Cost Optimization Of Arrival And Service Control Queueing Model With Single Server In Fuzzy Environment Of Uncertainty

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Abstract: In this article, the analysis of cost optimization for queuing system with single server in fuzzy environment of uncertainty has been studied. The purpose of the investigation is to build the total cost function along with the total optimal cost of the queuing system in fuzzy paradigm under uncertainty. The fuzzy analysis is carried out to provide more realistic solution of the problems under consideration as compared to usual crisp solutions for the model. The theoretical developments for both crisp and the fuzzy systems have been derived for the model separately and computed costs are clearly perceptible to verify and compare. At the last, the sensitivity analysis has also been carried out with the help of numerical study to finally judge the theoretical findings of the model under investigation..

Keywords: Fuzzy Optimization, Queuing model, Cost Function, Sensitivity Analysis

1. Introduction

In many situations, we come across the problems where we are not in position of very clear decision such as if the water is dirty so whether it is drinkable means to which extent it is dirty, what is the severity of the accident even grade system of marking. Fuzziness precises the imprecise information in such a way that, we can make better decisions in different areas, where ambiguity sustains. Fuzziness makes use of fuzzy set theory and logic which has been discussed by various authors. Bellman and Zadeh [2] have discussed in detail the problems and solutions of decision making in fuzzy environment while Kaufmann [17] described the theory of fuzzy subsets for different areas of applications. Zimmermann [38] also discussed the fuzzy set theory and the applications of fuzzy sets in various areas of practical application in real life and Zadeh [39] developed and used the fuzzy set theory for the theory of possibility and he suggested that fuzzy set theory is the basis of the possibility theory. Markovian queues are of very much importance for the research in queuing theory.

Mishra and Yadav [22] developed and discussed the computational approach to cost and profit analysis of clocked queuing networks in queuing theory and Mishra and Shukla [23] described the computational approach to the cost analysis of machine interference model for the theory of queues. Priya and Sudhesh [28] described the problem of transient analysis of a discrete-time infinite server queue with system disaster while Sharma [32] discussed in details the optimal flow control of multi-server time sharing queuing network with priority in queuing theory. Singh *et al.* [34] worked out on the analysis of single server finite queuing model with reneging in queuing theory and Sundari and Palaniammal [35] discussed the problem of simulation of M/M/1 queuing system using artificial neural network program.

It is well established from the literature that the Morkovian queues are of paramount importance and the fuzzy queuing models are very much useful in it in comparison to usual crisp theory as fuzzy queuing models provides more realistic solutions in various practical applications. For instance the mean arrival rate or mean service rate or both appear more possibilistic rather than probabilistic and the occurrence of arrivals and services are totally probabilistic at service station while the numerical expressions are possibilistic. On the other hand, the fuzzy queuing models are closer to reality and have large number of applications as compared to crisp in different areas of applications. Prado and Fuente [26] discussed the problems and applications of queuing theory in Markovian decision processes using fuzzy set theory while Li and Lee [21] described the analysis of fuzzy queues and discussed their applications in various areas of applications and Buckley [3] discussed in details the elementary queuing theory based on possibility theory.

As for as the queuing models are concerned, these models may be either descriptive type or normative type respectively. The models which are observed in real life situations are actual are known as descriptive type queuing models while the models normative type models are those which are optimum models for the given situations. In normative type models, we optimize the arrival parameters, service, number of servers, queue discipline and controls etc for the given queuing models. Thus, normative models are the inspirational queuing model while descriptive models are real life situation queuing model. The descriptive type of queuing models is also known as queuing decision models or design and control models. Under this category of models, the parameters of the models are so computed that they should optimize the models. Negi and Lee [24] worked on such queuing models and analyzed them in fuzzy environment. Jo *et al.* [14] discussed the performance evaluation of networks based on Fuzzy Queuing System and Kao *et al.* [16] worked on the Parametric programming of such queuing models and analyzed them in fuzzy environment. The control models are of paramount importance in queuing theory and amongst these models, the service control depends on various measures such as service rate, number of servers, queue discipline, or a combination of these factors etc. The arrival control also plays an important role in queuing theory and is possible by distributing arriving customers or assigning them to some servers. The arrivals may be controlled through some toll devices or by some feasible constraints which may include designing the parameter for physical space and working shift etc. Buckley *et al.* [4] worked on devising such parameters in queuing theory and provided the solution through fuzzy expert system while Chen [6] described the bulk arrival queuing model with fuzzy parameters and varying batch sizes and given the solution of the problem through fuzzy set theory and Ke and Lin [18] worked on the fuzzy analysis of queuing systems with an unreliable server through a nonlinear programming approach.

Later, the new trend in queuing models came into the picture where the models are optimized through uncertain data inputs. This uncertainty of input data is used to develop the model through fuzzy expert system. In fuzzy expert system, the fuzzy optimization methods are used to optimize fuzzy coefficients and parameters of the queuing models. Chanas and Nowakowski [5] worked on the Single value simulation on Fuzzy variable in queuing theory and Fazlollahtabar and Gholizadeh [10] discussed the economic analysis of the M/M/1/N queuing system cost model in a vague environment. Prameela and Kumar [27] described the FM / FEk /1 queuing model with Erlang service under various types of fuzzy numbers while Palpandi and Geetharamani [25] worked on the evaluation of performance measures of bulk arrival queue with fuzzy parameters using robust ranking technique and Sanga *et al.* [31] worked on the FM/FM/1 double orbit retrial queue with customers' joining strategy through a parametric nonlinear programing approach.

The solutions of design and control models for performance measures are obtained using various methods when cost coefficients, arrival and service parameters are precise and known. On the other hand if these parameters of these models are imprecise and have variations over time such as waiting cost per unit may have variation over time, then the usual queuing decision models does not provide reliable estimates of the parameters of the models under consideration due to imprecision and ambiguity situations which are beyond human control. Under such situation, an intervention is needed to know the impact which may make the system to work. The fuzzy queuing decision models are suitable to handle such situations more effectively which may investigate and explore these models. Barak and Fallahnezhad [1] discussed the cost analysis of fuzzy queuing systems and Fathi-Vajargah and Ghasemalipour [9] worked on the simulation study of a random fuzzy queuing system with multiple servers. Enrique and Enrique [8] discussed the simulation of fuzzy queuing systems with a variable number of servers, arrival and service rates while Kannadasan and Sathiyamoorth [15] worked on the analysis of M/M/1 queue with working vacation in fuzzy environment and Gou *et al.* [11] described the hesitant fuzzy linguistic entropy and cross-entropy measures and alternative queuing method for multiple criteria decision making. Chen *et al.* [7] worked on the analysis of strategic customer behavior in fuzzy queueing systems and Ke and Lin [18] on the Fuzzy analysis of queuing systems with an unreliable server: A nonlinear programming approach. Keith and Ahner [20] carried out a survey of decision making and optimization under uncertainty while Qin *et al.* [29] worked on linguistic interval-valued intuitionistic fuzzy archimedean power muirhead mean operators for multiattribute group decision-making and Singh *et al.* [33] worked on the optimization of markovian arrival and service queuing model under fuzziness. Mishra et al. [36] have applied neural network and signed distance method to analyze Markovian queueing model with single server.

The present paper uses the both types of arrival and control queuing model with single server in fuzzy environment of uncertainty. The queuing model with one or more uncertain parameters which are to be optimized for designing and controlling the queuing model under consideration, the fuzzy environment of uncertainty provides better results as the fuzzy estimates of the parameters are more realistic and practical than the estimates of the parameters of crisp model. Crisp model M/M/1 characterizes ideal situation of waiting time analysis and its findings whereas fuzzy model FM/FM/1 represents and discusses real situations of waiting line and its results. It is also a well realized facts that ideal situation of waiting line or queue does not happen in reality. In the present investigation, we intend to construct the total cost function of arrival and control queuing model with single server in fuzzy environment of uncertainty as an important performance measure under control design of arrival and service of FM/FM/1model and its optimization. Here, we propose to apply a trapezoidal system of fuzzy numbers to fuzzify the total cost function and later an efficient method of signed distance method (SDM) is applied to defuzzify the model FM/FM/1. The optimization of total cost function provides us a system of non linear equations involving parameters of the model, which is soved using R software for the optimum performance measure of the model. The optimum performance measure is here the optimal total optimal cost of the queuing model FM/FM/1 under consideration. At last, the sensitivity analysis has been carried out through numerical demonstration of the model. The model FM/FM/1 under investigation is presumably cost-effective and efficient as compared to predecessor models.

Notations: Following notations are used in the paper.

TC = Optimal Total Cost (OTC) , k = Service cost per unit (SC), c = Waiting cost per unit (WC), λ = Arrival rate of customer (ARC), μ = Service rate. (SR), n = Number of customer (NC), $\tilde{k}$ = Fuzzified Service cost per unit (FSC), $\tilde{c}$ = Fuzzified Waiting cost per unit(FWC)

$\tilde{\lambda}$ = Fuzzified Arrival Rate (FAR), $\tilde{n}$ = Fuzzified Number of customer (FNC)

2.Arrival Control Model

In general, the arrival control queuing model is a stochastic input-output system, where input process is being controlled by accepting or rejecting arriving customers. Such models include various standard queuing systems as its special cases. Under these models, the form of individually and socially optimal acceptance policies are taken into account when there are rewards and waiting costs associated with accepted customers (Johansen and Stidham [13]). Johansen and Stidham [13] studied the control of arrivals to a stochastic input-output system while Chen [6] suggested the solution for a bulk arrival queuing model with fuzzy parameters and varying batch sizes. Ke *et al.* [19] suggested the solution for controlling arrivals for a markovian queuing system with a second optional service and Walker and Bright [37] discussed in detail the modeling arrival-to-departure sequence disorder in flow-controlled manufacturing systems. Itoh and Mitici [12] described the solution for queue-based modeling of the aircraft arrival process at a single airport while Samanta [30] studied the waiting-time analysis of D-BMAP/G/1 queuing system. The mathematical formulations of the arrival control system for crisp as well as for fuzzy paradigm are discussed below:

2.1 Arrival Control Model: Crisp Mathematical Formulation

Total cost function for the model is defined as $TC = k\mu + cE(n)$

This implies that $TC = k\mu + \frac{c\lambda}{\mu}$.

For minimum cost with respect to service, we have

$\frac{d}{d\mu}[TC] = \frac{d}{d\mu}\left(k\mu + \frac{c\lambda}{\mu}\right)$. For stationary values $\frac{d}{d\mu}[TC] = 0$, this ultimately yields $\mu = \left(\frac{c\lambda}{k}\right)^{1/2}$ along with having satisfied $\frac{d^2}{d\mu^2}[TC] = \frac{2c\lambda}{\mu^3} > 0$ for minimum cost.

2.2 Fuzzy Mathematical Model

Further, we define a trapezoidal fuzzy number $\tilde{A} = (a,b,c,d)$ with membership function $\mu_A(x)$ as

$$\mu_A(x) = \begin{cases} L(x) = \frac{x-a}{b-a}, when\ a \le x \le b \\ 1, when\, b \le x \le c \\ R(x) = \frac{d-x}{d-c}, when\, c \le x \le d \\ 0, otherwise \end{cases}$$

Now, we wish to fuzzify cost coefficients and arrival rates k, c, λ with the help of trapezoidal fuzzy numbers as $\tilde{k}, \tilde{c}$ and $\tilde{\lambda}$ respectively. We have further as

$$\tilde{k} = (k_1, k_2, k_3, k_4), \quad \tilde{c} = (c_1, c_2, c_3, c_4), \; \tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$$

Thus, fuzzified total cost turns out to be as $\tilde{TC} = \tilde{k}\,\mu + \frac{\tilde{c}\,\tilde{\lambda}}{\mu}$ which implies that

$$\tilde{T}C = \left(k_1\mu + \frac{c_1\lambda_1}{\mu},\; k_2\mu + \frac{c_2\lambda_2}{\mu}, k_3\mu + \frac{c_3\lambda_3}{\mu}, k_4\mu + \frac{c_4\lambda_4}{\mu} \right)$$

which is finally expressed as $\tilde{T}C = (W, X, Y, Z)$

$$W = k_1\mu + \frac{c_1\lambda_1}{\mu}, \qquad X = k_2\mu + \frac{c_2\lambda_2}{\mu}, \qquad Y = k_3\mu + \frac{c_3\lambda_3}{\mu} \qquad Z = k_4\mu + \frac{c_4\lambda_4}{\mu}$$

Now, we define followings as

$$C_L(\alpha) = W + (X - W)\alpha \quad = \quad k_1\mu + \frac{c_1\lambda_1}{\mu}\left[\left(k_2\mu + \frac{c_2\lambda_2}{\mu} - \left(k_1\mu + \frac{c_1\lambda_1}{\mu} \right)\right)\right]\alpha$$

and,

$$C_R(\alpha) = Z - (Z - Y)\,\alpha \;=\; \left(k_4\mu + \frac{c_4\lambda_4}{\mu} \right) - \left(k_4\mu + \frac{c_4\lambda_4}{\mu} - k_3\mu + c_3 \right)\alpha$$

Next, we define SDM and apply for present model.

$$\breve{T}C_{ds} = \frac{1}{2}\int_0^1 [\quad C_L(\alpha) + C_R(\alpha)]\,d\alpha \quad , \quad \breve{T}C_{ds} \;=\; \frac{1}{4}(k_1 + k_2 + k_3 + k_4)\mu \;+\; \frac{1}{\mu}(c_1\,\lambda_1 + c_2\,\lambda_2 + c_3\,\lambda_3 + c_4\,\lambda_4)$$

In order to attain minimum fuzzified cost, we have now stationary value

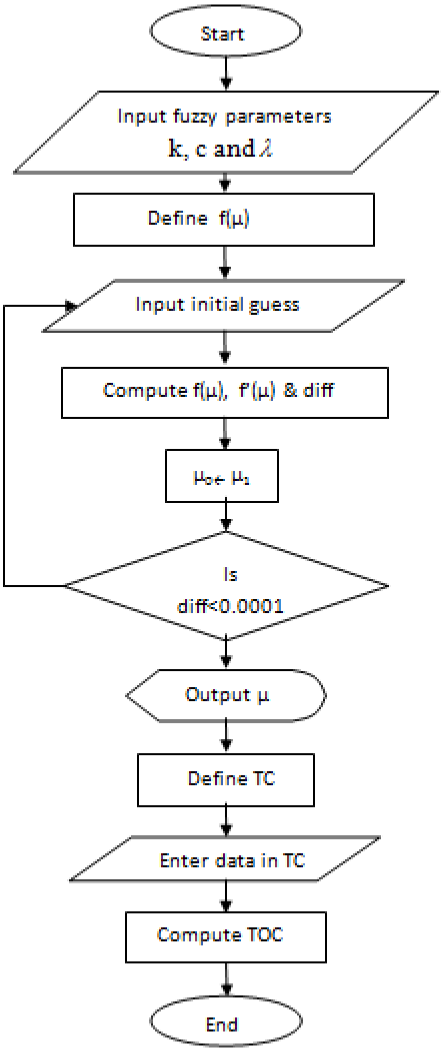
$$\frac{d\breve{T}C_{ds}}{d\mu} = \frac{1}{4}(k_1 + k_2 + k_3 + k_4) - \frac{1}{\mu^2}(c_1\,\lambda_1 + c_2\,\lambda_2 + c_3\,\lambda_3 + c_4\,\lambda_4) = 0,$$ yielding result as

$$\mu = \left(\frac{c_1\,\lambda_1 + c_2\,\lambda_2 + c_3\,\lambda_3 + c_4\,\lambda_4}{k_1 + k_2 + k_3 + k_4} \right)^{1/2}$$ having satisfied the condition as

$$\frac{d^2}{d\mu^2}\tilde{T}C_{ds} = \frac{1}{2}\left(\frac{c_1\,\lambda_1 + c_2\,\lambda_2 + c_3\,\lambda_3 + c_4\,\lambda_4}{\mu} \right) > 0$$

2.3 Computing Algorithm

The following is a flow chart of computing algorithm for optimal service rate and total optimal cost of the model under consideration.



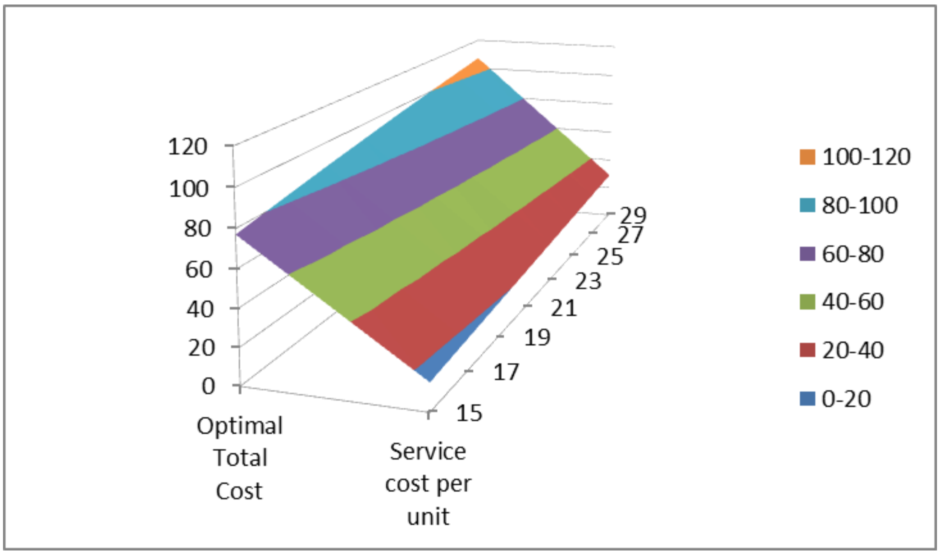
2.4 Arrival Control Model: Crisp Computation:

Following table 1 represents computation of cost function under crisp environment.

Table-1 (Computation table for k, TC)

k	c	λ	μ	TC
15	11	9	2.56	77.07
17	11	9	2.41	82.04
19	11	9	2.28	86.74
21	11	9	2.17	91.12
23	11	9	2.07	95.43
25	11	9	1.98	99.49
27	11	9	1.91	103.39
29	11	9	1.84	107.16

Figure-1: Variation of optimal total cost and service cost per unit



2.5 Arrival Control Model: Fuzzy Computation:

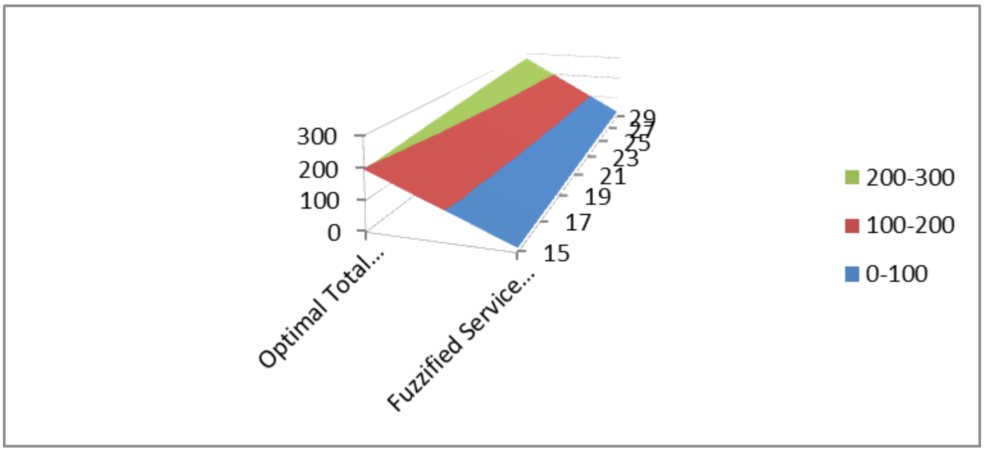
Following tables from table 2 represents computation of cost function under fuzzy environment

Table: 2 (Computation table for $\tilde{k}$,$\tilde{T}C$)

$\tilde{k}$				$\tilde{c}$			
k_1	k_2	k_3	k_4	c_1	c_2	c_3	c_4
12	14	16	18	8	10	12	14
14	16	18	20	8	10	12	14
16	18	20	22	8	10	12	14
18	20	22	24	8	10	12	14
20	22	24	26	8	10	12	14
22	24	26	28	8	10	12	14
24	26	28	30	8	10	12	14
6	28	30	32	8	10	12	14

$\tilde{\lambda}$				μ	$\tilde{T}C$
λ_1	λ_2	λ_3	λ_4		
6	8	10	12	2.63	197.48
6	8	10	12	2.47	210.24
6	8	10	12	2.39	222.26
6	8	10	12	2.22	233.66
6	8	10	12	2.12	244.98
6	8	10	12	2.03	255.67
6	8	10	12	1.96	265.16
6	8	10	12	1.89	274.91

Figure-1: Optimal total cost and fuzzified service



3. Service Control Model: Crisp Mathematical Formulation

Total cost function for the model is defined as $TC = k\,n\,\mu + c\,E(n)$. This implies that $TC = k\,n\,\mu + c\,\dfrac{\lambda}{\mu}$

For minimum cost with respect to service, we have

$\dfrac{d}{d\mu}[TC] = \dfrac{d}{d\mu}\left(k\,n\,\mu + \dfrac{c\,\lambda}{\mu}\right)$. For stationary values $\dfrac{d}{d\mu}[TC] = 0$, this ultimately yields $\mu = \left(\dfrac{c\lambda}{k\,n}\right)^{1/2}$ alongwith having satisfied $\dfrac{d^2}{d\mu^2}[TC] = \dfrac{2c\lambda}{\mu^3} > 0$ for minimum cost.

3.1 Service Control Model: Fuzzy Mathematical Model

Now, we wish to fuzzify cost coefficients and arrival rates k, c, λ, and n with the help of trapezoidal fuzzy numbers (defined by function A) as $\tilde{k}$, $\tilde{c}$, $\tilde{\lambda}$ and $\tilde{n}$ respectively depicted as

$$\tilde{k} = (k_1, k_2, k_3, k_4),\quad \tilde{c} = (c_1, c_2, c_3, c_4),\quad \tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)\quad \tilde{n} = (n_1, n_2, n_3, n_4)$$

Now, we define fuzzified total cost as

$\tilde{TC} = \tilde{k}\ \tilde{n}\ \mu + \dfrac{\tilde{c}\,\tilde{\lambda}}{\mu}$, which implies that

$$\tilde{TC} = \left(k_1 n_1 \mu + \frac{c_1\lambda_1}{\mu},\ k_2 n_2 \mu + \frac{c_2\lambda_2}{\mu}, k_3\, n_3 \mu + \frac{c_3\lambda_3}{\mu}, k_4 n_4 \mu + \frac{c_4\lambda_4}{\mu}\right)$$

This is further simplified as

$\tilde{TC} = (W, X, Y, Z)$, where

Now, we define left and right cuts as

$$C_L(\alpha) = W + (X - W)\alpha \quad = k_1 n_1 \mu + \frac{c_1\lambda_1}{\mu}\left[\left(k_2 n_2 \mu + \frac{c_2\lambda_2}{\mu} - \left(k_1 n_1 \mu + \frac{c_1\lambda_1}{\mu}\right)\right)\right]\alpha \text{, and}$$

$$C_R(\alpha) = Z - (Z - Y)\,\alpha \quad = \left(k_4 n_4 \mu + \frac{c_4\lambda_4}{\mu - \lambda_4}\right) - \left(k_4 n_4 \mu + \frac{c_4\lambda_4}{\mu - \lambda_4} - k_3 n_3 \mu + \frac{c_3\lambda_3}{\mu}\right)\alpha$$

Now, we apply SDM as

$$\breve{T}C_{ds} = \frac{1}{2}\int_0^1 [\ C_L(\alpha) + C_R(\alpha)]d\alpha\ ,$$

$$\breve{T}C_{ds} = \frac{1}{4}(k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4)\mu + \frac{1}{\mu}(c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3 + c_4\lambda_4)$$

Now, in order to attain minimum fuzzified cost, we have now stationary value

$$\frac{d\breve{T}C_{ds}}{d\mu} = \frac{1}{4}(k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4) - \frac{1}{\mu^2}(c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3 + c_4\lambda_4) = 0$$, yielding result as

$$\mu = \left(\frac{c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3 + c_4\lambda_4}{k_1 n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4}\right)^{1/2}$$, having satisfied the condition as

$$\frac{d^2}{d\mu^2}\tilde{T}C_{ds} = \frac{1}{2}\left(\frac{c_1\lambda_1 + c_2\lambda_2 + c_3\lambda_3 + c_4\lambda_4}{\mu}\right) > 0$$

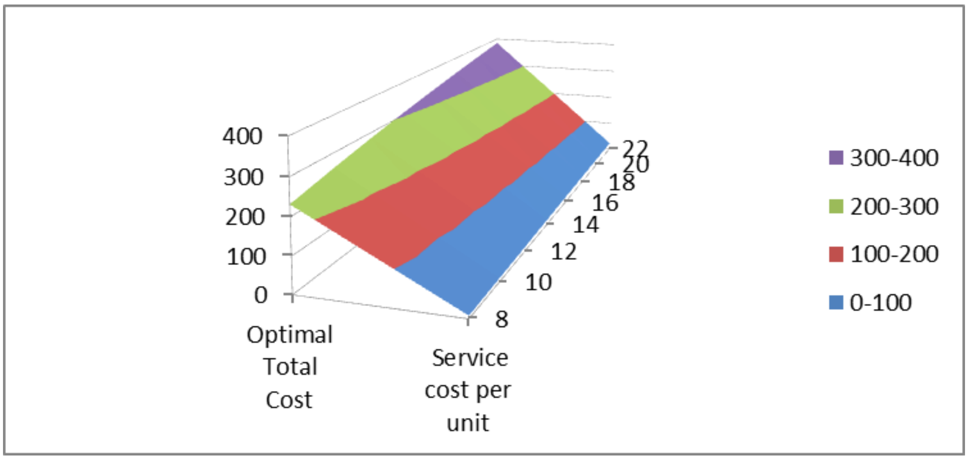
3.2 Service Control Model: Crisp Computation

Following tables 3 represents computation of different cost functions under crisp environment.

Table-3 (Computation table for k, TC)

k	c	λ	n	μ	TC
8	14	12	10	1.44	231.86
10	14	12	10	1.29	259.23
12	14	12	10	1.18	283.97
14	14	12	10	1.09	306.72
16	14	12	10	1.02	327.89
18	14	12	10	0.96	347.79
20	14	12	10	0.91	366.63
22	14	12	10	0.87	384.45

Figure-3: Variation of Optimal Total Cost and Service cost per unit



3.3 Service Control Model: Fuzzy Computation

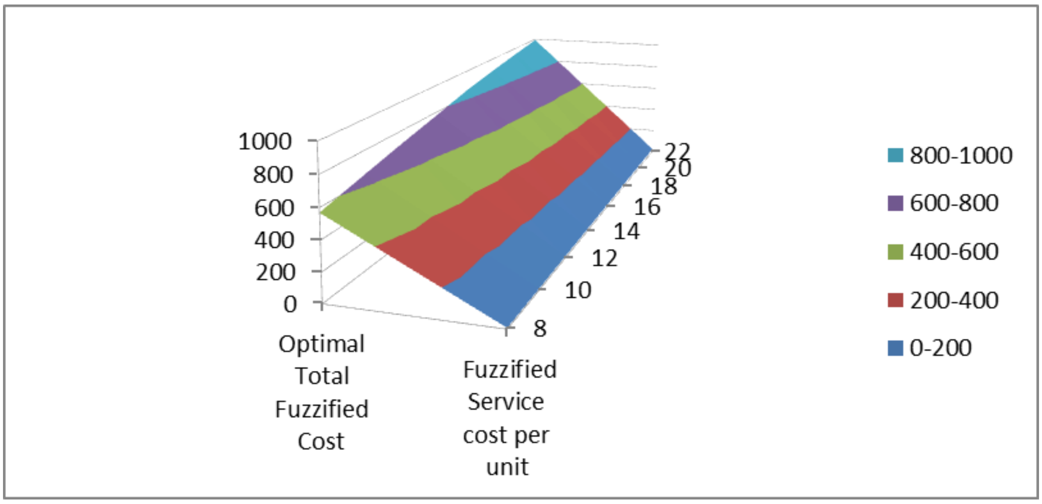
Following tables 4 represents computation of different cost functions under fuzzy environment.

Table: 4 (Computation table for $\tilde{k}$, $\tilde{T}C$)

$\tilde{\lambda}$				$\tilde{c}$			
λ_1	λ_2	λ_3	λ_4	c_1	c_2	c_3	c_4
9	11	13	15	11	13	15	17
9	11	13	15	11	13	15	17
9	11	13	15	11	13	15	17
9	11	13	15	11	13	15	17
9	11	13	15	11	13	15	17
9	11	13	15	11	13	15	17
9	11	13	15	11	13	15	17
9	11	13	15	11	13	15	17

$\tilde{k}$				$\tilde{n}$				μ	$\tilde{T}C$
k_1	k_2	k_3	k_4	n_1	n_2	n_3	n_4		
5	7	9	11	7	9	11	13	1.42	570.05
7	9	11	13	7	9	11	13	1.28	644.15
9	11	13	15	7	9	11	13	1.17	713.23
11	13	15	17	7	9	11	13	1.09	778.53
13	15	17	19	7	9	11	13	1.02	846.73
15	17	19	21	7	9	11	13	0.96	898.43
17	19	21	23	7	9	11	13	0.91	946.98
19	21	23	25	7	9	11	13	0.87	991.15

Figure-4: Optimal total fuzzified cost and fuzzified service cost



4.Arrival and Service Control Model: Crisp Mathematical Model

Total cost function for the model is defined as $TC = k\,n\mu + \dfrac{c\,\lambda}{\mu-\lambda}$.

For minimum cost with respect to service, we have

$\dfrac{d}{d\mu}[TC] = \dfrac{d}{d\mu}\left(k\,n\,\mu + \dfrac{c\,\lambda}{\mu-\lambda}\right)$. For stationary values $\dfrac{d}{d\mu}[TC] = 0$, this ultimately yields $\mu = \lambda + \left(\dfrac{c\lambda}{k\,n}\right)^{1/2}$ (for max service), along with satisfied $\dfrac{d^2}{d\mu^2}[TC] = \dfrac{2c\lambda}{(\mu-\lambda)^3} > 0$ for minimum cost.

4.1 Arrival and Service Control Model: Fuzzy Mathematical Model

We construct fuzzified total cost function as $T\tilde{C} = \tilde{k}\ \tilde{n}\ \mu + \dfrac{\tilde{c}\ \tilde{\lambda}}{\mu - \tilde{\lambda}}$

Now, we wish to fuzzify cost coefficients and arrival rates k, c, λ, and n with the help of trapezoidal fuzzy numbers as $\tilde{k}$, $\tilde{c}$, $\tilde{\lambda}$ and $\tilde{n}$ respectively.

$$k \to \tilde{k}, \qquad c \to \tilde{c}, \qquad \lambda \to \tilde{\lambda} \qquad n \to \tilde{n}$$

$$\tilde{k} = (k_1, k_2, k_3, k_4), \quad \tilde{c} = (c_1, c_2, c_3, c_4), \quad \tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4) \quad \tilde{n} = (n_1, n_2, n_3, n_4)$$

Further, we have $T\tilde{C} = \tilde{k}\ \tilde{n}\ \mu + \dfrac{\tilde{c}\ \tilde{\lambda}}{\mu - \tilde{\lambda}}$, which implies that

$$\tilde{T}C = (k_1, k_2, k_3, k_4)(n_1, n_2, n_3, n_4)\mu + \frac{(c_1, c_2, c_3, c_4)(\lambda_1, \lambda_2, \lambda_3, \lambda_4)}{\mu - (\lambda_1, \lambda_2, \lambda_3, \lambda_4)}$$

This is expressed as

$\tilde{T}C = (W, X, Y, Z)$, where, $W = k_1 n_1 \mu - c_1, X = k_2 n_2 \mu - c_2, Y = k_3 n_3 \mu - c_3, Z = k_4 n_4 \mu - \dfrac{c_4 \lambda_4}{\mu - \lambda_4}$

Now, we define left and right cuts as

$C_L(\alpha) = W + (X - W)\alpha = (k_1\, n_1 \mu - c_1) + [(k_2\, n_2 \mu - c_2 - (k_1\, n_1 \mu - c_1))]\alpha$ and

$CR(\alpha)=Z-(Z-Y)\ \alpha = \left(k_4 n_4 \mu + \dfrac{c_4 \lambda_4}{\mu - \lambda_4}\right) - \left(k_4 n_4 \mu + \dfrac{c_4 \lambda_4}{\mu - \lambda_4} - k_3 n_3 \mu + c_3\right)\alpha$

Next, we apply SDM as $\breve{T}\,C_{ds} = \dfrac{1}{2}\displaystyle\int_0^1 [\,C_L(\alpha) + C_R(\alpha)]\ d\alpha$

$$\tilde{T}C_{ds} = \frac{1}{4}\left[(k_1\, n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4)\mu - (c_1 + c_2 + c_3) + \frac{c_4 \lambda_4}{\mu - \lambda_4}\right]$$

Now, for minimum cost with respect to μ $\dfrac{d}{d\mu}[TC_{ds}] = \dfrac{1}{4}\left[(k_1\, n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4) - \dfrac{c_4 \lambda_4}{(\mu - \lambda_4)^2}\right]$ =0, with sufficient condition $\dfrac{d^2}{d\mu^2}[TC_{ds}] = \dfrac{1}{2}\dfrac{c_4 \lambda_4}{(\mu - \lambda_4)^3} > 0$, which ultimately gives us $(k_1\, n_1 + k_2 n_2 + k_3 n_3 + k_4 n_4)(\mu - \lambda_4)^2 - c_4 \lambda_4 = 0$

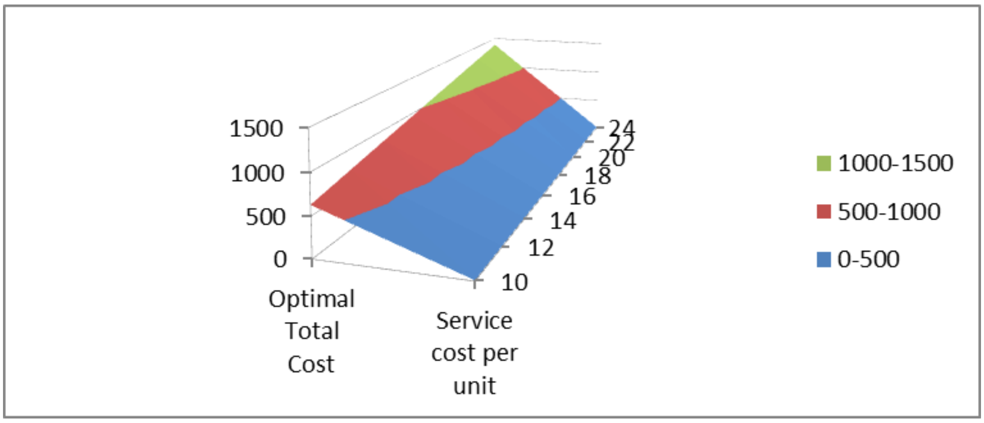
4.2 Arrival and Service Control Model: Crisp Computation

Following tables 5 represents computation of different cost functions under crisp environment

Table-5 (Computation table for k, TC)

k	c	λ	n	μ	TC
10	12	8	6	9.26	631.79
12	12	8	6	9.15	742.27
14	12	8	6	9.06	851.60
16	12	8	6	9.00	960.00
18	12	8	6	8.94	1067.64
20	12	8	6	8.89	1174.66
22	12	8	6	8.85	1281.14
24	12	8	6	8.81	1387.15

Figure-5: Variation of Optimal Total Cost and Service cost per unit



4.3 Arrival and Service Control Model: Fuzzy Computation

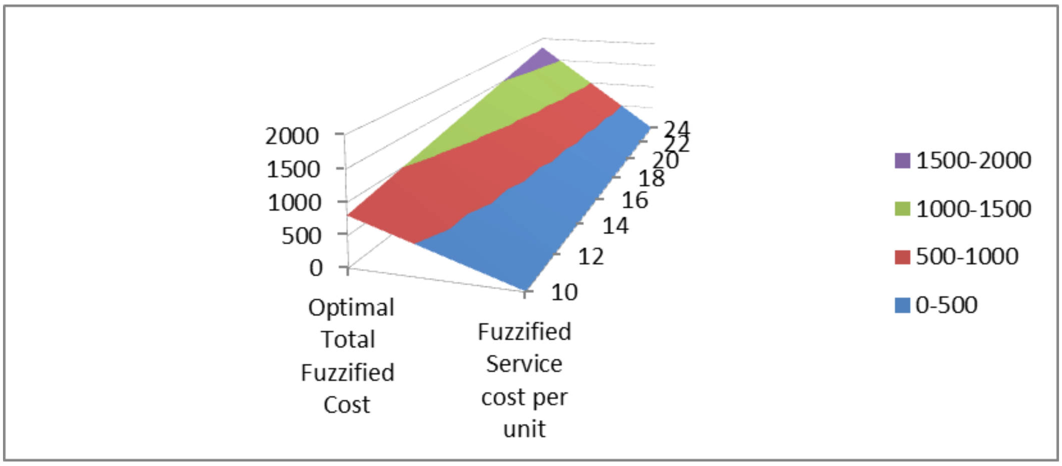
Following tables 6 represents computation of different cost functions under fuzzy environment.

Table 6: (Computation table for $\tilde{k}$, $\tilde{TC}$)

$\tilde{\lambda}$				$\tilde{c}$			
λ_1	λ_2	λ_3	λ_4	c_1	c_2	c_3	c_4
5	7	9	11	9	11	13	15
5	7	9	11	9	11	13	15
5	7	9	11	9	11	13	15
5	7	9	11	9	11	13	15
5	7	9	11	9	11	13	15
5	7	9	11	9	11	13	15
5	7	9	11	9	11	13	15
5	7	9	11	9	11	13	15

$\tilde{k}$				$\tilde{n}$				μ	$\tilde{TC}$
k_1	k_2	k_3	k_4	n_1	n_2	n_3	n_4		
7	9	11	13	3	5	7	9	11.79	810.31
9	11	13	15	3	5	7	9	11.73	951.46
11	13	15	17	3	5	7	9	11.68	1091.92
13	15	17	19	3	5	7	9	11.63	1231.84
15	17	19	21	3	5	7	9	11.60	1371.29
17	19	21	23	3	5	7	9	11.57	1510.36
19	21	23	25	3	5	7	9	11.54	1649.09
21	23	25	27	3	5	7	9	11.52	1787.54

Figure-6: Variation of Optimal Total Fuzzified Cost and Fuzzified Service cost/unit



5. Conclusion

In the present investigation, we constructed a queuing model in fuzzy environment as it provides more realistic solution when measurements are beyond human control. The total optimal cost of considered queuing model with single server in fuzzy environment has given an enhanced solution to the queuing model in crisp environment. It may be observed that the results of section 3-5 are easily comparable for both the environments, the crisp and the fuzzy. The further extension of considered queuing model to the fuzzy environment may give various applications with uncertainty. The present work may be very informative to researchers working in the field and may give robust results which may be of paramount importance in the construction of queuing models for fuzzy environment. Such queuing models are very important in establishing the more efficient waiting line system to meet out staffing needs and settlement, pricing, management of arrivals, service quality, reduction of customer' waiting time and increasing serviced customers etc. These models may also be useful in providing queuing performance measures as operational management strategies for scheduling and inventory control to elevate customer-service in the organizations with natural queues. The present Markovian queuing model techniques is also important for six sigma practitioners for enhancing the customer-service different organizations. Fuzzy neuro and intuitionistic fuzzy approaches may also be more realistic to study such queuing models for further studies.

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