Research Article

Vibrational Analysis of Damped Non-Homogeneous Annular Plate Whose Thickness Changes Exponentially with Winkler's Type Elastic Foundation

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Abstract: The present paper deals with the mathematical model on vibrational analysis of damped non-homogeneous annular plate considering parameters of changing thickness and Winkler's type elastic foundation. Here thickness and non-homogeneity are considered to vary exponentially. Numerical simulation is performed through QSIT (Quintic spline interpolation technique) which provides approximate results as per desired accuracy for two different combination of edge conditions CC & CS and we obtained first three modes of frequency parameter. Accuracy for the solution to our assumed mathematical model is affirmed by comparing obtained results with those available in previous literature. MATLAB (2015) is used to produced and present the results of above mathematical model.

Keywords – Annular plate, damping parameter, Winkler's foundation, exponential thickness and non-homogeneity.

1. Introduction

Annular plate is widely used in design of stream turbine, high speed aircraft structures, racing sports, automobiles, nuclear plant structures etc. to providing structural component with high strength. Annular plate with composite material have created lighter and stronger structures which can racist high temperature environment. So various effects and parameters have been applied in previous studies & in this present paper also, which will helps to improves structures dynamics of a mechanical structures. For practical applications, in making mechanical structures which consist of highways, pavement, damps, building foundation, airport runways etc., researchers have used different type of elastic foundation parameters and varying thickness parameter for plate structures. Winkler's, Pasternak and Vlasov's are generally used by researchers as elastic foundation parameter. Various researchers have applied Winkler's foundation in their research for studying frequency in vibration of different type plate structures.

Lal et al. [10] analysed and studied annular plate with non-homogeneity parameter & finds its vibrational behaviour for changing thickness & Winkler's type elastic foundation. Gupta et al. [4] in their research studied polar orthotropic annular plate axisymmetric vibrations whose thickness varies linearly with Pasternak foundation. Bhattacharya [1] have studied in detail effect of Vlasov's foundation on free vibration of plates. In addition to these Robin et al. [12], Sharma et al. [13], Gupta et al. [14] have also considered Winkler's foundation on behaviour of frequency of different type of plate structures. Non- homogeneity of the materials provides flexibility to mechanical structures for their operators as in case of switches, pressure capsules etc. Non-homogeneity provides us materials which are strong and light in weight in comparison to the old materials used previously, so it is necessary to study vibrations of annular plate structures considering non-homogeneity parameters. The non-homogeneity parameters have been considered by the various researchers [9, 11, 15] in their mathematical model of various shapes of plate structures. Singh & Jaiman [7] and Khare & Mittal [8] discussed on different type of boundary conditions of a thin annular circular plate.

The vibrational frequencies in mechanical structures should always be considered as damped frequencies since free vibrations are the ideal case and are practically not possible. This study of damping in structural dynamics is an important concept to be considered as they effect the vibrational behaviour of a structures [2]. The damping effect could be so small to effect the vibrational behaviour of plate or so large to effect the whole structure [5].

Thus considering the importance of ring shaped (especially annular plates) plates with various parameters, a mathematical model which consist of a fourth order partial differential equation is formulated for vibrational analysis of damped non-homogeneous annular plate considering thickness changing exponentially and tacking Winkler's elastic foundation. QSIT is used for numerical simulation of results to fetch first three modes of frequency parameter for two edge conditions viz. C-C and C-S respectively.

2. Methodology of The Problem

For the formulation of our problem a thin isotropic annular plate of exponentially varying thickness *h(r)* is considered and parameters of damping & foundation are incorporated in mathematical model which is available in previous literature [6] and we obtained our mathematical model as fourth order partial differential equation (PDE) given by:

$$D\frac{\partial^4\phi}{\partial r^4}+\frac{2}{r}\left[D+r\frac{\partial D}{\partial r}\right]\frac{\partial^3\phi}{\partial r^3}+\frac{1}{r^2}\left[-D+r(2+\nu)\frac{\partial D}{\partial r}+r^2\frac{\partial^2 D}{\partial r^2}\right]\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r^3}\left[D-r\frac{\partial D}{\partial r}+r^2\nu\frac{\partial^2 D}{\partial r^2}\right]\frac{\partial\phi}{\partial r}+$$
$$\rho h\frac{\partial^2\phi}{\partial t^2}+d_k\frac{\partial\phi}{\partial t}+f_e\phi=0 \qquad (1)$$

Where $D=\frac{Eh^3}{12(1-\upsilon^2)}$, $D, \phi, E(r), d_k$ and f_e are flexural rigidity, transverse deflection, young's modulus, damping parameter and foundation parameter respectively.

To obtain a harmonic solution, deflection function takes the form $\phi=Z(r)e^{-\mu t}\cos\omega t,$ where ω is called circular frequency of the plate and Assuming thickness change exponentially as $H=H_O e^{\alpha x}$, Material non-homogeneity expressed as $E=E_0e^{\mu x}$, $\rho=\rho_0 e^{\mu x}$ where μ is non-homogeneity parameter, ρ_0 is density and E_0 is Young's modulus at the inner edge. We put assumed deflection function and taking dimensional less variables $r/a=x$, $H=h/a$ in equation (1).
So the dimensionless form of the mathematical model (fourth order PDE) after suitable mathematical calculation is obtain as,

$$W_0\frac{\partial^4 Z}{\partial x^4}+W_1\frac{\partial^3 Z}{\partial x^3}+W_2\frac{\partial^2 Z}{\partial x^2}+W_3\frac{\partial Z}{\partial x}+W_4 Z=0, \qquad (2)$$

$$W_0=1$$
$$W_1=\frac{2}{x}[1+x(3\alpha+\beta)]$$
$$W_2=\frac{1}{x^2}[-1+(2+\nu_\theta)(3\alpha+\beta)x+x^2(3\alpha+\beta)^2$$
$$W_3=\frac{1}{x^3}[1-x(3\alpha+\beta)+\upsilon x^2(3\alpha+\beta)^2$$
$$W_4=-[12(1-\upsilon^2)D_k{}^2I^{*2}\exp(-2x(2\alpha+\beta))+\Omega^2\exp(-2\alpha x)-12(1-\upsilon^2)F_e\exp(-x(3\alpha+\beta))C^*]$$

Here $d_k{}^2=\frac{3k^2}{a^2E_0\rho_0}, \Omega^2=\frac{12(1-\upsilon^2)a^2\omega^2\rho_0}{E_0}, F=\frac{af_e}{E_0}$

The solution to the PDE (2) is obtained using 'quintic spline interpolation technique' with suitable computer programming through MATLAB to obtain desired degree of accuracy. The method used by Verma et al. [6] to find results for their problem and accurate results were obtained.
For this method;

$$Z(X)=a_0+\sum_{i=1}^{4}a_i(X-X_0)^i+\sum_{t=0}^{m-1}b_t(X-X_t)_+^5,$$

$$where\ (X-X_t)_+=\begin{cases}0, & if\ X\le X_t\\ (X-X_t), & if\ X>X_t\end{cases}, \qquad (3)$$

Here we consider a substitution $X=(x-c/a)/(1-c/a)$ and after necessary changes quintic spline interpolation technique is applied.

Now putting the value of equation (3) in equation (2);
The j^{th} knot, reduced equation take the form:

$$\begin{aligned}&W_4 a_0+\left[W_4\left(X_j-X_0\right)+W_3\right]a_1+\left[W_4\left(X_j-X_0\right)^2+2W_3\left(X_j-X_0\right)+2W_2\right]a_2\\&+\left[W_4\left(X_j-X_0\right)^3+3W_3\left(X_j-X_0\right)^2+6W_2\left(X_j-X_0\right)+6W_1\right]a_3\\&+\left[W_4\left(X_j-X_0\right)^4+4W_3\left(X_j-X_0\right)^3+12W_2\left(X_j-X_0\right)^2+24W_1\left(X_j-X_0\right)+24W_0\right]a_4\\&+\sum_{t=0}^{m-1}b_t\left[\begin{array}{l}W_4\left(X_j-X_t\right)_+^5+5W_3\left(X_j-X_t\right)_+^4+20W_2\left(X_j-X_t\right)_+^3\\+60W_1\left(X_j-X_t\right)_+^2+120W_0\left(X_j-X_t\right)_+\end{array}\right]=0.\end{aligned} \tag{4}$$

Equation (4) provides us undetermined system of homogeneous liner equation of the form

$$[A][B]=0, \tag{5}$$

Where A is coefficient matrix of type $(m+1)\times(m+5)$ and B is column matrix of type $(m+5)\times 1$.
Now, boundary conditions at X=c/a, X=1,

(i) C-C (Clamped - Clamped): $Z=\frac{dZ}{dX}=0.$

(ii) C-S (Clamped - Simply supported): $Z=\frac{d^2Z}{dX^2}+(\nu_\theta/X)\left(\frac{dZ}{dX}\right)=0.$

Using these boundary conditions four additional equation are obtained in (m+5) unknowns for both the boundary conditions which on combining with equation (5), provides us two system of linear homogeneous equations as:

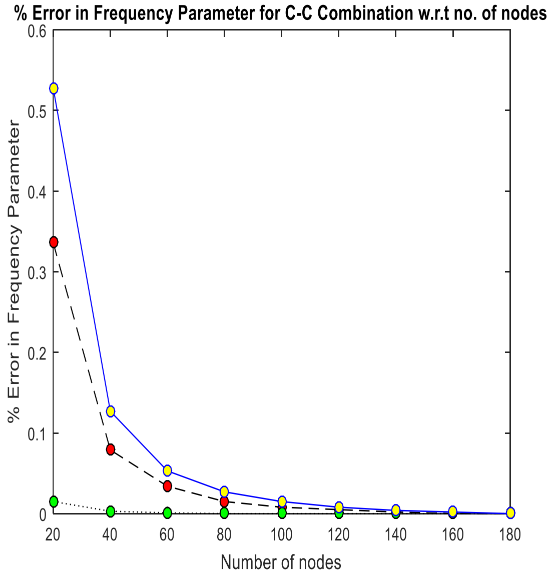
$$\left[\frac{A}{T^{CC}}\right]\left[B\right]=0 \tag{6}$$

$$\left[\frac{A}{T^{CSS}}\right]\left[B\right]=0 \tag{7}$$

For two edge conditions C-C and C-S, two equations (6, 7) provides a non-zero solution when the characteristic determinant of the equations becomes zero.

3. Results and discussion

Equations (6) & (7) provide two different frequency equations for edge combinations, C-C and C-S and we obtain three mode values of frequency parameter with our assumed plate parameters up to four decimal places.



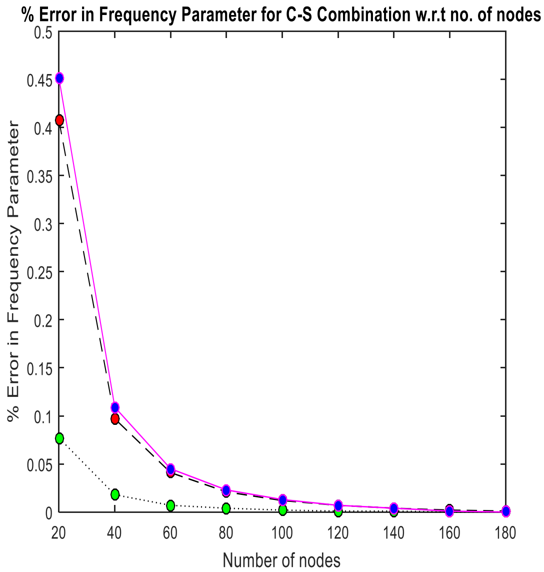


Fig. (i) **Fig. (ii)**

Fig. (i) **Fig. (ii)**

Figure 1. % Error in Ω for (i) C-C Combination (ii) C-S Combination for I , II and III modes , for $\alpha = 0.5,\ K = 0.02,\ d_k = 0.01, b/a = 0.3,\ \beta = 1,\ \upsilon = 0.3$. ------I mode; ……II mode; — — III mode. % error $= \left(\frac{|\Omega_m - \Omega_{180}|}{\Omega_{180}} \right) \times 100$

To obtained appropriate interval we have considered $\Delta X = \frac{1}{m}$ where $m = 20(20)200$ but for $m \geq 180$, the results shows no improvement. The results for convergence for number of increasing nodes for C-C & C-S combinations taking; $\alpha = 0.5, F = 0.02, D_k = 0.02, c/a = 0.3,\ \upsilon = 0.3$ are presented in Figure 1. The variations in different parameters used for the present problem are considered as Taper constant $\alpha = -0.5, -0.3, 0.0, 0.3, 0.5$, Foundation parameter $F = 0.0(0.01)0.03$ Damping Parameter $d_k = 0.0(0.01)0.05$ and Non-homogeneity parameter $\beta = -0.5, 0.0, 0.5, 1.0$. Also, assumed fixed values for calculation the above variations in plate parameters are considered as $\upsilon = 0.3$, $h = 0.1$ and $m = 180$.

Table 1 shows the damping parameter effect on frequencies of non-homogeneous annular plate with assumed fixed plate parameters as $F = 0.02,\ c/a = 0.3,\ \upsilon = 0.3, \beta = 1, \alpha = 0.5$ for C-C and C-S edge conditions. Figure 2 shows the frequency parameter behaviour for three modes of vibration with varying damping parameter values which is taken in increasing order. It is observed that from fig. 2 that, as we increase damping parameter value then its corresponding vibrational frequencies decreases continuously in all the three modes and for both the edge conditions. The linear change which occurs in frequencies is same for every modes and for both the limiting conditions. The calculated results for frequency parameter Ω for the variation on the value of taper parameter (α) for a damped $(d_k = 0.02)$ and non-damped $(d_k = 0.0)$ annular plate for both boundary conditions that is C-C plate position & C-S plate position are shown in table 2and fig. 3(a) & 3(b) respectively with fixed values $F = 0.02,\ c/a = 0.3,\ \upsilon = 0.3, \beta = 1$ for vibration analysis of three modes. For the variation in values of taper constant $\alpha = -0.5, -0.3, 0.0, 0.3, 0.5$, the frequency parameter shows the exponentially increment in fig. 1(a) and this increasing order continue when values of taper constant increases for non-damped annular plate. Similar is the nature of graphical representation of fig. 1(b) as in fig. 1(a) for damped annular plate under both boundary conditions i.e. C-C and C-SS respectively. It is also observed that the increment for both boundary condition are same for all three modes of vibration for both non-damped and damped annular plate.

Table 3 and fig. 4(a), 4(b) shows the change for frequency parameter by taking different values of foundation parameter under assumed fixed plate parameters such as $\alpha = 0.5,\ c/a = 0.3,\ \upsilon = 0.3, \beta = 1, h = 0.1$ for both non-damped and damped annular plate respectively. We observe that all the value of frequencies are increasing corresponding to both edge conditions for all modes and this increment found same with respect to damping effect. Further, we note that all the frequency corresponding to foundation parameter increases as its value increases. The nature of increment from graph of frequencies is observed to be linear for both non-damped and damped annular plate respectively.

In the series of result discussion of present paper, now we consider outcome of non-homogeneity parameter on frequency of vibration of non-damped and damped annular plate for assumed boundary conditions and plate parameters. From table 4 we analysed that, frequencies corresponding to C-C plate position are increasing but for C-S plate position it decreases continuously for non-damped and damped annular plate respectively. After observing this behaviour of frequency parameter, fig. 5 (a) & 5 (b) shows the linear change in values of frequency parameter corresponding to all three modes for both boundary conditions respectively.

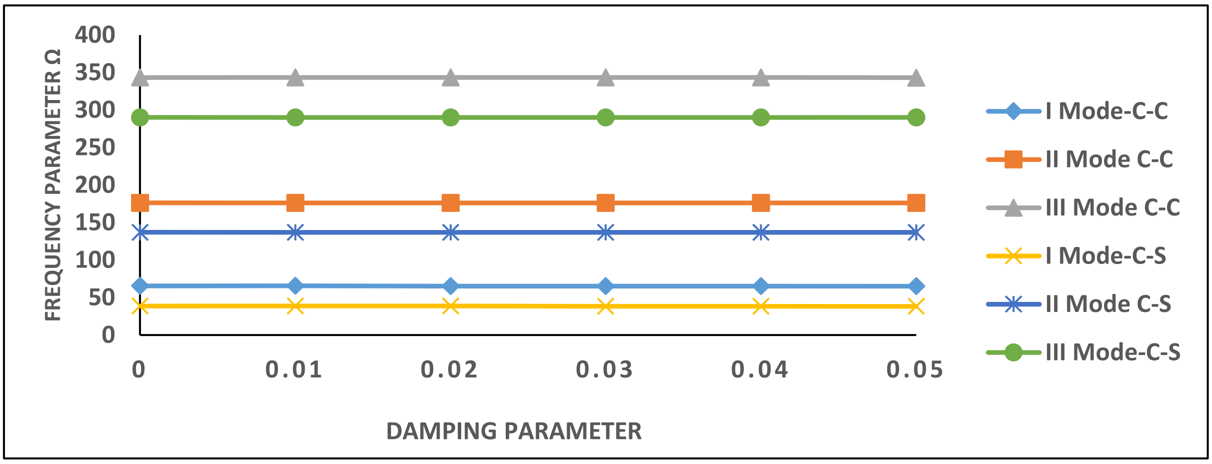
Validation of results for our present paper shows that results are well compared with those obtained by [6] and different researcher through their findings which are given in references [6,10,12] and different validation are presented in the form of tables given by tables 5 and 6 respectively.

T**able 1.** Table for Numerical Values of Ω for isotropic annular plate for different values of damping parameter (d_k) tacking υ=0.3, $c/a = 0.3$, $m = 180$, $F = 0.02$, $\alpha = 0.5$, $\beta = 1$, $h = 0.1$,

	d_k (Damping parameter)

Modes	0.0	0.01	0.02	0.03	0.04	0.05
I Mode-C-C	65.4443	65.4313	65.3923	65.3273	65.2362	65.1188
II Mode C-C	176.2953	176.2902	176.2748	176.2493	176.2134	176.1674
III Mode C-C	343.2686	343.2659	343.2579	343.2450	343.2255	343.2013
I Mode-C-SS	38.6570	38.6386	38.5832	38.4906	38.3606	38.1929
II Mode C-SS	137.0700	137.0636	137.0443	137.0121	136.9670	136.9090
III Mode-C-SS	290.2309	290.2278	290.2183	290.2025	290.1803	290.1520

Figure 2: Graph for Ω for isotropic annular plate for different values of damping parameter (d_k)



tacking υ=0.3, $c/a = 0.3$, $m = 180$, $F = 0.02$, $\alpha = 0.5$, $\beta = 1$, $h = 0.1$

Table 2. Table for Numerical Values of Ω for different values of taper constant (α) for non-damped annular plate $(d_k = 0.0)$ and damped annular plate $(d_k = 0.02)$ tacking υ=0.3, $c/a = 0.3$, $m = 180$, $F = 0.02$, $\beta = 1$, $h = 0.1$

	Non-Damped condition $(d_k = 0.0)$					Damping effect $(d_k = 0.02)$				
Modes	$\alpha = -0.5$	$\alpha = -0.3$	$\alpha = 0.0$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = -0.5$	$\alpha = -0.3$	$\alpha = 0.0$	$\alpha = 0.3$	$\alpha = 0.5$
I Mode-C-C	34.9047	39.1009	47.0062	57.1719	65.4443	34.5793	38.8756	46.8767	57.0971	65.3923
II Mode C-C	91.0659	103.7535	126.4272	154.2905	176.2953	90.9405	103.6674	126.3776	154.2615	176.2748
III Mode C-C	177.6891	202.7471	247.0878	301.0186	343.2686	177.6246	202.7027	247.0622	301.0034	343.2579
I Mode-C-SS	24.8595	26.7581	30.4013	35.0179	38.6570	24.4228	26.4508	30.2210	34.9125	38.5832
II Mode C-SS	73.3561	82.9731	100.0494	120.8239	137.0700	73.2021	82.8671	99.9879	120.7876	137.0443
III Mode-C-SS	152.5714	173.5754	210.5871	255.3478	290.2309	152.4966	173.5239	210.5571	255.3310	290.2183

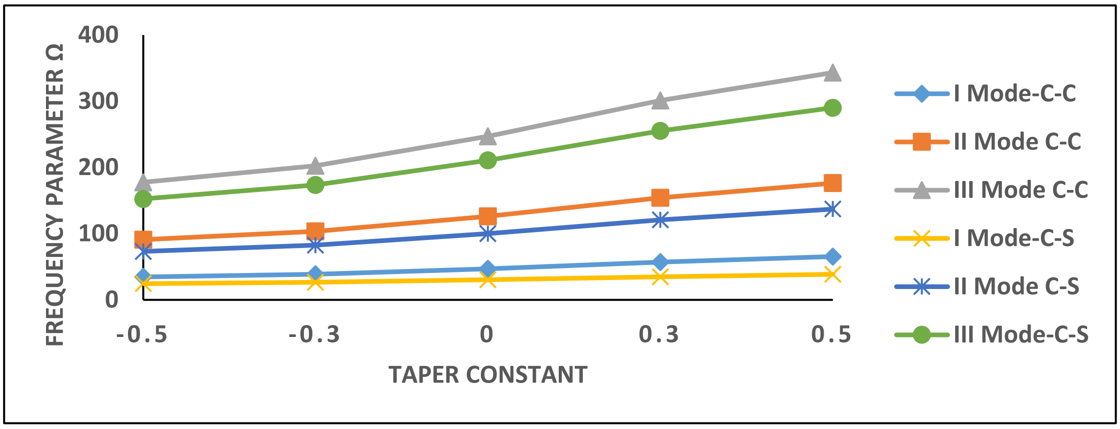
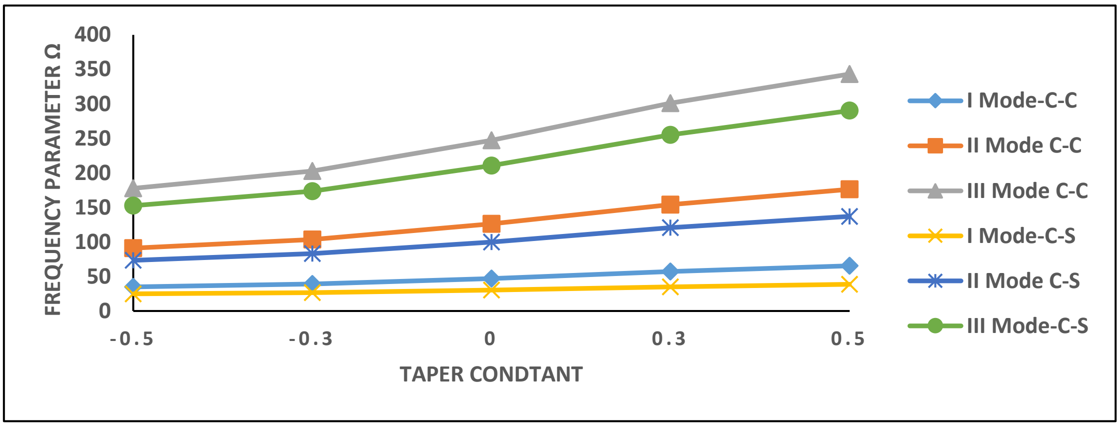


Figure 3(a): Graph for Ω for different values of taper constant (α) for non-damped annular plate $(d_k = 0.0)$



tacking $\upsilon = 0.3$, $c/a = 0.3$, $m = 180$, $F = 0.02$, $\beta = 1$, $h = 0.1$

Figure 3(b): Graph for Ω for different values of taper constant (α) for damped annular plate $(d_k = 0.02)$ tacking $\upsilon = 0.3$, $c/a = 0.3$, $m = 180$, $F = 0.02$, $\beta = 1$, $h = 0.1$

Table 3. Table for Numerical Values of Ω for different values of foundation parameter (F) for a non-damped annular plate $(d_k = 0.0)$ and damped annular plate $(d_k = 0.02)$ tacking $\upsilon = 0.3$, $c/a = 0.3$ $m = 180$, $\alpha = 0.5$, $\beta = 1$, $h = 0.1$

	Non-Damped condition $(d_k = 0.0)$				Damping effect $(d_k = 0.02)$			
Modes	$F = 0.0$	$F = 0.01$	$F = 0.02$	$F = 0.03$	$F = 0.0$	$F = 0.01$	$F = 0.02$	$F = 0.03$
I Mode-C-C	64.7905	65.1182	65.4443	65.7687	64.7380	65.0660	65.3923	65.7170
II Mode C-C	176.0502	176.1728	176.2953	176.4177	176.0297	176.1523	176.2748	176.3973
III Mode C-C	343.1421	343.2054	343.2686	343.3319	343.1313	343.1946	343.2579	343.3211
I Mode-C-SS	37.6379	38.1509	38.6570	39.1565	37.5620	38.0760	38.5832	39.0836
II Mode C-SS	136.7588	136.9145	137.0700	137.2254	136.7330	136.8887	137.0443	137.1997
III Mode-C-SS	290.0820	290.1565	290.2309	290.3054	290.0693	290.1438	290.2183	290.2927

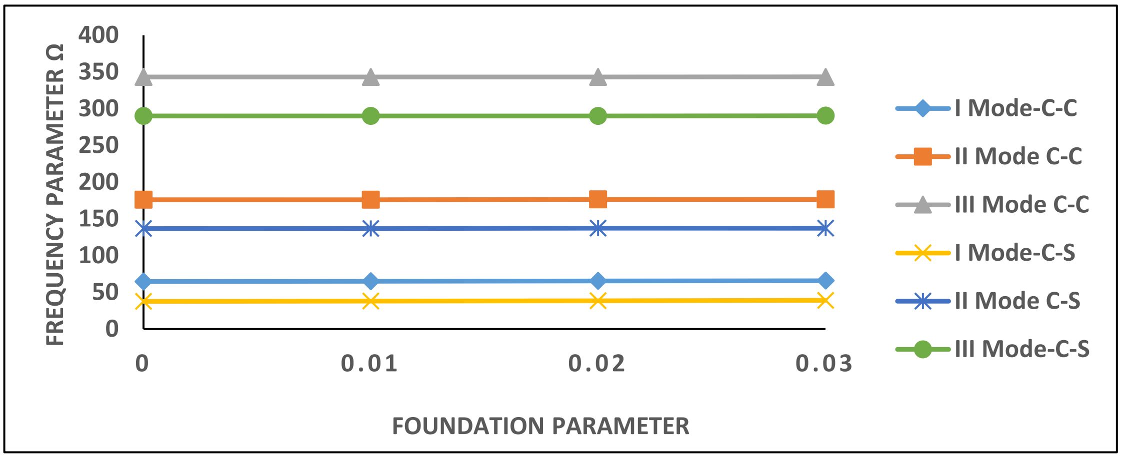


Figure 4(a): Graph for Ω for different values of foundation parameter (F) for non-damped annular plate $(d_k = 0.0)$ tacking $\upsilon = 0.3$, $c/a = 0.3$ $m = 180$, $\alpha = 0.5$, $\beta = 1$, $h = 0.1$

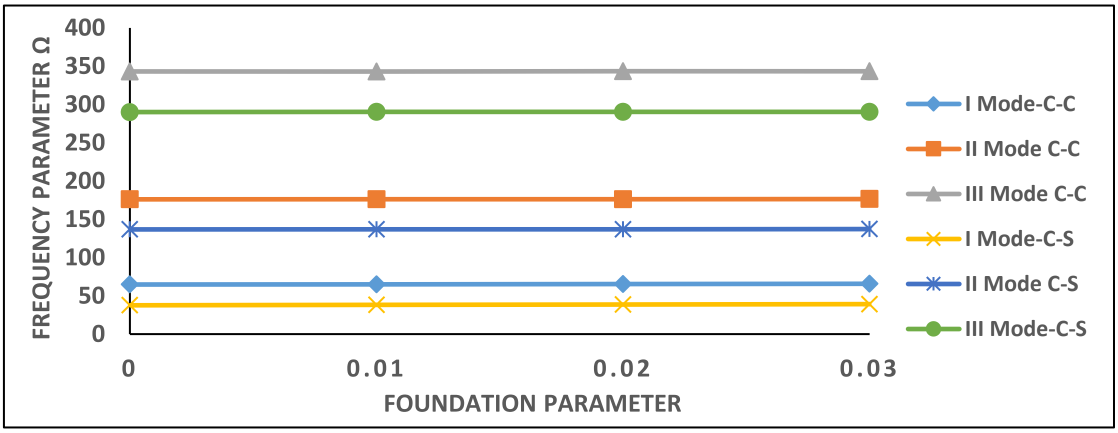


Figure 4(b): Graph for Ω for different values of foundation parameter (F) for damped annular Plate $(d_k = 0.02)$, tacking $\upsilon = 0.3$, $c/a = 0.3$, $m = 180$, $\alpha = 0.5$, $\beta = 1$, $h = 0.1$

Table 4. Numerical Values of Ω for different values of non-homogeneity parameter (β) for a non-damped annular plate $(d_k = 0.0)$ and damped annular plate $(d_k = 0.02)$ tacking $\upsilon = 0.3$, $c/a = 0.3$, $m = 180$, $F = 0.02$, $\alpha = 0.5$, $h = 0.1$

	Non-Damped condition $(d_k = 0.0)$				Damping effect $(d_k = 0.02)$			
Modes	$\beta = -0.5$	$\beta = 0.0$	$\beta = 0.5$	$\beta = 1$	$\beta = -0.5$	$\beta = 0.0$	$\beta = 0.5$	$\beta = 1$
I Mode-C-C	64.8288	64.9785	65.0237	65.4443	64.4910	64.6289	64.9275	65.3923
II Mode C-C	174.4433	174.8845	175.5062	176.2953	174.3180	174.8177	175.4697	176.2748
III Mode C-C	341.0129	341.5999	342.3542	343.2686	340.9489	341.5656	342.3352	343.2579
I Mode-C-SS	43.1017	41.4506	39.9863	38.6570	42.5920	41.1845	39.8465	38.5832
II Mode C-SS	138.9217	138.1536	137.5420	137.0700	138.7644	138.0699	137.4961	137.0443
III Mode-C-SS	291.4724	290.9126	290.5014	290.2309	291.3974	290.8725	290.4793	290.2183

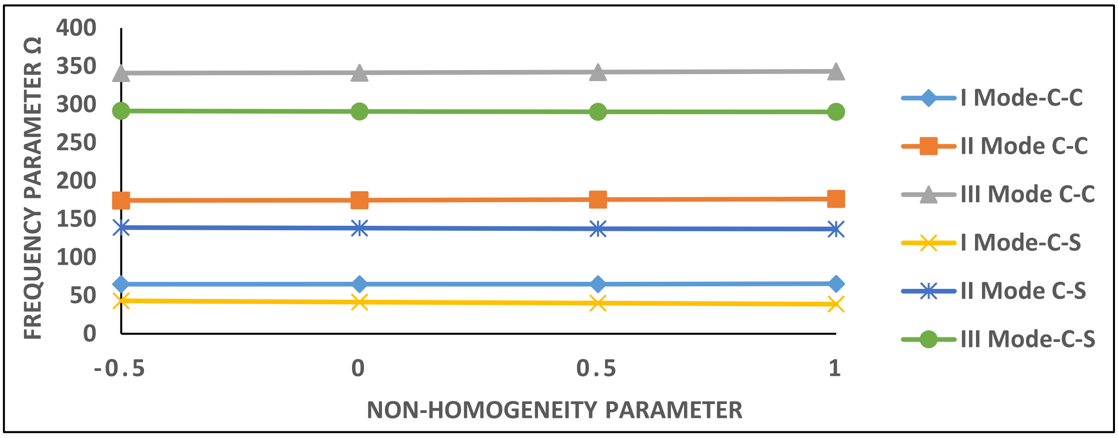


Figure 5(a): Graph for Ω for different values of non-homogeneity parameter (β) for non-damped plate $(d_k = 0.0)$ tacking $\upsilon = 0.3$, $c/a = 0.3$, $m = 180$, $F = 0.02$, $\beta = 1$, $h = 0.1$

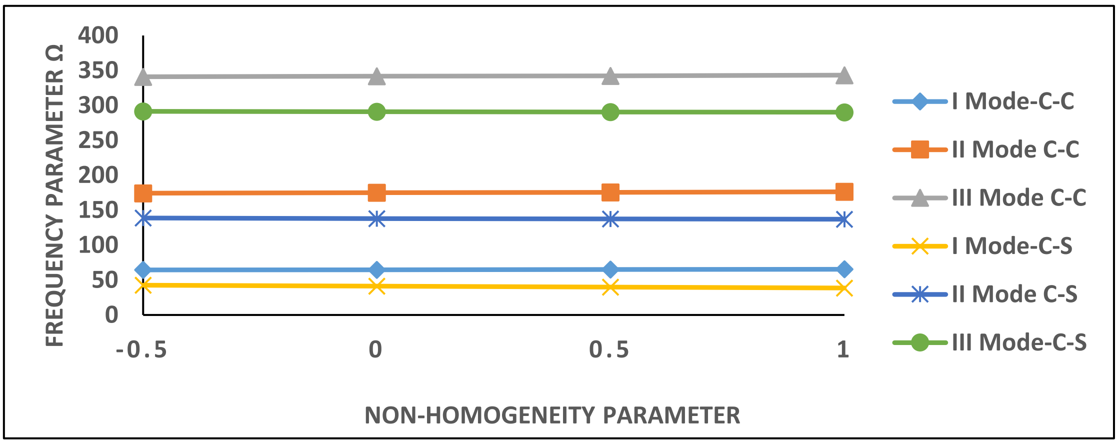


Figure 5 (b): Graph for Ω for different values of non-homogeneity parameter (β) for damped plate $(d_k = 0.02)$ tacking $\upsilon = 0.3$, $c/a = 0.3$, $m = 180$, $F = 0.02$, $\beta = 1$, $h = 0.1$

Table 5- Validation of results for frequency parameter Ω considering an isotropic homogeneous $(\beta = 0.0)$ annular plates of uniform thickness $(\alpha = 0.0)$ for $c/a = 0.3, \upsilon = 0.3$.

	Edge conditions	C-C			C-S		
		Mode					
	Value taken from reference	I	II	III	I	II	III
$F = 0.0$	Lal R. [1987]	45.3371	125.6191	246.6944	29.9689	100.6065	211.562 9
	Sharma S. [2006]	45.3462	125.3621	246.1573	29.9777	100.4228	211.1294
	Dhiman [2019]	45.3461	125.3631	246.1626	29.9776	100.4235	211.1336
$D_k = 0.0$	Present	**45.3461**	**125.3612**	**246.1838**	**29.9776**	**100.4264**	**211.1508**

Table 6- Validation of results for frequency parameter Ω considering an isotropic homogeneous $(\beta = 0.0)$ damped annular plates of uniform thickness $(\alpha = 0.0)$ for $c / a = 0.3, \upsilon = 0.3$

	Edges	C-C			C-S		
		Mode					
	Reference	I	II	III	I	II	III
	Lal R. [1987]	46.5259	126.0531	246.9155	31.7385	101.1478	211.8208
$F = 0.01$	Sharma S. [2006]	46.5347	125.7969	246.379	31.7469	100.9651	211.3878
	Dhiman [2019]	46.5346	125.7979	246.3843	31.7468	100.9657	211.3921
$D_k = 0.01$	Present	**46.4172**	**125.7585**	**246.3834**	**31.5743**	**100.9146**	**211.3834**

4. Conclusion: Present study results has been carried out by using computer program on MATLAB 2015 to obtain results in permissible range and desired accuracy. The effect of damping on non-homogeneous isotropic annular plate with thickness varying exponentially and also considering Winkler's foundation effect have been discussed with three modes of vibration under two boundary conditions C-C and C-S. After suitable mathematical formulation we form tabulated data and its graphical representations, observe that that damping effect reduced vibrational frequencies with different plate parameters. So damping phenomenon in the present study decreases the vibration frequencies when we continuously increase its value with assumed plate parameters. The results so obtained for present problem helps us to obtain desired frequency by changing various parameters and help to make design structure strong and shock proof.

Conflict of Interest and Acknowledgement

There is no conflict of interest between authors for present publication. The authors acknowledges the contribution of various researchers whose research articles have been studied in completion of this paper.

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