How We Teach: Inquiry in Teaching and Learning Mathematics

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Abstract

The author of this paper as a teacher of mathematics shares her views on mathematics teaching based on her professional experiences over many years. In this paper the author mainly focuses on how teachers create opportunity for pupils to engage with mathematics seriously and enjoyably. Being a good teacher and teaching mathematics successfully is not easy process and task. From a constructivist perspective the author as a teacher of mathematics and mathematics educator suggests that the question of "how we teach" should be central to a process of being good teachers and developing teaching.

Key Words: Teaching mathematics, teaching development, constructivist approach

1. Introduction

In the *School of Mathematics* at Loughborough University, the *Mathematics Education Centre* (MEC) runs a series of seminars focusing on "How we Teach". Their purpose is to enable those who teach mathematics in the university to share their perspectives on teaching and to discuss teaching practice. We have tried to open up a teaching discourse in which we can get beyond the more managerial aspects of teaching and really discuss in depth what we do, how we do it, and how we think about it in relation to our students. We can get good ideas from each other. If we find that we do not all think about teaching in the same ways, this can raise issues which enable us to look critically at our practices and perhaps come to teaching with new images and visions.

This article began its life as a presentation in the *How we Teach* series. It is a personal reflection on what it means to teach mathematics and how, as a teacher of mathematics, one develops teaching to provide the best learning opportunity for the students being taught. I draw here on professional experience over many years and research I have engaged with during this time. Most of my practice and research has been into mathematics teaching at school level (mainly secondary) and the education of teachers of mathematics. I also tutored for the Open University for some years. In joining the School of Mathematics at Loughborough, I became a mathematics teacher at university level which was a new challenge. I needed to think what this should mean in terms of how I approached both mathematics and my students. This sent me back to reflections on how I approached

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mathematics as a school teacher, in my Open University tutoring and in working with teachers in a teacher education context.

Perhaps one concern for any teacher of mathematics at any level is how to enable students to develop both mathematical understanding and good practice in working mathematically. One of my interests has been in how to use investigational tasks to enable students to learn and understand mathematics. To illustrate what I mean by this, I offer a task below.

An investigational task

> Starting with any rectangle, fill it with squares of side equal to the length of the shorter side of the rectangle until no more will fit. Repeat with any rectangle remaining.

For example, in the rectangle ABCD in Figure 1, we see seven squares, then two, and we still have not finished. How many squares fit into the tiny rectangle that remains on the bottom right? Are we guaranteed to finish?

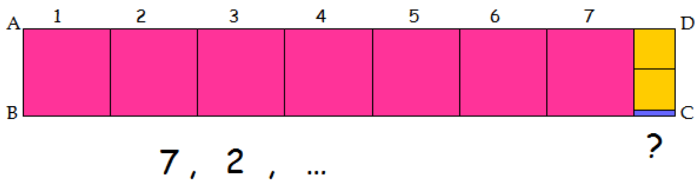


Figure 1. Filling a rectangle with squares and starting to generate an associated sequence.

You can explore further by trying the process with a specific rectangle, for example

- Try it with a rectangle of dimensions 5x19.
- What sequence of squares do you get?
- Can you find a rectangle that will give a *longer* sequence of squares?
- Are such sequences always finite?
- Can you find a rectangle that will generate any given sequence? e.g., 3, 1, 5?

I don't want to give too much away here, because you might enjoy exploring this for yourself and I don't want to pre-empt whatever you might find[1]. I therefore go on to consider the value of such a task and why we might want to use it with students -- what can such a task offer to those who are interested to engage with it?

[1] A colleague in Norway, Byrge Birkeland, was kind enough to programme this for me, and you can find an animated version on his website http://home.hia.no/~byrgeb/bb/classes/English/Euclid.html. You might also consult David Fowler's book *Mathematics of Plato's Academy : a new Reconstruction* (Fowler, D. H. (1987)

- *Ready engagement*: it is easy to get involved, easy access – everyone can draw a rectangle and start to explore. Perhaps doing this engages interest or curiosity and stimulates further exploration.
- *Questions to tackle:* some basic questions are provided with which to get going, to stimulate activity and guide initial engagement. There is potential to ask further questions as a result of initial engagement.
- *Possibility to engage with serious mathematical concepts* such as geometric and numeric relationships, algebraic formulation, and standard results (such as Euclid's algorithm). Opportunity is provided for generalisation, convincing and proving.

Ready engagement means that all students can get involved with mathematics and at least make a start with something that looks interesting and challenging. Having questions to tackle provides starting points for engagement which experience has shown can lead to further questions and directions of exploration. Experience also shows that the task can lead to important areas of mathematics with which the student needs to engage as part of the mathematics curriculum. Thus the task can bring students into important mathematical concepts and through their engagement with the questions, and asking their own questions, they can come to understandings of the mathematics with which they engage.

Of course, the task itself is only a part of the teaching approach. A teacher using such a task has to be very clear what she is trying to achieve, both in mathematics and in developing mathematical processes. There are also important pedagogic questions: for example, how will the teacher organise the students? Will she encourage collaborative working? What kinds of interventions will she make? And so on.

Investigational work in learning and teaching mathematics

It is worth being aware of how such tasks fit into the development of mathematical thinking and research. In the 1980s, internationally, there was great interest in the use of *problem-solving approaches* for teaching mathematics in classrooms. Many of these drew on the work of George Polya (e.g.1945) who introduced the idea of heuristic processes in mathematical problem solving. Polya offered, famously, a four step model for tackling a problem:

1. Understanding the problem
2. Devising a plan
3. Carrying out the plan
4. Looking back. (Polya, 1945; pp. xvi-xvii)

Each of these steps was broken down into instructions or questions (e.g., Draw a figure. Introduce suitable notation. What is the unknown? What are the data?). The idea of *heuristic* "serving to discover" (p. 113) permeated Polya's discussion of these steps in relation to problems that he introduced and discussed. More recently, publications from John Mason, Leone Burton and Kaye Stacey (1982) and from Alan Schoenfeld (1985), among many others, took further these ideas about heuristic. Mason et al. used the terms *specialising*, *generalising*, *conjecturing* and *convincing* to describe the process of working on a problem, seeking and expressing patterns, offering conjectures and proving or disproving them, and convincing yourself, a friend or an enemy. Schoenfeld offered a table of "Frequently used heuristics" (p. 109), starting with *analysis* and *exploration* involving "try to simplify the problem" or "relax a condition and then try to re-impose it". Overall, these sets of heuristics identified strategies in the problem solving process that might be helpful to students tackling a problem. For the teacher using a problem solving approach they offered a structure from which to guide and evaluate students' activity in problem solving.

In the UK in the 1970s and 1980s we saw many teachers using problems or investigational tasks in informal ways in their mathematics classrooms, some of them documented in journals such as *Mathematics Teaching* (e.g., Curtis 1975; Favis, 1975). Eric Love (1988) refers to *mathematical activity*, "In contrast to tasks set by the teacher – doing exercises, learning definitions, following worked examples – in mathematics activity the thinking, decisions, projects undertaken were under the control of the learner. It was the learners' activity" (p. 249). John Mason (1978), reflecting on the introduction of investigations to the summer school activities of an Open University mathematics course, wrote "the main aim, in my view, is to reach a state where the initiative to ask questions rests with the student". A major purpose behind such investigational work has been to encourage students to take increasing responsibility for their own learning.

In 1982, a major UK government report, the *Cockcroft Report,* overtly encouraged investigational work as a part of the general approach to teaching of mathematics in schools (pp. 72-75). This led to more teachers trying out investigational tasks and later (in 1988) the inclusion of investigational work as part of the new National Curriculum in mathematics. It was in this context that I carried out my PhD research, addressing the question *How can we use investigational tasks to teach mathematics more generally*? The study characterised investigational approaches to teaching mathematics and led to insights into the development of mathematics teaching as teachers explored the use of investigational work in their classrooms (Jaworski, 1991; 1994).

We can set such investigational work as a comparison with what Skovsmose and Säljö (2008) refer to "an exercise paradigm" as dominant in the culture of mathematics classrooms widely (p. 40). They write:

> This [the *exercise paradigm*] implies that the activities engaged in the classroom to a large extent involve struggling with pre-formulated exercises that get their meaning through what the teacher has just lectured about. An exercise traditionally has one, and only one, correct answer, and finding this answer will steer the whole cycle of classroom activities and the obligations of the partners involved … (p. 40).

The exercise culture is born and is reinforced through traditional testing and examinations. In contrast, an *enquiry culture* might be envisaged. Skovsmose and Säljö write further,

> The ambition of promoting mathematical inquiry can be seen as a general expression of the idea that there are many educational possibilities to be explored beyond the exercise paradigm (2008, p. 40).

The use of investigational tasks can be seen as promoting mathematical inquiry as expressed here. An inquiry mode involves seeking new visions for classroom mathematics through collaborative activity which leads to more open questions and tasks and less concentration on narrowly focused instruction as in the exercise paradigm.

Working with mathematics teachers for teaching development

My PhD research had brought me into contact with a number of dedicated teachers who were interested in exploring the development of their own teaching, despite an exercise culture still being common in schools. In all cases, these teachers were seriously concerned about their students' development of mathematical thinking; of being able to understand mathematics conceptually and relationally rather than merely instrumentally, that is just following rules or applying formulae (Skemp, 1976). The use of investigational tasks was a step in this direction, but there were many issues to contend with including the school environment, expectations of students, and curriculum and examination requirements The experience has led to my research focus for the next 10 years: what it means to teach mathematics to enable students' conceptual learning. Central questions have been

- What does it mean for students to engage with mathematics with fluency and (conceptual) understanding?
- How as teachers do we create opportunity for this?
- How do we know what sense students are making of what we offer?
- How can we develop teaching practice in informed ways?

For example, we can set and mark homework, set and mark tests. We can sit with students (watch their work, listen to them, ask probing questions) for enough time to glean what they are doing and how they are thinking. We can set up group activity in which students work on and discuss mathematics together, fostering attention to key mathematical ideas and mathematical argument. Research in mathematics educationover many years has studied such processes and how they contribute to mathematical understanding.

For example, research, internationally, has involved:

- Clinical interviews with one or more students, recording and probing their understanding through getting them to work on tasks and articulate their thinking (e.g., Steffe, 1977)

- Classroom studies in which the reality of the classroom is preserved (as far as research will allow) and students' classroom participation is recorded and analysed (e.g., Cobb, Wood and Yackel, 1990)
- Teacher education studies in which teachers' and student teachers' developmental activity has been studied by researchers, often their own tutors in teacher education programmes (e.g., Cooney, 1994; Jaworski & Gellert, 2003).

In teacher education, studies have focused on how teachers create opportunity for pupils to engage with mathematics seriously and enjoyably. Research into continuing professional development programmes (CPD) and initial teacher education programmes (ITE) has opened up explorations into how differing models of education for teachers contribute to better understandings of mathematics teaching and how teaching develops. The *Journal of Mathematics Teacher Education*, initiated in 1998, documents much of this research. In more recent years we have seen a first *International Handbook of Mathematics Teacher Education* (Wood, Jaworski, Krainer, Sullivan Tirosh 2008) and an ICMI study (number 15) into the professional development of teachers of mathematics (Even & Ball, 2009). These publications document the processes of mathematics teacher education and teaching development and emphasise the importance of research in these areas.

During this time we have seen socio-political conflict in mathematics education between the theories of educators about what constitutes good mathematics teaching and the demands of politicians to see higher achievement in schools, perhaps as evidenced in international surveys such as the TIMSS (e.g., Mullis, Martin, Gonzalez, & Chrostowski, 2004]). For example, in the USA, on the one hand we have seen the NCTM standards and the "reform" movement and on the other, "the math wars" and the President-led "no child left behind" project (e.g. Wilson, 2003; Boaler, 2008). On the surface of things, the position looks paradoxical: surely *good mathematics teaching* is what is required for *higher achievement in schools*. The conflict arises over what exactly we mean by "good mathematics teaching" and how this is achieved. Mathematics educators widely, as evidenced in the publications referenced above, seek teaching that engages students in mathematics and emphasizes conceptual understanding. Politicians, sometimes supported by mathematicians, seek fluency with arithmetic and algebraic skills, and success in tackling standard questions in fundamental topics. For example, in the UK, since the introduction of a National Curriculum in 1988, we have seen growing imposition of *tests* for students at all levels of education. This has put huge pressure on teachers to teach to the test and correspondingly to narrow students' mathematical experience. The exercise culture is alive and well. It is a challenge for educators as to how to work with teachers to enable both mathematical understanding and higher achievement for students.

In parallel with these socio-political forces, in mathematics education we have seen a shift theoretically from a largely constructivist way of thinking about mathematics learning and teaching towards a more sociocultural perspective embracing complexity. The problem-solving movement focused largely on the conceptual development of the individual learner – a constructivist position – which can be seen to marginalize sociocultural factors in society and schooling. It has become clear that influences of gender, race and social class, alongside cultural, institutional and systemic factors in schooling and society, dominate what occurs in classrooms and how classroom teaching is perceived by students, parents and teachers. A focus on individual cognition, however soundly based theoretically, is only a part of the complex educational setting. So, the challenge for educators includes the embracing of this complexity.

Research and its contribution to teaching development

Acknowledgement of sociocultural complexity challenges teachers and educators to reconceptualise teaching processes and re-address what it means to teach. Much research is focused on knowledge and practice in teaching, the needs of teachers and ways in which teachers can work with students for understanding and achievement. Often, research studies developmental programmes, charting the progress of such programmes and evaluating outcomes. For example, in the USA, there has been a focus on curriculum materials that support the reform movement (e.g. Remillard, Herbel-Eisenmann & Lloyd, 2009). Educators have worked with teachers to support their use of such materials and have simultaneously studied teachers' development of new ways of working fostered by the materials. In such projects, the involvement of teachers is central, since it is only the teachers who work directly with students – educators, in the main, stand outside this possibility. Therefore the ways in which teachers engage in development and research seems crucial to what is learned and what development occurs.

It has become increasingly clear to me that research into teaching development in very many cases does not stand outside the developmental process, but rather contributes to that process. It has therefore seemed important to consider ways in which research itself can be a developmental tool (Jaworski, 2003). I speak here of research in which teachers are centrally engaged, either in *clinical partnership* with educators, or *co-learning agreements* (Wagner, 1997). In the former (CP), the educators take the lead in conducting the research, with teachers working very closely to achieve research aims. Here teachers are insider researchers, looking into their own teaching and their students' learning, whereas educators act as outsider researchers studying the teaching of others. In the latter (CLA) both teachers and educators are central to proposing and promoting the research. They may take insider and outsider roles respectively, as in clinical partnership; however, one or both might be *both* insiders and outsiders, exploring their own teaching as well as that of others (Jaworski, 2004, 2008).

Since 1994, I have worked in partnership with teachers and academic colleagues in different projects using both of these modes. For example, a project with Elena Nardi and Stephen Hegedus studying the teaching of mathematics in university tutorials fits into the clinical partnership mode (Nardi, Jaworski & Hegedus, 2004); a project in Norway, *Learning Communities in Mathematics* with many colleagues at the University of Agder, shows teachers and didacticians[2] entering into co-learning agreements in which they worked together to promote teaching development (Jaworski, Fuglestad, Bjuland, Breiteig, Goodchild, & Grevholm, 2007). Very important in such projects has been the collegial relationships that have developed between those teaching mathematics whose practice is the focus of study and the mathematics educators who have been primarily responsible for the research.

Central to much of this research has been a study of inquiry approaches towards knowing more about issues in teaching and learning mathematics. These have included inquiry for students doing mathematics in the classroom, inquiry for teachers in developing their teaching and inquiry for academics (or didacticians, or teacher educators) working with teachers to develop teaching. So, for example, teachers design inquiry-based tasks (such as the task above on filling rectangles with squares) to engage students with mathematics and promote understanding of mathematical concepts. Teachers engage in inquiry themselves into aspects of teaching and their pupils' learning, with questions about the nature of tasks, or about the pedagogies of questioning or of group work, or into formative assessment processes. Teacher educators engage in inquiry into processes in teacher education; for example, into ways in which their work with teachers can promote more effective ways of educating students in mathematics. Such inquiry may be conducted at very informal levels in which professionals learn from inquiry in practice. However, when it is conducted in systematic ways and its outcomes documented, it becomes research which can contribute to knowledge more widely (Stenhouse, 1984). So, for example, such research might address what it means to teach mathematics to enable students' conceptual learning. This might include processes and strategies in teaching mathematics, design of tasks for mathematical work, use of resources, including ICT, or influences of the real world of schools and classrooms. It might address how as teachers we *develop* mathematics teaching, or how research can be *integral* in promoting development and charting its progress. All such studies try to take account of the full sociocultural complexity of classrooms and the many factors influencing what is possible for teachers and students.

Until 2007, my research focused mainly on mathematics education in schools and involved research relations/partnerships with school teachers (e.g. Jaworski, 1998; 2008a). However, since this time I have been working in a School of Mathematics, in a Mathematics Education Centre where our research focus is mainly on the teaching and learning of mathematics at University level. Thus, along with other colleagues, I teach both mathematics and mathematics education to undergraduates, as well as supervising research students, and I conduct research in these areas. It has been extremely interesting, and challenging, to apply my previous experiences from school teaching and associated

[2] Didacticians are academics and/or teacher-educators who work in didactics, the transposition between a subject such as mathematics and the teaching-learning interface in which the subject is taught and learned.

theoretical perspectives to teaching and research in university level mathematics. This is my focus for the remainder of this paper.

New practices and new challenges

Since 1984 when I stopped being a school teacher of mathematics, I have worked in university departments in which my teaching largely has been with students who would become teachers (pre-service teachers), or with practising teachers. The focus of this teaching has been on how to teach mathematics. Now, I am again a mathematics teacher. I teach mathematics to undergraduate students in mathematics and engineering programmes. The environment is very different from a school environment; the resource is handled differently, the environment is different, expectations are different, the culture is different. In a very real way, as I began this teaching, I felt de-skilled. Despite the knowledge and long experience of thinking about issues in learning and teaching mathematics, I began to experience new kinds of practical situations for which I do not have finger-tip practical knowledge, different kinds of issues to consider, and a new culture of practice.

Teachers' interactions with learners are of a different kind from a school environment and I have to become accustomed to new norms and expectations. A major difference involves working with large number of students in a lecture – I cannot easily know what individual students are doing and where they stand with the mathematical content. Also, some students do not come to lectures, and some do not do the requisite study to support lecture material. Overall, there feels to be a much greater distance between teaching and learning than I have been accustomed to in school practice. Over the three years I have been in this post I have a sense of becoming acculturated – being drawn into a new culture of practice, of university teaching and learning, with its own complexity: norms, characteristics, expectations and ways of being.

Part of the acculturation process has involved becoming familiar with how those who have been used to working within this environment think about what they do. Just talking with my colleagues one to one has provided insights into how they think about teaching and what they actually *do* in teaching situations. I have attended lectures given by some of my colleagues, discussed my observations with those teaching and reflected on outcomes. In 2007, as I mentioned above, we initiated the series of seminars entitled *How we Teach*. These have afforded considerable opportunity to hear the perspectives of colleagues and discuss issues in learning and teaching. I have also engaged in associated research as I shall discuss below.

As an accustomed reflective practitioner with long experience of working with teachers to conduct research into teaching and hence develop teaching, I now found myself thinking very much about my own teaching practice. How would I develop new knowledge, how would my cultural awareness develop, and what issues and challenges would I face? Thinking critically is an important part of such questioning and reflection. Based in Wenger's (1998) concept of *Community of Practice*, and his constructs of *engagement*, *imagination* and *alignment* as part of belonging to a community of practice, I had developed the concept of "critical alignment" related to earlier research (Jaworski, 2006, 2008). Now, I found myself faced with critical alignment in my new role. I needed to engage with the practices in university teaching; I needed to use my imagination to see how I could interpret this role, and I needed to align with norms and expectation in the role. For example, I am expected to offer lectures and tutorials, and to fit into ways of doing and being in the School of Mathematics. While it would clearly be impossible to fulfil my new responsibilities without such alignment, I owe it to myself to bring a critical way of looking at what I am doing in relation to norms and expectations. Which of my ways of thinking about mathematics learning and teaching would need to change and how would this affect my own theories and beliefs about learning and teaching?

For example, if we think about formative processes in the Mathematics-Learner-Teacher relationship, the engagement involves interaction between learners and teacher from which the teacher

1. creates (with students) norms for ways of working, thinking and interaction
2. can listen closely to learners' use of mathematical language and articulation of mathematical concepts
3. designs tasks to challenge problematic conceptualisation and create opportunity for students to engage

We might label 1, 2 and 3 as respectively Management of Learning (ML), Sensitivity to Students (SS), and Mathematical Challenge (MC). These three elements form the *Teaching Triad*, a construct that emerged from my PhD research with secondary school teachers (Jaworski, 1994). One of these teachers suggested that his whole teaching approach could be seen as *management of learning*, with the other two elements (SS and MC) overlapping within. Subsequent research (Potari & Jaworski, 2002) suggested that the conjunction between SS and MC, in which the degree of challenge was seen to be appropriate for both cognitive and affective student needs/dispositions, led to *harmony*. *Harmony* is a state in which the challenges offered are appropriate for students, in order to achieve mathematical progress (see Figure 2).

In my new role, it became necessary to think about relationships between the three elements in working with university students. Thinking in terms of the TT is one way of bringing *critical alignment* into teaching practice. A key question is, what does critical alignment *look like* in practice?

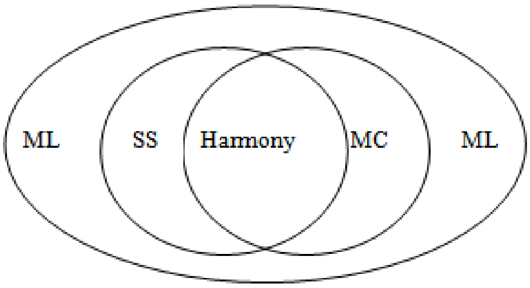


Figure 2. The Teaching Triad. The positioning of circles representing SS and MC within ML is crucial for the achievement of harmony: too little overlap and harmony becomes minimal or non-existent.

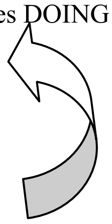
Developing Teaching

Developmental cycles of activity

So, the question arises, What does one do to develop teaching within the constraints of existing norms and expectations? I have proposed in school teaching situations that the use of *an inquiry cycle* is one way to think about and promote development. We might think first in terms of a *teaching cycle*:

A teaching cycle involves DOING

- Plan
- Act
- Reflect
- Feedback



The teaching cycle can be seen as a part of the 'normal' teaching process. One focus of critical alignment is to discern what the 'normal' process looks like in my new environment; either for myself, or for my colleagues more generally.

An inquiry cycle is a modification of the teaching cycle and involves overtly *inquiring into doing – and learning from the process.* Thus, there is an explicit intention to question and to analyse:

An inquiry cycle involves DOING, OBSERVING and ANALYSING

- Critical questions about teaching
- Plan/replan
- Act and Observe
- Reflect and Analyse
- Feedback to planning

Here, as well as the stages of *act* and *reflect* we see elements of *observe* and *analyse*. Initial questions about teaching lead to specific aspects of teaching which are being studied. So as well as reflecting and feeding back to the planning stage, as part of the normal process of teaching, specific *observation* is planned which results in some form of data which can then be *analysed* alongside reflection (Jaworski, 2008b)

One example can be seen in my development of teaching in a first year module on *Mathematics for Materials Engineering Students*.

An example: Developing teaching with first year engineering students

In my second year of teaching this module, the specification for the module was revised to include more topics and I was allocated two lectures and one tutorial per week. 73 students were registered, with a wide range of experience ranging from GCSE (taken 2-3 years previously) to a grade A at A level. This meant a considerable difference in learning needs. The situation held certain initial problems including difficulties of access to students, difficulties of directing input and difficulties in pedagogy. Regarding access, how would I get to know 70+ students, their experiences and needs, in a lecture environment? Regarding my own input, I had to consider, what are the key concepts and how much depth is needed? What will most students be familiar with from their school work? For example, could I assume that most students will be familiar with basic functions and equations? Pedagogical questions included what approaches to use – how to interact with students and gauge their degrees of understanding. There were questions about where to *prove* results and where to focus just on how to use and apply. For example, as part of trigonometry, students had to know the sine and cosine rules. Certain texts for engineering students introduced these rules and demonstrated their use without showing their origin or proving their validity. There were also questions about resources and how these would contribute to students' learning. For example, what are the pros and cons of using PowerPoint in lectures; how could I make the most of the LEARN server (the local VLE); what software could be valuable?

In a spirit of inquiry, I made some initial decisions with a view to observing and reflecting on outcomes. I took into account feedback from students in the evaluation forms from my previous year's module with a smaller group of students. These students had liked clear examples worked through on the OHP, although they had not been against presentation by PowerPoint. I therefore decided to use the animation possibilities of PowerPoint to explain concepts but also to work through examples using the OHP. Later in the year, I discovered the value of a tablet PC in which I could combine to some extent both of these modes. After a lecture, I placed PowerPoint slides with hand-written annotations onto the LEARN server for general access.

More challenging decisions concerned how I would *engage* students with the mathematical concepts I was presenting; get them really involved with the mathematics. In the previous year, with the smaller group of students, I had used a selection of investigative tasks of the style discussed above to encourage thinking and involvement. This had taken time, and although students had shown interest and evidence of engaging with the tasks outside lectures it was clear that the chosen tasks had not addressed many of the conceptual difficulties that students were experiencing (Jaworski, 200x). With more than 70 students, I needed to consider what was possible in a lecture environment and what kinds of tasks would address concepts more directly. I wanted to get students inquiring at an appropriate level – to offer tasks that could be tackled at several levels, providing easy access and support, and more depth and challenge for the more experienced. Also, it seemed important to provide back-up materials for all, especially those who need more basic emphasis and practice (The HELM materials – *Helping Engineers Learn Mathematics* – http://www.lboro.ac.uk/research/helm/). In addition, I designed, CAA tests (Computer Aided Assessment) to provide periodic formative and summative opportunities.

I decided to use *GeoGebra* (a free geometry/algebra graphical package – http://www.geogebra.org/cms/) to create an environment for exploration. I asked for our weekly tutorial to be held in a computer laboratory and prepared tutorial sheets with a range of tasks to involve students in mathematical inquiry and conceptual activity. For a variety of reasons, these did not achieve my aims for them. I had not been satisfied with students' engagement and evaluation feedback suggested that students had not found them a helpful use of time (Jaworski, in press). Thus, in teaching the module in the following year (my third year of teaching at this level), I modified both the sheets themselves and my mode of using them with students. I felt that engagement was better than in the previous year, although still not what I hoped for; feedback comments were mainly positive, although students did not give much detail of what they had found valuable. I discuss some of the issues below.

Developmental issues

I will mention briefly issues for development at two levels: first, at the local level of my own teaching in the example above; second in more general terms.

In current student culture, students expect to be told clearly what to learn and provided with the essential resources. It is hard, therefore, to develop an inquiry culture in which students take appropriate levels of responsibility in working with the materials and resources offered. The structure of lectures and tutorial with the whole cohort makes it hard to facilitate group work and discussion between students, and to get in-depth feedback about student understandings. The computer lab setting goes some way towards providing possibilities for discussion with groups of students about mathematical concepts, but more is needed. It is too easy to get sucked into giving good presentations/explanations at the expense of engaging and challenging students. When this happens, student engagement becomes “following”, “keeping with” rather than grappling with key ideas.

I am aware of my own acculturation to ways of being in university learning and teaching and of my corresponding critical alignment. It is unrealistic to expect to be able to create learning environments as in school classrooms. Much of the responsibility for approaches to learning has to rest with students. So, the questions then are what a teacher can do within this setting. Over my first three years in this position, my engagement with the inquiry cycle has included design of materials, trialling of GeoGebra as a means for inquiry-based learning, becoming aware of the possibilities and limitations of lectures and tutorials, and gaining feedback from students through limited face-to-face contact, questionnaires, CAA tests and examinations. Much of what I discuss here concerns my *management of learning* (ML) within the module. *Mathematical challenge* (MC) for students comes, generally, in the challenging nature of concepts they need to understand and specifically in the tasks and questions I design for tutorials. *Sensitivity to students* (SS) can be seen in the ways in which I make provision for the learning of the whole group and take account of feedback. However, I am not satisfied that I am achieving harmony. I still feel distant from students at an individual level, and need to consider how to gain a more in-depth perception of their learning needs and achievements that will inform my planning better

My developmental intention for the coming year in which I will teach this module for the fourth time is to continue to focus on fostering conceptual learning. I intend to try to make lectures more interactive, to continue with GeoGebra in tutorials and to design tutorials to achieve small group discussion of key concepts. The latter will require some reorganisation of the structure of the module, and I hope to involve key colleagues from mathematics and engineering to discuss possibilities. One of the problems I have experienced in my developing process with this module is that I have been working largely alone. The *How we Teach* seminars provide a collegial basis for discussion of issues broadly and I need to explore further how an inquiry community might be developed to look with colleagues at specific issues in more depth.

In parallel with my teaching in this module, of course, I have engaged with other teaching and research. One research project, together with a doctoral student, is to explore the teaching of linear algebra in first year mathematics. We have worked very closely with the mathematician who lectures in linear algebra. Here we form a small research team of three, and in effect, a small community of inquiry. I do not have space here to discuss this project in detail (however, see Jaworski, Treffert-Thomas & Bartsch, 2009) but I mention it as a contrast with my own research discussed above. The linear algebra team has one insider researcher and two outsiders and has demonstrated considerable potential for discussion of practices and issues. In particular, the lecturer has been supported and encouraged to think critically about his teaching and to consider and promote changes in teaching. My research into the engineering module has involved only myself as an insider researcher. The research has not been rigorous. It would have been valuable to have another insider or outsider researcher working with me to support momentum and encourage systematic engagement. As I have noticed in research in schools, it is very hard for a teacher to engage in insider research alone. There are too many pressures from normal day to day practice, and not enough support to encourage a critical approach. Thus, the inquiry community seems an essential factor amongst any group of teachers who seek to develop the teaching of their students.

In conclusion, considering "how we teach" is central to a process of being good teachers and developing teaching. Developing teaching is not straightforward or comfortable – it requires addressing hard questions, making compromises, being critical. It is helpful to know more about its impact on students, which raises questions as to how we can achieve this. Such questions can be addressed valuably within a community of inquiry.

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