Mathematical Modeling Of Closed Loop Boost Converter

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Abstract: Dc-Dc converters are electronic power circuits that convert a voltage of Dc to another voltage level of Dc. These circuits are often referred to as switching power supplies or switchers. The mathematical modeling of basic boost converter circuit is defined in this paper. High-quality, low or light weight, strong and effective control supplies are needed for cutting edge electronic frameworks. Linear power controllers working on the basis of a voltage or current divider are inefficient since they are limited to output voltages lower than the input voltage and also have a very low power density. In their active (linear) modes, linear regulators are bringing operated, but switching regulators are used at higher power levels. In turn-on and turn-off states, switching regulators use power-electronic semiconductor switches. There is a slight power loss in these states due to this, but switching regulators can achieve high efficiencies in energy conversion. It is possible to operate modern electronic power switches at high frequencies. The size of transformers, filter inductors and capacitors is reduced at the higher operating frequency, along with enhancements in the dynamic characteristics of converters with increasing operating frequencies. The closed loop converter has been designed for the stepping up from 12V to 30V with switching frequency of 25KHz.

Keywords: Boost Converter, Compensation, transfer function

1. Introduction

A boost converter is a DC to DC converter which is involved in stepping up voltage from its input. It basically consists of an inductor, output capacitor, diode and the load. The transistor and the diode are the semiconductor devices and the capacitor or the inductor are storage elements[1]. The capacitor is involved in reducing the ripple in the voltage. The supply for the boost converters may be the sources from the solar panels, batteries, rectifier, DC generators. The output voltage of the DC -DC converter is greater than its input voltage. By being completely on or completely off, the transistor acts as a switch. In battery powered systems the cells or the batteries are stacked up in series to achieve higher voltage rates but it's not possible to stack up batteries in many applications so in these areas the boost converters are modelled for these applications[9]. The converters are also used in hybrid electric vehicles and lighting systems where space is a constraint.[3] The basic switching operation of the converter is shown in Fig1.
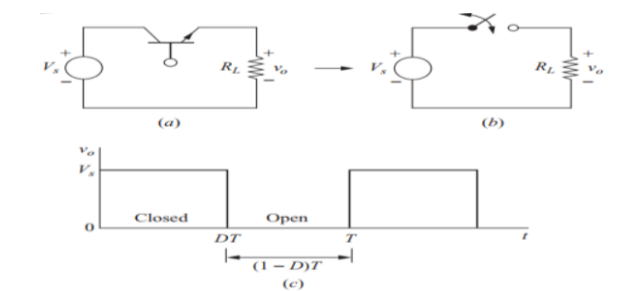

Fig. 1 (a) Simple converter for dc-dc switching; (b) Equivalent switching; (c) Output voltage;

Section II gives the implementation of boost converter, section III gives the details on closed loop control and compensation design, finally section IV gives the simulation results for the design.

2. Implementation of boost converter

The equation gives the average or dc component   of the output voltage, as in equation 1

$$Vo = \frac{1}{T}\int_0^{DT} Vs(t)dt = VsD \longrightarrow \quad 1$$

The average value of the output voltage is controlled by adjusting the duty cycle D. The boost converter is shown in Fig 2. The boost converter is a switching converter which operates at regular intervals by opening and

closing an electronic switch[7]. When switch is closed during the operation, The diode is in reverse. The Kirchhoff voltage law around the path containing the source, inductor, and closed switch is given by the equation 2.

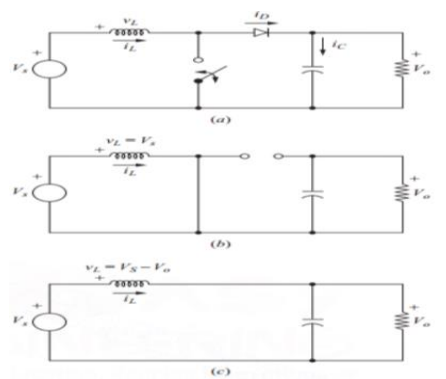$$V_L = Vs = L\frac{diL}{dt} \; ; \; \frac{dil}{dt} = \frac{Vs}{L} \longrightarrow 2$$

Fig 2: The boost converter. (a) Circuit; (b) The equivalent circuit for the closed switch; (c) The equivalent circuit for the open switch

The rate of change of current is a constant, hence the current increases linearly when the switch is closed. The change in inductor current is computed from equation 3.

$$(\Delta il)closed = \frac{VsDT}{L} \longrightarrow 3$$

The diode becomes forward biased when the switch is opened, and the equation for the switch is given equation 4,

$$(\Delta il)open = \frac{(Vs-Vo)(1-D)T}{L} \longrightarrow 4$$

The net change in inductor current should be zero. Therefore

$$(\Delta il)closed + (\Delta il)open = 0$$

Vo is given by equation 5,

$$Vo = \frac{Vs}{(1-D)} \longrightarrow 5$$

Output power is given by equation 6.

$$Po = VoIo \longrightarrow 6$$

The input power is shown in equation 7

$$Pi = VsIs \longrightarrow 7$$

Equating input and output powers as in equation 8,

$$VsIl = \frac{Vs/(1-D)^2}{R} = \frac{Vs^2}{(1-D)^2}R \longrightarrow 8$$

Maximum and minimum inductor currents are determined by using the average value and the change in current from 9 and 10,

$$Imax = Il + \frac{\Delta Il}{2} = \frac{Vs}{(1-D)^2R} + \frac{VsDT}{2L} \longrightarrow 9$$

$$Imin = Il - \frac{\Delta Il}{2} = \frac{Vs}{(1-D)^2R} - \frac{VsDT}{2L} \longrightarrow 10$$

In order to have continuous inductor current Imin positive as in equation 11.

$$Imin = 0 = \frac{Vs}{(1-D)^2R} - \frac{VsDT}{2L} \longrightarrow 11$$

The minimum combination of inductance and switching frequency for continuous current in the boost converter is therefore given by equation 12.

$$(Lf)\,min = \frac{D(1-D)^2R}{2} \longrightarrow 12$$

3. Closed loop controland compensation design

In order to provide good noise immunity and optimum flexibility in the selection of inductor values and capacitor types, the voltage regulators use a PWM voltage mode control system with external loop compensation. The switching frequency can be configured from 25KHz to above 1.5MHz to provide the ability to customize the configuration in terms of size and efficiency. The parameters of the compensation network have to be properly calibrated to achieve the desired efficiency. Fig 3 shows a typical voltage-mode control and voltage-mode error-amplifier synchronous boost converter with error amplifier.[4]
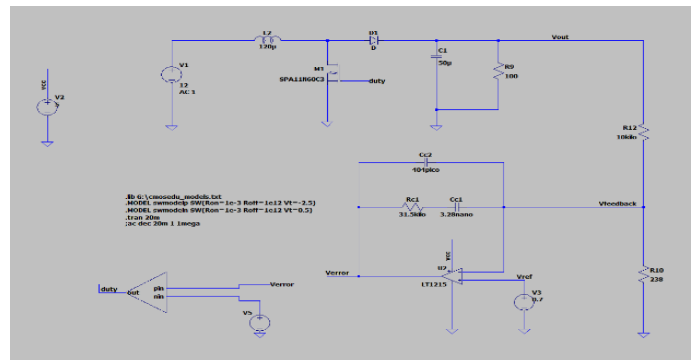

Fig 3: Typical synchronous boost converter

The boost converter is designed with three basic blocks, Power stage (Gp(s)), PWM generator is, the compensator block (H(s)) consisting of the error-amplifier as shown in fig 4[11].
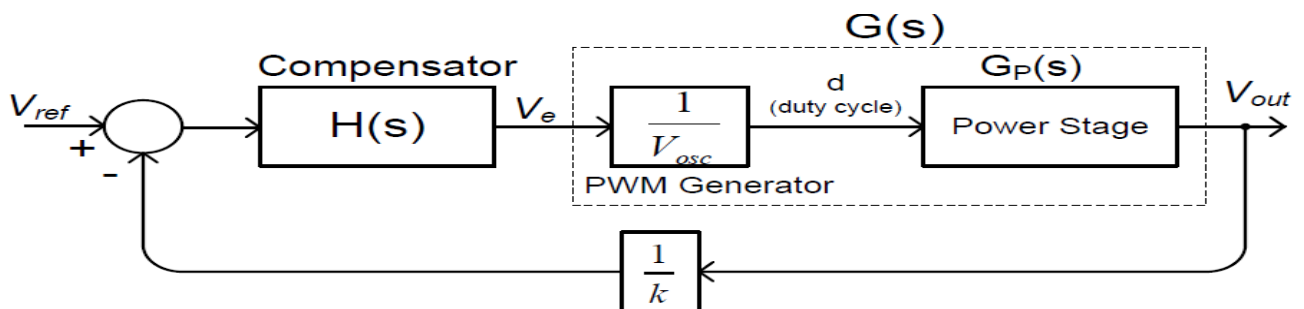

Fig 4: The synchronous boost converter block diagram.

To obtain the equation as below, the transfer function of the PWM generator and the power stage are combined as in 13.

$$G(s) = Gp(s) * \frac{1}{Vosc} \longrightarrow 13$$

Where the transfer function of the power stage is as in 14:

$$G_p(s) = \frac{V_{out}}{d}(s) = \frac{R_{Load}(C_o \cdot ESR \cdot s + 1)}{L_o C_o \cdot s^2 (R_{Load} + ESR) + s \cdot (L_o + R_{Load} \cdot C_o \cdot ESR) + R_{Load}} \times V_{in} \longrightarrow 14$$

Fig 5 displays the Bode plot of the power stage with a slope of -40dB/dec., The double pole causes the gain to fall to the zero frequency that compensates for one of the Zero[8]
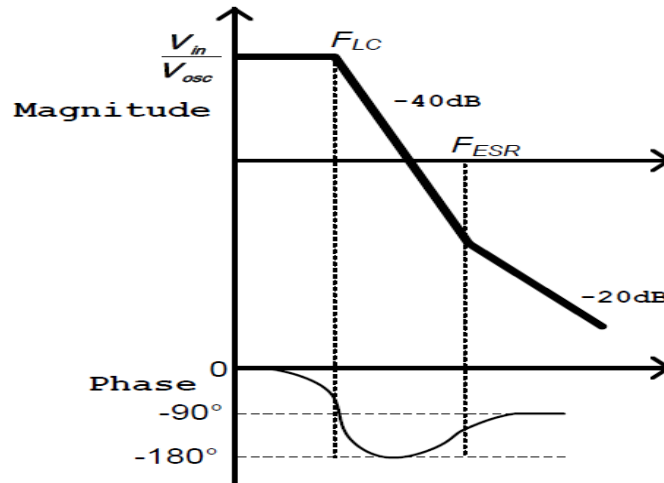
Fig 5: The Bode plot of the boost converter's power stage.

The system's loop gain is the product of transfer functions along the loop of the closed power. The loop gain, therefore, is given by equation 14.

$$M(s) = \frac{1}{K} H(s)G(s) \qquad \longrightarrow 15$$

There are three types of compensation depending on the crossover and power stage frequencies[5]. The type of compensation is choosen based on the table given below. For applications where the zero frequency induced by the output capacitor and its ESR (FESR) is lower than the closed loop bandwidth (F0)[10], as shown in table 1 above, type II compensation is used. Fig 6 displays the diagram of the Type II compensator.

Table 1 : The type of compensation and zero crossover frequency position

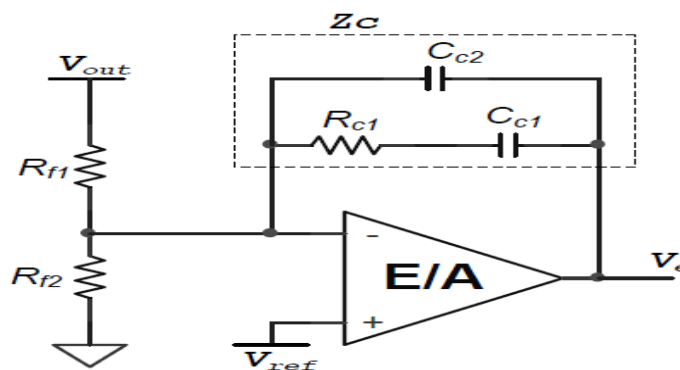| Compensator Type	Relative location of the crossover and power-stage frequencies
Type II (PI)	FLC < FESR < F0 < FS/2
Type III-A (PID)	FLC < F0 < FESR < FS/2
Type III-B (PID)	FLC < F0 < FS/2 < FESR

Fig6:Type II compensator

The following equation 16 gives the transfer function of the compensator.

$$H(s) \approx -\frac{1 + R_{C1} \cdot C_{C1} \cdot s}{R_{f1} \cdot s \cdot C_{C1} \cdot (R_{C1} \cdot C_{C2} \cdot s + 1)} \qquad \longrightarrow 16$$

As   given below, the compensator has a pole at the origin

and another pole and one zero as in equation 17 and 18.

$$Fz1 = \frac{1}{2\pi Rc1Cc1} \quad \longrightarrow 17$$

$$Fp2 = \frac{1}{2\pi Rc1Cc2} \quad \longrightarrow 18$$

The phase of the loop must never be 360º/0º and the phase at crossover frequency should be at least 45º[6] in order to provide a stable device. The compensator has one pole at origin and the other is given by equation 18, the zero of the compensator should be located at a frequency lower than the double poles of the LC as in equation 19, filter to ensure that the loop phase does not drop near 0º.

$$Fz1 = 0.75 * FLC \quad \longrightarrow 19$$

It is important to position the second pole in higher value than the cross-over frequency, so that equation 20, of the compensation network at a its phase is not less than 45º.

$$Fp2 = \frac{Fs}{2} \quad \longrightarrow 20$$

The transfer function from the error amplifier output to the output voltage is provided by by equation 21

$$G(s) = \frac{V_{out}}{V_e}(s) = \frac{V_{in}}{V_{osc}} \cdot \frac{R_{Load}(C_o \cdot ESR \cdot s + 1)}{L_o C_o \cdot s^2(R_{Load} + ESR) + s \cdot (L_o + R_{Load} \cdot C_o \cdot ESR) + R_{Load}} \quad \longrightarrow \quad 21$$

And RC1 is found by solving the transfer function and is given in the below equation 22

$$R_{C1} = \frac{R_{f1} \cdot F_{ESR} \cdot V_{osc} \cdot F_0}{V_{in} \cdot F_{LC}^2} \quad \longrightarrow 22$$

As Fz1 was chosen and Rc1 is determined, it is possible to calculate Cc1 as in 23

$$C_{C1} = \frac{1}{2\pi \cdot R_{C1} \cdot F_{Z1}} = \frac{1}{1.5\pi \cdot R_{C1} \cdot F_{LC}} \quad \longrightarrow \quad 23$$

Similarly, CC2 can be calculated as in equation 24

$$C_{C2} = \frac{1}{2\pi \cdot R_{C1} \cdot F_{P2}} = \frac{1}{\pi \cdot R_{C1} \cdot F_S} \quad \longrightarrow \quad 24$$

4. Results and discussion

The boost converter has been designed for the following specifications. Vin = 12V, Vout = 30V, the load is a resistance of 50 ohm, switching frequency is at 25 kHz.The duty ratio is D = 0.6. The minimum inductance for continuous current is determined is Lmin= 96μH. To provide a margin for continuous current to be ensured, let
L = 120 μH.
Imax = 1.5 + 1.2 = 2.7 A
Imin = 1.5 – 1.2 = 0.3 A
Fesr = 2.25KHz
Fo = 2.5KHz
Fz1 = 1.56KHz
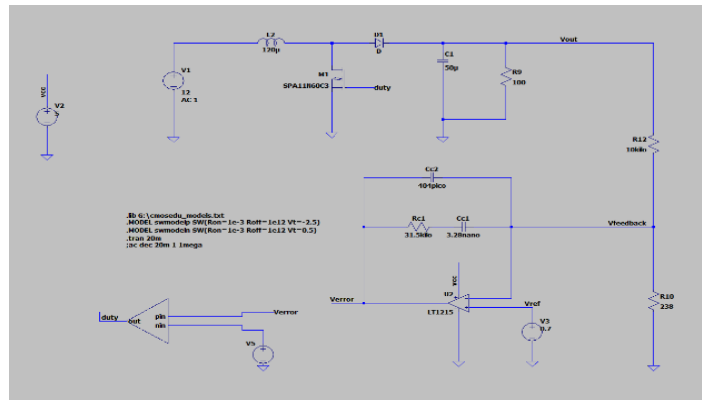Fp2 = 12.5KHz
Rc1 = 31.5Kohm
Cc1 = 3.28nF
Cc2 = 404Pf

Fig 7: Boost converter schematic in LTspice

Fig 7 shows the complete schematic of the boost converter which has been simulated in LTSpice. It includes the compensation network as well.
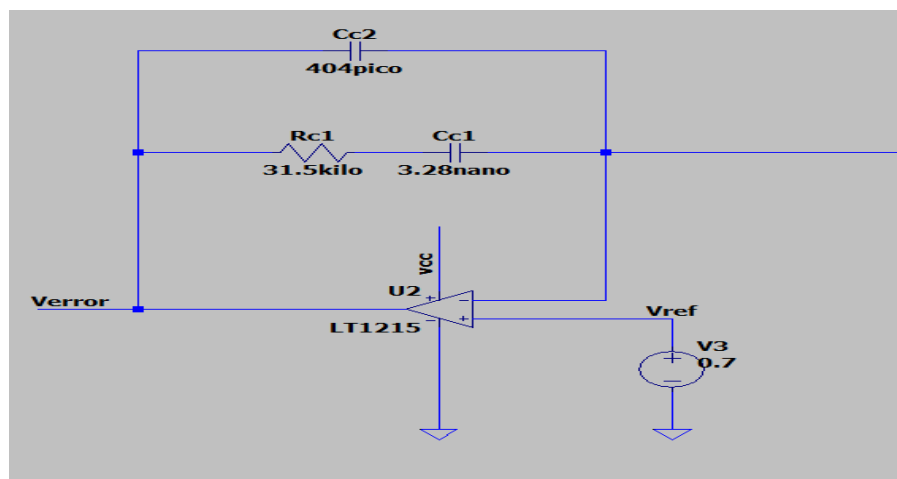

Fig 8: Compensation Network

Fig 8 represents the compensation network where one of the input to error amplifier is from the output of the power stage and the other voltage is set to 0.7V which is the reference voltage to error amplifier.
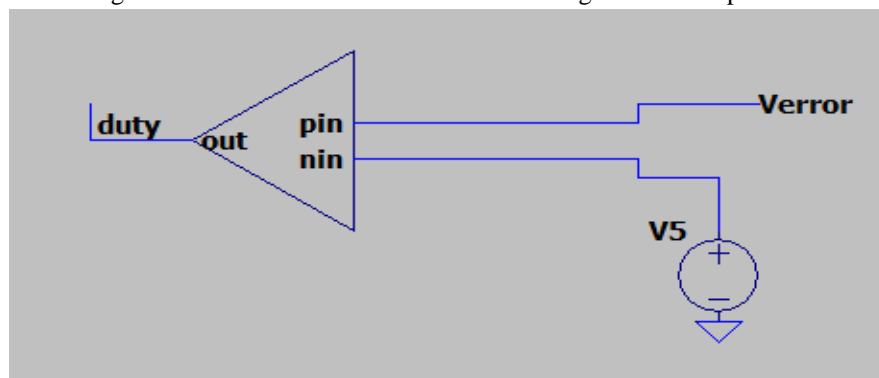

Fig 9: PWM generator

Fig 9 is the PWM generator. It takes one of the inputs from the error amplifier and other input is the sawtooth voltage whose peak amplitude is 5V which is involved in generating the duty for MOSFET.

Fig 10 shows the output of the boost converter whose value has been stepped up to 30V and Fig 11 represents the charging and discharging of the inductor over the duty Cycle. Fig 12 gives the output of the resistor divider network whose value is adjusted to 0.7V to match with the reference voltage which is one of the input to error amplifier.
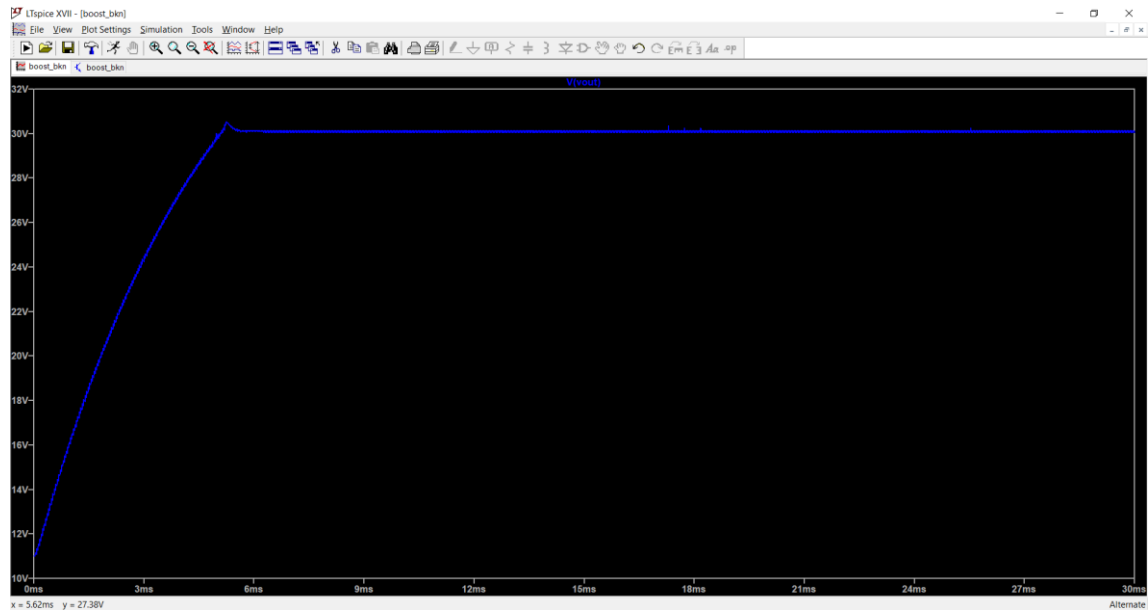

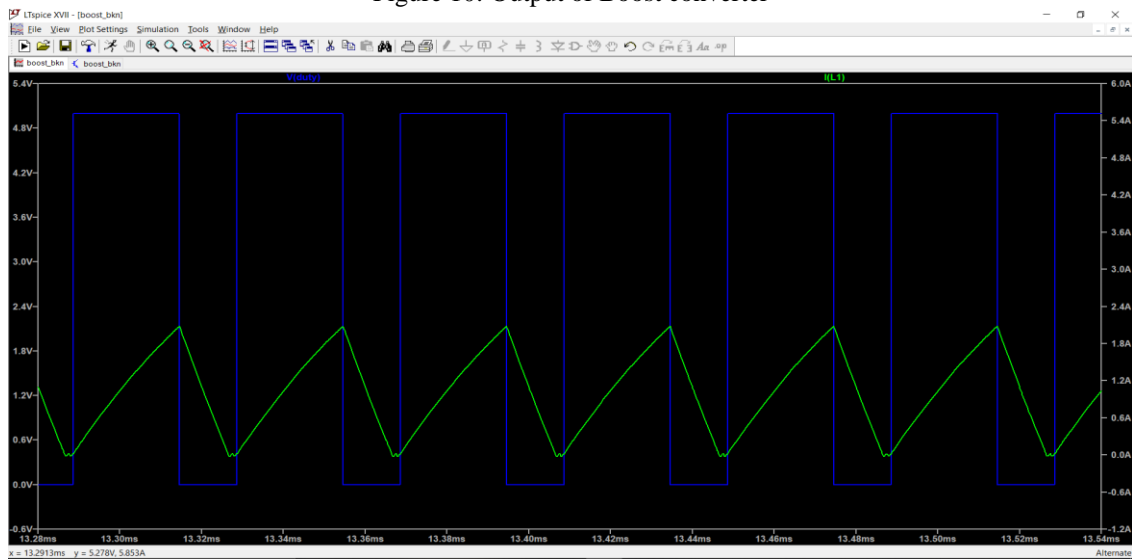Figure 10: Output of Boost converter

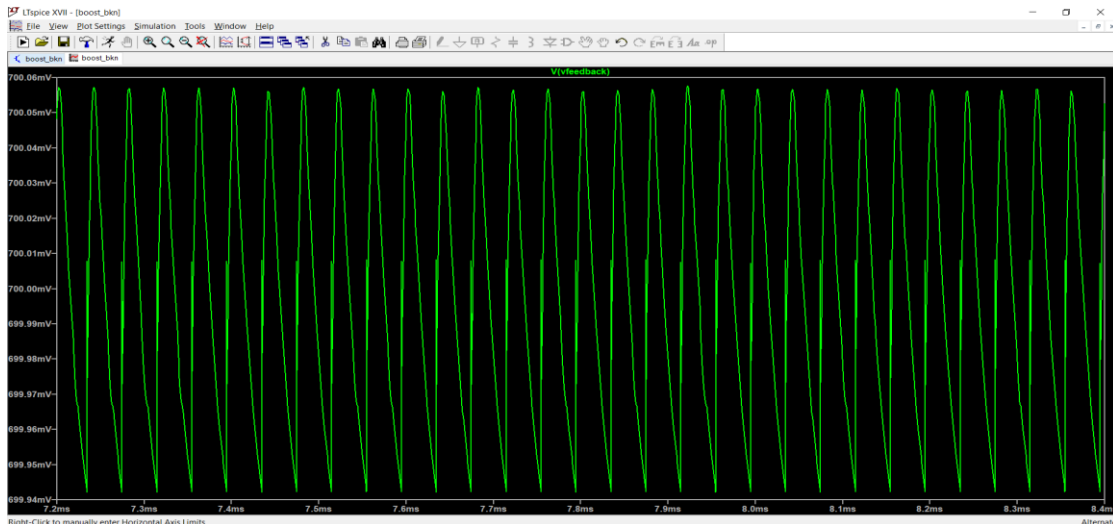Fig 11: Charging and Discharging of Inductor.

Fig 12: Output of resistor divider network.

5. Conclusion

It is evident from the results obtained that the boost converter raises the voltage from 12 to 30 volts in accordance with the parameters previously obtained, satisfying the desired output voltage specifications of 30 V at a frequency of 25 kHz. This mathematical modelling of boost converter design has met all of the requirements specified previously. Simulations of LTSpice using measured parameters have been carried out and corresponding waveforms have been obtained. With a maximum output ripple of 1 percent, the output voltage around the output capacitor is 30V. An additional constraint needs to be imposed on the load, however, boost converter simulations have been carried out and it has been observed that the output voltage of varying duty cycles often varies.

6. Acknowledgment

References

1. Muhammad H. Rashid, "Power Electronics, Circuits, Devices, and Applications", Third Edition, Pearson Education, Inc., 2004.
2. Carl Nelson & Jim Williams, "Linear Technology, LT1070 Design Manual", 1986.
3. Marty Brown, "Practical Switching Power Supply Design", New York: Academic Press,Inc., 1990.
4. Tasi-Fu Wu, Yu-Kai Chen, "Modeling PWM DC/DC Converters Out of Basic Converter Units", IEEE Transactions on Power Electronics, Vol 13, Iss 5, 1998.
5. B. M Hasaneen, Adel A. Elbaset Mohammed, "DESIGN AND SIMULATION OF DC/DC BOOST CONVERTER", 12th International Middle-East Power System Conference, 2008.
6. Arnab Ghosh, Subrata Banerjee, Member, "Third International Conference on Computer, Communication, Control and Information Technology (C3IT)," 2015.
7. Lijie Qi, Maosen Xiao, "Robust Control of Boost Converter", 2nd International Conference on Cybernetics, Robotics and Control, 2017.
8. Mummadi Veerachary, Malay Ranjan Khuntia, "Design and Analysis of Enhanced Gain Buck-Boost Converter", 2019 International Conference on Computing, Power and Communication Technologies (GUCON) Galgotias University, Greater Noida, UP, India. Sep 2019.
9. Seyed Hossein Hosseini, Reza Ghazi, Sina Kowsari Movahhed, "A Novel High Gain Single-Switch DC-DC Buck-Boost Converter with Continuous Input and Output Power", Electrical Power Distribution Conference (EPDC) 2019 24th, pp. 10-15, 2019.
10. Irving M. Gottlieb, "Power Supplies, Switching Regulators, Inverters, & Converters", New York: McGraw-Hill, 1993.
11. ASPECTS OF SECURITY ENHANCEMENT OF WIRELESS NETWORKS AND MOBILE COMMUNICATIONS SYSTEMS, Seema Sinha, International Journal Of Advance Research In Science

And Engineering http://www.ijarse.com IJARSE, Volume No. 10, Issue No. 03, March 2021 ISSN-2319-8354(E).

12. Anita Soni, "DC-DC Switching Boost Converter", ILLINOIS College of Engineering, 1999.

13. K M Sudharshan, B P Divakar " Simulation of Single Stage Boost PFC using PWM Average Model", International Journal of Computing and Applications Volume 13, Number 1, (January-June 2018), pp 195-200

14. K M Sudharshan, B P Divakar, "Frequency Analysis of Single Stage Boost PFC using Average Model", Journal of Advanced Research in Dynamical & Control Systems, Vol. 10, 04-Special Issue, February 2018

15. K M Sudharshan, B P Divakar, "Design and Simulation of Error amplifier used power management chips", International Journal of Recent Technology and Engineering" ISSN: 2277-3878.