] \ominus \frac{1}{s}\mu(0) = (sA\left[\underline{\mu}\,(x,\kappa)\right] - \frac{1}{s}\,\underline{\mu}(0,\kappa), sA[\,\overline{\mu}\,(x,\kappa)] - \frac{1}{s}\,\overline{\mu}(0,\kappa))$$

Since

$sA\left[\underline{\mu}(x,\kappa)\right] - \frac{1}{s}\underline{\mu}(0,\kappa) = A\left[\underline{\mu}^{`}(x,\kappa)\right]$ and $sA[\overline{\mu}(x,\kappa)] - \frac{1}{s}\overline{\mu}(0,\kappa) = A\left[\overline{\mu^{`}}(x,\kappa)\right]$

$\hat{A}\left[\mu^{`}(x)\right] = (A\left[\underline{\mu}^{`}(x,\kappa)\right], A\left[\overline{\mu^{`}}(x,\kappa)\right])$

$s\hat{A}[\mu(x)] \ominus \frac{1}{s}\mu(0) = (A\left[\underline{\mu}^{`}(x,\kappa)\right], A\left[\overline{\mu^{`}}(x,\kappa)\right])$

$\hat{A}\left[\mu^{`}(x)\right] = s\hat{A}[\mu(x)] \ominus \frac{1}{s}\mu(0)$.

Proof (b)

$(-\frac{1}{s}\mu(0)) \ominus (-s\hat{A}[\mu(x)] = (-\frac{1}{s}\overline{\mu}(0,\kappa) + (-sA[\overline{\mu}(x,\kappa)], -\frac{1}{s}\underline{\mu}(0,\kappa) + (-sA\left[\underline{\mu}(x,\kappa)\right])$

Since $(-\frac{1}{s}\overline{\mu}(0,\kappa)) + (-sA[\overline{\mu}(x,\kappa)] = A\left[\overline{\mu^{`}}(x,\kappa)\right]$ and $(-\frac{1}{s}\underline{\mu}(0,\kappa)) + (-sA\left[\underline{\mu}(x,\kappa)\right] = A\left[\underline{\mu}^{`}(x,\kappa)\right]$

$\hat{A}\left[\mu^{`}(x)\right] = (A\left[\overline{\mu^{`}}(x,\kappa)\right], A\left[\underline{\mu}^{`}(x,\kappa)\right])$

$(-\frac{1}{s}\mu(0)) \ominus (-s\hat{A}[\mu(x)] = (A\left[\overline{\mu^{`}}(x,\kappa)\right], A\left[\underline{\mu}^{`}(x,\kappa)\right])$

$\hat{A}\left[\mu^{`}(x)\right] = (-\frac{1}{s}\mu(0)) \ominus (-s\hat{A}[\mu(x)]$.

Example 1

Consider a fuzzy initial value problem $\upsilon^{`}(x) = \upsilon(x),\ \ \upsilon(0,\kappa) = (\kappa - 1, 1 - \kappa),\quad 0 \le \kappa \le 1$.

Solution:

Using fuzzy Aboodh transform on both sides, to get

$\hat{A}\left[\upsilon^{`}(x)\right] = \hat{A}[\upsilon(x)]$.

Case (1)

$\upsilon(x)$ be (i)-differentiable ,

$\hat{A}\left[\upsilon^{`}(x)\right] = s\hat{A}[\upsilon(x)] \ominus \frac{1}{s}\upsilon(0)$, $\hat{A}[\upsilon(x)] = s\hat{A}[\upsilon(x)] \ominus \frac{1}{s}\upsilon(0)$

Using upper and lower functions, to have

$A\left[\underline{\upsilon}(x,\kappa)\right] = sA\left[\underline{\upsilon}(x,\kappa)\right] - \frac{1}{s}\underline{\upsilon}(0,\kappa)$, $A[\overline{\upsilon}(x,\kappa)] = sA[\overline{\upsilon}(x,\kappa)] - \frac{1}{s}\overline{\upsilon}(0,\kappa)$

$sA\left[\underline{\upsilon}(x,\kappa)\right] - A\left[\underline{\upsilon}(x,\kappa)\right] = \frac{1}{s}\underline{\upsilon}(0,\kappa)$, $sA[\overline{\upsilon}(x,\kappa)] - A[\overline{\upsilon}(x,\kappa)] = \frac{1}{s}\overline{\upsilon}(0,\kappa)$

$A\left[\underline{\upsilon}(x,\kappa)\right](s-1) = \frac{1}{s}\underline{\upsilon}(0,\kappa)$, $A[\overline{\upsilon}(x,\kappa)](s-1) = \frac{1}{s}\overline{\upsilon}(0,\kappa)$

$A\left[\underline{\upsilon}(x,\kappa)\right] = \frac{1}{s^2-s}\underline{\upsilon}(0,\kappa)$, $A[\overline{\upsilon}(x,\kappa)] = \frac{1}{s^2-s}\overline{\upsilon}(0,\kappa)$

$\left[\underline{\upsilon}(x,\kappa)\right] = A^{-1}\left[\frac{1}{s^2-s}\right]\underline{\upsilon}(0,\kappa)$, $[\overline{\upsilon}(x,\kappa)] = A^{-1}[\frac{1}{s^2-s}]\overline{\upsilon}(0,\kappa)$

$\underline{\upsilon}(x,\kappa) = (\kappa-1)e^x, \overline{\upsilon}(x,\kappa) = (1-\kappa)e^x$.

Case (2)

$\upsilon(x)$ be (ii)-differentiable,

$\hat{A}\left[\upsilon^{`}(x)\right] = (-\frac{1}{s}\upsilon(0)) \ominus (-s\hat{A}[\upsilon(x)], \hat{A}[\upsilon(x)] = (-\frac{1}{s}\upsilon(0)) \ominus (-s\hat{A}[\upsilon(x)]$

Using upper and lower functions, to have

$A[\overline{\upsilon}(x,\kappa)] = -\frac{1}{s}\underline{\upsilon}(0,\kappa) + sA\left[\underline{\upsilon}(x,\kappa)\right], A\left[\underline{\upsilon}(x,\kappa)\right] = -\frac{1}{s}\overline{\upsilon}(0,\kappa) + sA[\overline{\upsilon}(x,\kappa)]$

$A[\overline{\upsilon}(x,\kappa)] = -\frac{1}{s}(\kappa-1) + sA\left[\underline{\upsilon}(x,\kappa)\right], A\left[\underline{\upsilon}(x,\kappa)\right] = -\frac{1}{s}(1-\kappa) + sA[\overline{\upsilon}(x,\kappa)]$

Using Cramer's rule to get

$A[\overline{\upsilon}(x,\kappa)] = (1-\kappa)\frac{1}{s^3-s} + (\kappa-1)\frac{1}{s^2-1}$, $A\left[\underline{\upsilon}(x,\kappa)\right] = (\kappa-1)\frac{1}{s^3-s} + (1-\kappa)\frac{1}{s^2-1}$

$\overline{\upsilon}\,(x,\kappa) = (1-\kappa)A^{-1}\left[\frac{1}{s^3-s}\right] + (\kappa-1)A^{-1}\left[\frac{1}{s^2-1}\right], \underline{\upsilon}\,(x,\kappa)$

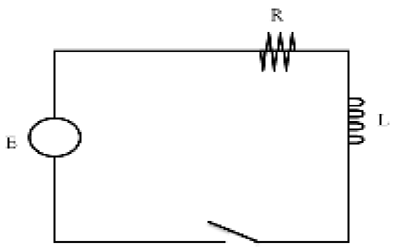
$= (\kappa-1)A^{-1}[\frac{1}{s^3-s}] + (1-\kappa)A^{-1}[\frac{1}{s^2-1}]$

$\overline{\upsilon}\,(x,\kappa) = (1-\kappa)(\sinh x - \cosh x), \underline{\upsilon}\,(x,\kappa) = (\kappa-1)\,(\sinh x - \cosh x)$.

Example 2 (Application of $\widehat{A}$-transform of first-order differential equation)

Consider an RL circuit with $R = 10, L = 2, E_0 = 0$ Suppose that the initial charge on the capacitor is $I(0,\kappa) = (\kappa-1, 1-\kappa)$, $0 \le \kappa \le 1$. Find the charge $I(t)$ for $t \ge 0$.

Solution:



$LI`(t) + RI(t) = E_0$ , $2I`(t) + 10I(t) = 0$

Using fuzzy Aboodh transform on both sides, to get

$\widehat{A}[2I`(t) + 10I(t)] = \widehat{A}[0], 2\widehat{A}[I`(t)] + 10\widehat{A}[I(t)] = 0$.

Case (1)

$I(t)$ be (i)-differentiable

$\widehat{A}[I`(t)] = s\widehat{A}[I(t)] \ominus \frac{1}{s}I(0)$

Using upper and lower functions, to have

$sA[\underline{I}\,(t,\kappa)] - \frac{1}{s}\underline{I}(0,\kappa) + 5A[\underline{I}\,(t,\kappa)] = 0$, $sA[\overline{I}\,(t,\kappa)] - \frac{1}{s}\overline{I}(0,\kappa) + 5A[\overline{I}\,(t,\kappa)] = 0$

$A[\underline{I}\,(t,\kappa)](s+5) = \frac{1}{s}(\kappa-1), A[\overline{I}\,(t,\kappa)](s+5) = \frac{1}{s}(1-\kappa)$

$A[\underline{I}\,(t,\kappa)] = \frac{1}{s(s+5)}(\kappa-1), A[\overline{I}\,(t,\kappa)] = \frac{1}{s(s+5)}(1-\kappa)$

$A[\underline{I}\,(t,\kappa)] = \frac{1}{(s^2+5s)}(\kappa-1), A[\overline{I}\,(t,\kappa)] = \frac{1}{(s^2+5s)}(1-\kappa)$

$\underline{I}\,(t,\kappa) = (\kappa-1)e^{-5t}, \overline{I}\,(t,\kappa) = (1-\kappa)e^{-5t}$.

Case (2)

$I(t)$ be (ii)-differentiable:

$\widehat{A}[I`(t)] = (-\frac{1}{s}I(0)) \ominus (-s\widehat{A}[I(t)]$

Using upper and lower functions, to have

$sA[\underline{I}\,(t,\kappa)] - \frac{1}{s}\underline{I}(0,\kappa) + 5A[\overline{I}\,(t,\kappa)] = 0$, $sA[\overline{I}\,(t,\kappa)] - \frac{1}{s}\overline{I}(0,\kappa) + 5A[\underline{I}\,(t,\kappa)] = 0$

$sA[\underline{I}\,(t,\kappa)] + 5A[\overline{I}\,(t,\kappa)] = \frac{1}{s}(\kappa-1), sA[\overline{I}\,(t,\kappa)] + 5A[\underline{I}\,(t,\kappa)] = \frac{1}{s}(1-\kappa)$

Using Cramer's rule to get

$$A\left[\underline{I}(t,\kappa)\right] = (\kappa-1)\frac{1}{s^2-25} - (1-\kappa)\frac{5}{s^3-25s}, A\left[\bar{I}(t,\kappa)\right] = (1-\kappa)\frac{1}{s^2-25} - (\kappa-1)\frac{5}{s^3-25s}$$

$$\underline{I}(t,\kappa) = (\kappa-1)(\cosh 5t + \sinh 5t), \bar{I}(t,\kappa) = (1-\kappa)(\cosh 5t + \sinh 5t).$$

Conclusion:

The main aim of the paper is to solve first -order linear fuzzy differential equations using proposed fuzzy Aboodh transform. Two numerical examples are given to illustrate the efficiency of the proposed method.

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