Research Article

Cost Analysis of M/M/1 and MF/MF/1 Queuing Model using ANN and Signed Distance Method

S S Mishra[a], Sandeep Rawat[b], S Ahmad Ali[c], B B Singh[d], Abhishek Singh[e]

[b,c] Department of Mathematics & Computer Science, Babu Banarasi Das University, Lucknow-226028, U.P., INDIA
[a,d,e]Department of Mathematics & Statistics (Centre of Excellence) Dr. Rammanohar Lohia Avadh University, Ayodhya-224001, U.P., INDIA
[a] smssmishra5@gmail.com, [e] abhi.rmlau@gmail.com

Article History: Received: 11 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 23 May 2021

Abstract: In this paper, we attempt to analyze total cost of Markovian queuing models M/M/1 and MF/MF/1by using artificial neural network that models the complex and non-linear relationship between input and output and signed distance method respectively. Total costs of the models have been subjected to variation with the change in depending parameters of arrival and service rates. An efficient algorithm has been developed in order to compute the results. Models' respective results have been presented through tables and graphs.

Keywords: Artificial Neural Network (ANN), Markovian Queuing System, Modelling and Computing, cost, M/M/1, MF/MF/1

1. Introduction

Queuing theory is the mathematical study of waiting lines or "queue". This technique provide basis of decision theory making about the resources needed to provide a service. The formation of waiting lines is a common phenomenon which occurs whenever the current demand for a service exceeds the current capacity to provide that service. A queue is formed whenever a customer is made to wait due to the fact that numbers of customers are more than the service provider. Recourses are optimized among the customers by applying queuing theory.

Mathematical analysis using queuing is frequently used in (i) toll-plaza (ii) telephone-industry (iii) software-industry (iv) service-centers (v) call centers (vi) hospital and airports etc.; vide Baskett[5], Bunday[7], Chen H and Yao[8], Hiller & Liebermann[17], Jain and Smith[19], Kendall[22], Koenigsberg[23], Mishra & Shukla[26], Mishra & Yadav[28], Priya, and Sudhesh [33], Taha[44], Udagawa & Nakamura[45], Wagner[49], Sharma [39], Singh *et al.* [40] and Sivakami and Palaniammal [41].

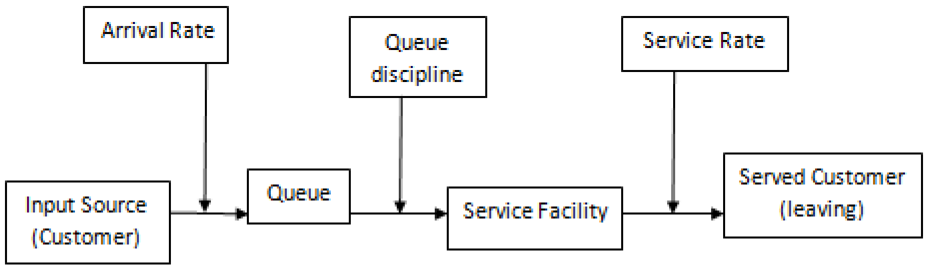


Fig.1 Basic Components of Queuing System

Implementing ANN approach to reach solution for infinite capacity and single server queue of Markovian model. We also need various results to be compared which are obtained by applying traditional methods of Mathematics. ANN approach is a bio-inspired or nature inspired approach which aims to train data set so that it can take decision at own when forecasting or prediction is intended. It is detailed at relevant point of the section. The fundamental concept of Neural-Network (NN) Design and traffic management have been developed by Daganzo [10], Hagan et al [15], Amdeo [2], Jain & Mao[18], Kalogirou [20], Kelly [21], Nagel [31], Namdeo [32], Raheja [38], Specht[42].

Among execution estimates associated with Markovian queues, traffic congestion and its study has been serious concern for the professionals and researchers engaged in the field. Several researchers have endeavored to examine the different aspects of Markovian queue and its measures applying various analytic and computational techniques; see for references Mala and Verma [24] , Sivakami and Palaniammal [41] , Vandaele *et al* [47], Jain and Smith [19], Van Woensel & Vandaele [46].

First, there are several analytical techniques including method of iteration and method of generating function which are utilized to examine the model to furnish different execution estimates associated with Markovian queue with infinite capacity having single server. They can just give the current solution to the model viable. But if any future prediction for any performance measure is required to improve or adjust the service channel's capacity, these techniques are not adequate to fix such issue. Second, we lack adequate size of real time data inputs to form the training set of managed learning, consequently prediction process for any working attributes of queueing model is not possible to be predicted proficiently. At the same time, to compute the total cost of the model in fuzzy environment is another useful and interesting aspect of the model to supplement the utility of the model for the future application. These analyses of the models under considerations are part of frontier research of the field and need of the current time. Therefore, total cost of the models M/M/1 and MF/MF/1 has been discussed in this paper as an important performance measure. Also, related variational analysis has been displayed and their connected graphical facets have been also presented to make the insight into the model more lucid.

Notations and Assumptions: Notations used frequently are the following.

λ : Average Arrival rate of customer.

μ : Average Service rate.

E(n) : average number of customers in the system.

$p_n(t)$: probability of exactly n customers in queuing system (waiting + service).

$p_0(t)$: probability of exactly no customer in the system at time t.

$p_1(t)$: probability of exactly one customer in queue.

L_s : Average customers in queue.

L_q : Average customers in queue.

W_s : Average waiting time in system (includes service time) for each individual customer or time a customer spends in the system.

W_q : Average waiting time in queue (excludes service time) for each individual customer or Expected time a customer spends in a queue.

TC : Total Cost

V(n): the variance of the queue size (fluctuation in queue).

2. Model and Methods

Following essential valuable material and relevant techniques are used in this section. **Queueing Model**

Here, we consider Markovian queueing model of which both arrival and service follow Poisson probability law and it has single server with first come and first served discipline as well as infinite capacity. There are several operating characteristics or performance measures of the queuing model namely traffic congestion, expected number of customers in queue and system; expected waiting time in queue and system; and service utilization factor or busy period.

M/M/1 Model

The M/M/1 system is made of a Poisson arrival (Arrival rate λ), one exponential server (Service rate μ), unlimited FIFO (or not specified queue), and unlimited customer population. Because both arrival and service are Poisson processes, it is possible to find probabilities of various states of the system that are necessary to compute the required quantitative parameters. System state is the number of customers in the system. It may be any non-negative integer number.

The following are the three cases:

Time t and units	Arrival	Service	Time (t + h) No. of units
N n-1 n+1	0 1 0	0 0 0	n n n

According to law of compound probabilities, the system yields at time (t+h) as

$p_n(t+h) = p_n(t)[1-(\lambda+\mu)h] + [1-\mu h] + p_{n-1}(t)\lambda h + p_{n+1}(t)\mu h + 0(h)$, giving us

$\frac{p_n(t+h)- p_n(t)}{h} = -(\lambda+\mu)p_n(t) + \lambda p_{n-1}(t) + \mu\, p_{n+1}(t) + \frac{0(h)}{h}$, which further turns out to be

$\lim\limits_{h\to 0}\frac{p_n(t+h)- p_n(t)}{h} = \lim\limits_{h\to 0}\left[-(\lambda+\mu)p_n(t) + \lambda p_{n-1}(t) + \mu\, p_{n+1}(t) + \frac{0(h)}{h}\right]$, implying

$$\frac{dp_n(t)}{dt} = -(\lambda+\mu)p_n(t) + \lambda p_{n-1}(t) + \mu\, p_{n+1}(t)$$

where $n > 0$, $\left(\lim\limits_{h\to 0}\frac{0(h)}{h} = 0\right)$. Also, steady state mandates to produce as

$p_n(t) \to 0,\ p_n(t) = p_n$, this produces the following condition as

$$0 = -(\lambda+\mu)p_n + \lambda p_{n-1} + \mu\, p_{n+1} \tag{1}$$

Proceeding exactly in the same way, probability of no units in the system at time (t+h) is also given as

$p_0(t+h) = p_0(t)[1-\lambda h] + p_1(t)\mu h + 0(h)$. This implies that

$$\frac{p_0(t+h) - p_0(t)}{h} = -\lambda\, p_0(t) + \mu\, p_1(t) + \frac{0(h)}{h}$$

$$\lim_{h\to 0}\frac{p_0(t+h) - p_0(t)}{h} = -\lambda\, p_0(t) + \mu\, p_1(t) \quad , \qquad \text{for } n = 0$$

which finally gives us as

$$\frac{d\, p_0(t)}{d(t)} = -\lambda\, p_0(t) + \mu\, p_1(t)$$

Under steady state, we have

$$0 = -\lambda\, p_0 + \mu\, p_1 \tag{2}$$

Equation (2) turns to give $p_1 = \frac{\lambda}{\mu}\, p_0$ and from equation (1), we have

$p_2 = \frac{\lambda}{\mu}\, p_1 = \left(\frac{\lambda}{\mu}\right)^2 p_0$. In general, we get $p_n = \left(\frac{\lambda}{\mu}\right)^n p_0$

Next we know that, $\sum_{n=0}^{\infty} p_n = 1$, it implies that $p_0 + \frac{\lambda}{\mu}\, p_0 + \left(\frac{\lambda}{\mu}\right)^2 p_0 + \cdots\cdots\cdots = 1$

$p_0\left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \cdots\cdots\cdots\right] = 1$, which produces that $p_0\left(\frac{1}{1-\frac{\lambda}{\mu}}\right) = 1$

since $\frac{\lambda}{\mu} < 1$, sum of infinite G.P. is valid. Therefore, we have $p_0 = 1 - \frac{\lambda}{\mu} = 1 - \rho$

Also $p_n = \left(\frac{\lambda}{\mu}\right)^n p_0 = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$ and also we can have $p_n = \rho^n\,(1-\rho)$

The following important performance measures of model are derived from above results.

$$L_s = \sum_{n=0}^{\infty} n\, p_n = \sum_{n=0}^{\infty} n\left(\frac{\lambda}{\mu}\right)^n\left(1 - \frac{\lambda}{\mu}\right) = \frac{\rho}{1-\rho} \quad , \quad \rho = \frac{\lambda}{\mu} < 1$$

$$\mathrm{L_q} = \mathrm{L_s} - \frac{\lambda}{\mu} = \frac{\rho^2}{1-\rho} = \frac{\rho}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$\mathrm{W_q} = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{\rho}{\mu(1-\rho)}$$

$W_s = W_q + \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)} + \frac{1}{\mu} = \frac{1}{\mu-\lambda}$, Further

$$\mathrm{var}\,(\mathrm{n}) = \mathrm{E}(\mathrm{n}^2) - [\mathrm{E}(\mathrm{n})]^2$$

$$\sum_{n=1}^{\infty} n^2\, p_n (1-\rho)\rho^n - \left(\frac{\rho}{1-\rho}\right)^2 \;,\; \sum_{n=1}^{\infty} n\, p_n = E(L_s) = \frac{\rho}{1-\rho}$$

$$\mathrm{var}\,(\mathrm{n}) = \rho\,(1-\rho)\frac{(1+\rho)}{(1-\rho)^3} - \frac{\rho^2}{(1-\rho)^2} \qquad = \frac{\rho}{(1-\rho)^2}$$

Service Time: It is time duration in one servicing completed.

Busy Period: it is a time duration in which server is always engaged in providing the service. **Idle Period:** This is a time duration in which server has no customer to serve in the system **Total Cost of the System**: It is defined as

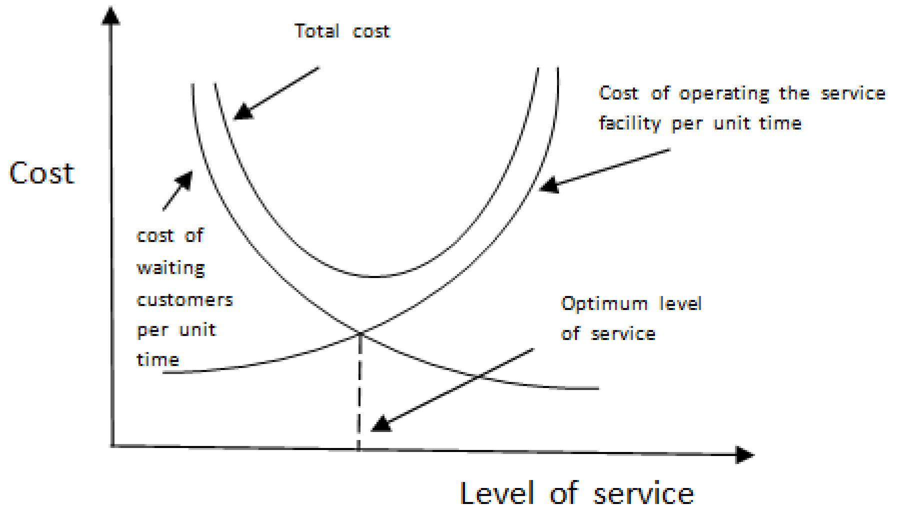
$$\mathrm{TC} = \text{waiting cost} + \text{service cost}$$

Let c_1 *and* c_2 be per unit cost of the waiting and service respectively.

Therefore, $TC = c_1\, E(n) + c_2\, \mu$

$$TC = c_1\,\frac{\lambda}{\mu-\lambda} + c_2$$

Fig. 2 Graph of Cost Model



Artificial Neural Network Approach

Artificial Neural Network (ANN) is a machine learning approach that models human brain and consists of a number of artificial neurons. Neuron in ANN's tend to have fewer connections than biological neurons vide Altiparmak et al [1], Badiru A.B., Sieger [4], Cortez et al [9], Demuth [11], Ding [12], Ding [13], Ertunc and Hosoz [14], Hensher and Ton [16].

Artificial Neural Network is computational implementation of human-brain designed by imitating neurons' working mechanism. Billions of neurons of human brain act as organic switches which are interconnected that form a Neural Network. A single neuron produces an output that depends on inputs taken from thousands of interconnected neurons vide Miguel [25], Mishra [27], Modestus et al [29], Modestus et al [30], Vlahogianni et al [48].

Continuous activation of certain connections of neuron which makes the human brain strong, this working is known as learning of a human brain. This is a key property of ANN. As per latest MIT report, ANN is among the top ten technologies in the world.

Mathematical framework of ANN is given as under:

i. Neural inputs are denoted by x, and its bias by x_0=+/-1 (computing process is affected by some of neurons).

ii. Synaptic connection is denoted by weights w.

iii. Activation function is given by $y = \text{sum}(wx) + b$, b is bias; y is an output.

iv. y is an outcome of activation function.

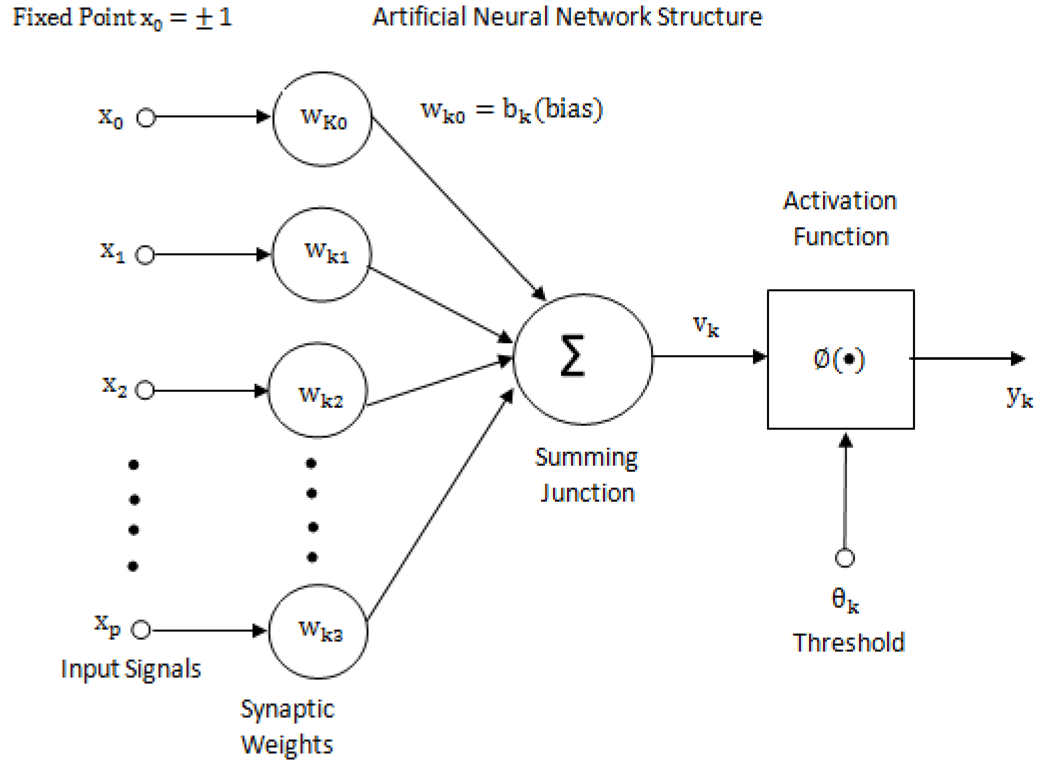
v. Activation maps as $y_{next} = f(y)$, y is an output.

vi. This value, through a synapse.

vii. $f(y) >= \theta$, $y=1$, $f(y)<\theta$, $y=0$ where θ is a threshold value.

viii. Activation function f is "linear" in nature at the input layer of ANN and f is "non-linear" at hidden layer. At output layer, it is both linear and non-linear.

Fig.3 Description of Activation Function



R Language

R is a programming language developed by Ross Ihaka and Robert Gentleman at the Department of Statistics of the University of Auckland in 1993. R possesses an extensive catalog of statistical and graphical methods. It includes machine learning algorithms, linear regression, time series, statistical inference to name a few. R is free software and is freely available under the GNU General Public License. Data analysis with R is done in a series of steps; programming, transforming, discovering, modelling and communicate the results.The computational algorithm is implemented on R (Core Team) [35], [36], [37].

Analysis and Computing

Simulation modeling and computing algorithm is used to compute and analyze the problem under consideration.

Simulation modeling

Simulation modelling is preferred when model is complex where experimentation and interaction between components and variables are not easily possible. Knowledge gained through simulation modelling can be used to improve the previous system by changing the input data and observing its consequential output. If any model is solved analytically and cost is more than saving, we don't prefer simulation modelling. Simulation modelling is essential for any system to simulate in order to model random input data. It is frequently recommended for any applied discipline where we need to introduce randomness or random occurrence of events; vide Averill [3] and Bernard [6].

For this simulation modelling, we need to choose some suitable probability distribution to fulfil the purpose of randomness property. Here, we choose Gaussian distribution and R software to implement the simulation modelling for arrival and service for Markovian queueing system with single server in which both arrival and service follow the property of randomness. We use input data for simulation modelling from Mala and Varma [24] and Rawat et al [34].

Computing Flow Chart

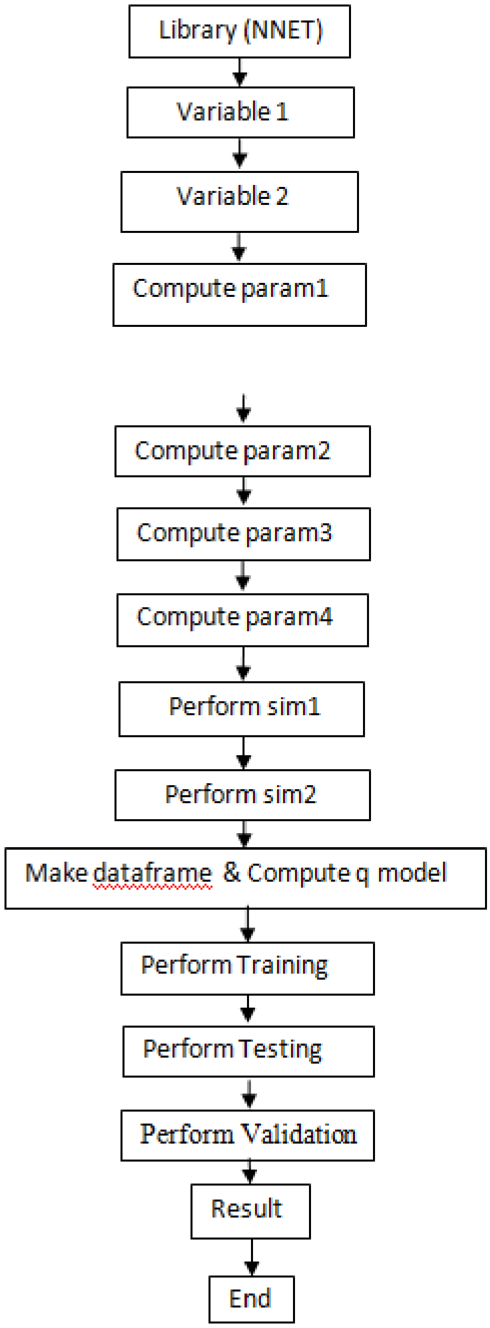


Table 1: Arrival and Service Rates

S. No.	Arrival	Service	S. No.	Arrival	Service

1	33.26885	38.19984	31	22.52678	40.83685
2	29.35008	35.08465	32	29.03038	43.67070
3	33.50372	42.50377	33	31.69670	45.57549
4	33.99833	41.35702	34	29.08163	38.17111
5	28.74941	44.80064	35	24.82413	37.82121
6	23.56832	41.85777	36	30.48325	41.05215
7	26.91346	38.52786	37	24.44203	38.24485
8	25.18780	38.29431	38	27.68503	39.06523
9	26.21402	37.28480	39	27.48972	39.72450
10	33.52164	41.92452	40	29.37367	40.76935
11	26.48595	42.01551	41	29.39765	43.59791
12	29.26787	45.89217	42	27.14805	37.38361
13	23.50952	42.37004	43	28.83287	39.32818
14	31.84922	44.97265	44	28.11183	43.07731
15	28.31358	41.95792	45	27.55532	42.20795
16	32.19199	38.61816	46	30.04771	43.25208
17	30.40682	39.53377	47	32.48268	39.64737
18	25.20710	40.83042	48	30.66482	34.85977
19	27.47929	43.23704	49	33.57852	43.56043
20	31.65242	43.74712	50	26.61578	42.33272
21	36.36383	42.53102	51	33.59845	41.05877
22	29.95785	41.24511	52	30.12467	40.90972
23	29.63227	38.02543	53	26.41421	39.86862
24	30.13362	40.70742	54	31.69022	38.85052
25	30.48719	43.99321	55	33.58213	38.98596
26	30.00760	42.37081	56	28.68472	43.79155
27	32.10908	44.91334	57	27.69915	36.52601
28	25.68893	40.48561	58	31.32093	39.75510
29	27.28678	34.90510	59	30.73140	40.57635
30	25.73413	45.02572	60	33.28239	47.38870

Table 2: Arrival, Service and Congestion Rates

S. No.	WC	SC	TC	S. No.	WC	SC	TC
1	13.493773	114.5995	128.0933	31	2.460589	122.5105	124.9711
2	10.236180	105.2540	115.4901	32	3.965812	131.0121	134.9779
3	7.445230	127.5113	134.9565	33	4.567645	136.7265	141.2941
4	9.240319	124.0710	133.3114	34	6.398966	114.5133	120.9123
5	3.582208	134.4019	137.9841	35	3.819957	113.4636	117.2836
6	2.577258	125.5733	128.1506	36	5.768480	123.1564	128.9249
7	4.634500	115.5836	120.2181	37	3.541600	114.7345	118.2761
8	3.843556	114.8829	118.7265	38	4.865475	117.1957	122.0612
9	4.735710	111.8544	116.5901	39	4.493703	119.1735	123.6672

10	7.978609	125.7736	133.7522	40	5.155230	122.3080	127.4633
11	3.411037	126.0465	129.4576	41	4.140439	130.7937	134.9342
12	3.521095	137.6765	141.1976	42	5.304657	112.1508	117.4555
13	2.492987	127.1101	129.6031	43	5.494425	117.9845	123.4790
14	4.853797	134.9179	139.7717	44	3.756892	129.2319	132.9888
15	4.150230	125.8738	130.0240	45	3.761144	126.6239	130.3850
16	10.019017	115.8545	125.8735	46	4.551177	129.7562	134.3074
17	6.663088	118.6013	125.2644	47	9.067440	118.9421	128.0095
18	3.226858	122.4913	125.7181	48	14.619888	104.5793	119.1992
19	3.487717	129.7111	133.1988	49	6.727871	130.6813	137.4092
20	5.234097	131.2414	136.4755	50	3.386889	126.9982	130.3851
21	11.792671	127.5931	139.3857	51	9.007251	123.1763	132.1835
22	5.308256	123.7353	129.0436	52	5.586379	122.7291	128.3155
23	7.061051	114.0763	121.1373	53	3.926476	119.6059	123.5323
24	5.699672	122.1223	127.8219	54	8.851645	116.5516	125.4032
25	4.514606	131.9796	136.4942	55	12.429006	116.9579	129.3869
26	4.854337	127.1124	131.9668	56	3.797584	131.3746	135.1722
27	5.015376	134.7400	139.7554	57	6.276102	109.5780	115.8541
28	3.472255	121.4568	124.9291	58	7.427156	119.2653	126.6924
29	7.163466	104.7153	111.8788	59	6.243076	121.7290	127.9721
30	2.667910	135.0772	137.7451	60	4.718796	142.1661	146.8849

Table 3: Normalized Rates

S. No.	**WC**	**SC**	**TC**	**S. No.**	**WC**	**SC**	**TC**
1	0.90738656	0.2665888	0.463191	31	0.00000000	0.4770623	0.374002
2	0.63947694	0.0179494	0.103164	32	0.12379193	0.7032467	0.659859
3	0.40994476	0.6101082	0.659249	33	0.17328763	0.8552784	0.840291
4	0.55757570	0.5185797	0.612252	34	0.32389833	0.2642958	0.258055
5	0.09224370	0.7934333	0.745736	35	0.11179658	0.2363681	0.154396
6	0.00959503	0.5585477	0.464827	36	0.27204620	0.4942465	0.486947
7	0.17878582	0.2927698	0.238224	37	0.08890403	0.2701813	0.182750
8	0.11373736	0.2741293	0.195615	38	0.19778158	0.3356602	0.290874
9	0.18710953	0.1935548	0.134586	39	0.16720649	0.3882799	0.336753
10	0.45381070	0.5638751	0.624845	40	0.22161156	0.4716746	0.445193
11	0.07816629	0.5711373	0.502163	41	0.13815354	0.6974373	0.658610
12	0.08721769	0.8805544	0.837534	42	0.23390060	0.2014410	0.159306
13	0.00266439	0.5994347	0.506321	43	0.24950748	0.3566477	0.331376
14	0.19682116	0.8071622	0.796802	44	0.10660999	0.6558851	0.603038
15	0.13895872	0.5665409	0.518344	45	0.10695967	0.5864976	0.528657
16	0.62161710	0.2999775	0.399779	46	0.17193327	0.6698344	0.640706
17	0.34562015	0.3730566	0.382379	47	0.54335785	0.3821236	0.460798
18	0.06301913	0.4765491	0.395340	48	1.00000000	0.0000000	0.209118
19	0.08447257	0.6686343	0.609038	49	0.35094803	0.6944457	0.729312
20	0.22809769	0.7093464	0.702639	50	0.07618033	0.5964560	0.528658
21	0.76748516	0.6122830	0.785775	51	0.53840787	0.4947747	0.580035
22	0.23419659	0.5096481	0.490337	52	0.25706992	0.4828782	0.469539
23	0.37834923	0.2526681	0.264484	53	0.12055683	0.3997829	0.332900
24	0.26638733	0.4667321	0.455439	54	0.52561054	0.3185230	0.386344
25	0.16892561	0.7289879	0.703175	55	0.81981840	0.3293331	0.500144
26	0.19686557	0.5994958	0.573842	56	0.10995655	0.7128923	0.665410
27	0.21010973	0.8024288	0.796336	57	0.31379384	0.1329916	0.113561
28	0.08320098	0.4490284	0.372801	58	0.40845833	0.3907220	0.423173
29	0.38677204	0.0036183	0.000000	59	0.31107772	0.4562705	0.459729
30	0.01705041	0.8113984	0.738908	60	0.18571852	1.0000000	1.0000000

Training Set: The set of data which enables the training is called the "training set." During the training of a network the same set of data is processed many times as the connection weights are ever refined.

Test Set: It is a set of data set on which trained ANN is implemented to predict the result which is further tested for its validation by applying certain statistical technique that is found popular for this validation.

Table 4: Training Set

S. No.	WC	SC	TC	S. No.	WC	SC	TC
1	0.90738656	0.2665888	0.4631911	21	0.7674851	0.6122830	0.7857757
2	0.63947694	0.0179494	0.1031640	22	0.2341965	0.5096481	0.4903378
3	0.40994476	0.6101082	0.6592496	23	0.3783492	0.2526681	0.2644843
4	0.55757570	0.5185797	0.6122529	24	0.2663873	0.4667321	0.4554394
5	0.09224370	0.7934333	0.7457369	25	0.1689256	0.7289879	0.7031758
6	0.00959503	0.5585477	0.4648275	26	0.1968655	0.5994958	0.5738424
7	0.17878582	0.2927698	0.2382244	27	0.2101097	0.8024288	0.7963360
8	0.11373736	0.2741293	0.1956152	28	0.0832009	0.4490284	0.3728012
9	0.18710953	0.1935548	0.1345863	29	0.3867720	0.00361837	0.0000000
10	0.45381070	0.5638751	0.6248450	30	0.0170504	0.81139841	0.7389081
11	0.07816629	0.5711373	0.5021633	31	0.0000000	0.47706235	0.3740022
12	0.08721769	0.8805544	0.8375348	32	0.1237919	0.70324678	0.6598599
13	0.00266439	0.5994347	0.5063214	33	0.1732876	0.85527842	0.8402916
14	0.19682116	0.8071622	0.7968026	34	0.3238983	0.26429588	0.2580558
15	0.13895872	0.5665409	0.5183442	35	0.1117965	0.23636819	0.1543962
16	0.62161710	0.2999775	0.3997799	36	0.2720462	0.49424658	0.4869479
17	0.34562015	0.3730566	0.3823795	37	0.0889040	0.27018130	0.1827503
18	0.06301913	0.4765491	0.3953406	38	0.1977815	0.33566021	0.2908747
19	0.08447257	0.6686343	0.6090383	39	0.1672064	0.38827995	0.3367535
20	0.22809769	0.7093464	0.7026396	40	0.2216115	0.47167462	0.4451935

Table 5: Test Set

S. No.	WC	SC	TC
1	0.13815354	0.6974374	0.6586107
2	0.23390060	0.2014410	0.1593067
3	0.24950748	0.3566478	0.3313764
4	0.10661000	0.6558851	0.6030386
5	0.10695967	0.5864976	0.5286572
6	0.17193327	0.6698345	0.6407062
7	0.54335785	0.3821236	0.4607987
8	1.00000000	0.0000000	0.2091182
9	0.35094803	0.6944457	0.7293122
10	0.07618033	0.5964561	0.5286587
11	0.53840788	0.4947747	0.5800351
12	0.25706993	0.4828783	0.4695395
13	0.12055684	0.3997829	0.3329008
14	0.52561054	0.3185230	0.3863448
15	0.81981841	0.3293332	0.5001444
16	0.10995655	0.7128923	0.6654108
17	0.31379384	0.1329916	0.1135619
18	0.40845833	0.3907220	0.4231739
19	0.31107772	0.4562705	0.4597297
20	0.18571852	1.0000000	1.0000000

Structure of Artificial Neural Network (ANN)

With the help of above classified data, we draw the ANN as under:

Fig. 7: Structure of ANN

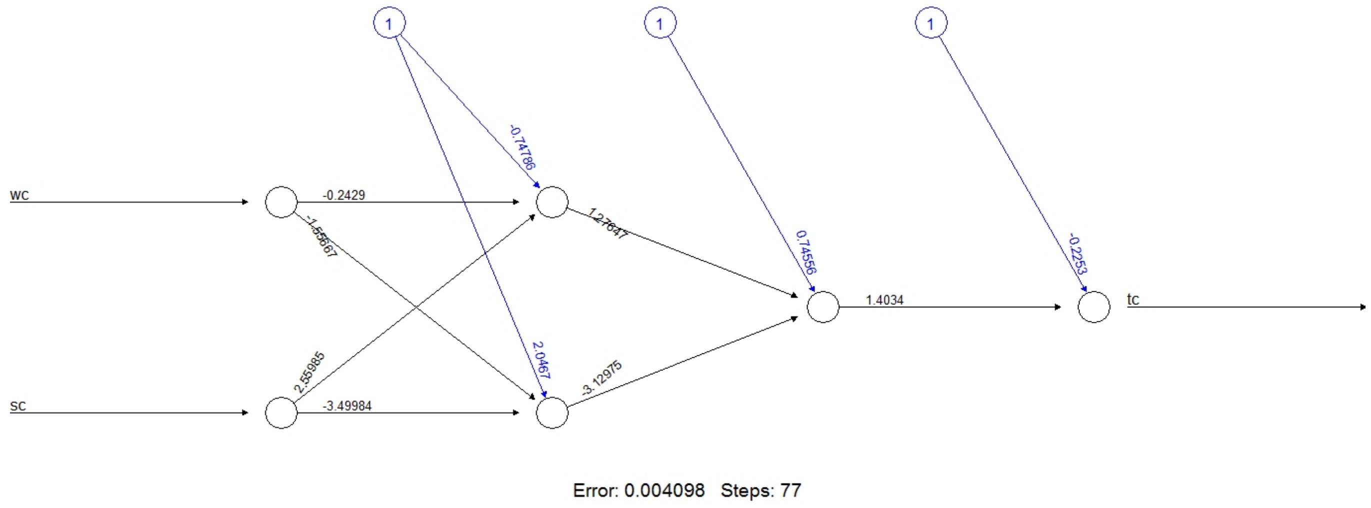


Table 5: Item wise Details of ANN

S. No.	Item	Value	S. No.	Item	Value
1	error	0.004098393	8	wc.to.1layhid2	-1.556666701
2	reached.threshold	0.004752941	9	sc.to.1layhid2	-3.499839491
3	steps	77.000000000	10	Intercept.to.2layhid1	0.745561456
4	Intercept.to.1layhid1	-0.747863586	11	1layhid1.to.2layhid1	1.276470346
5	wc.to.1layhid1	-0.242904807	12	1layhid2.to.2layhid1	-3.129747320
6	sc.to.1layhid1	2.559854522	13	Intercept.to.tc	-0.225296786
7	Intercept.to.1layhid2	2.046701583	14	2layhid1.to.tc	1.403397122

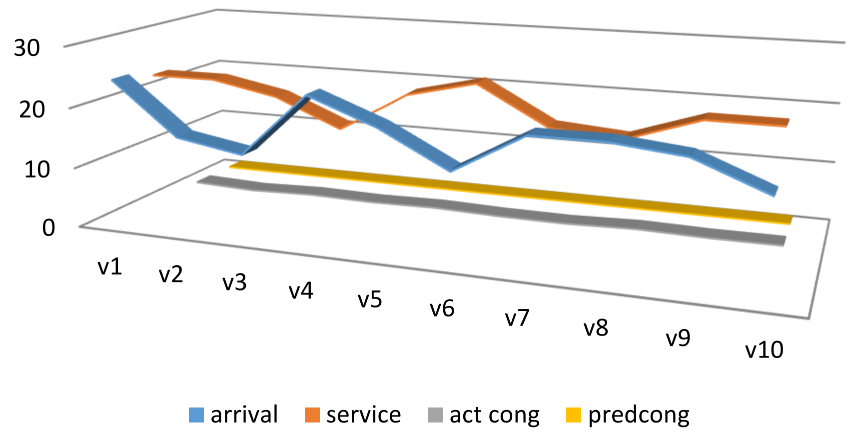
3. Prediction of Results

Using test set, we can predict results as under.

Table 6: Predicted values

S. No.	Actual Value	Predicted Value	S. No.	Actual Value	Predicted Value
1	0.6586107	0.9999981	11	0.5800351	0.7349983
2	0.1593067	0.0999983	12	0.4695395	0.5939983
3	0.3313764	0.1999981	13	0.3329008	0.4995981
4	0.6030386	0.7999983	14	0.3863448	0.4899981
5	0.5286572	0.7099998	15	0.5001444	0.6199982
6	0.6407062	0.7996983	16	0.6654108	0.7959981
7	0.4607987	0.5399983	17	0.1135619	0.2959981
8	0.2091182	0.3699982	18	0.4231739	0.6099981
9	0.7293122	0.8999981	19	0.4597297	0.5399982
10	0.5286587	0.7299983	20	1.0000000	0.9999981

Fig. 8: Arrival, srvice, actual congestion and predicted congestion



Validation: The current model is subjected to validation through fitting the general linear modeling for traffic congestion through popular parameter of coefficient of variation between actual and predicted values of total cost. Established and standard literature says that it permits to be up to five percent. Here, it comes out to be 0.002267938 which validates its legitimacy for the future use in different areas of application.

Fuzzy Mathematical Model

Further, we define a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ with membership function $\mu_A(x)$ as

$$\mu_A(x) = \begin{cases} L(x) = \dfrac{x-a}{b-c}, & \text{when } a \le x \le b \\ 1 & \text{when } b \le x \le c \\ R(x) = \dfrac{d-x}{d-c}, & \text{when } c \le x \le d \\ 0 & otherwise \end{cases}$$

Now, we wish to fuzzify cost coefficients and arrival rates c_1, λ, μ, c_2 with the help of trapezoidal fuzzy numbers as $\tilde{c_1}, \tilde{\lambda}, \tilde{\mu}, and\ \tilde{c}_2$ respectively.

$\tilde{c_1} = (c_{11}, c_{12}, c_{13}, c_{14})$, $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$, $\tilde{\mu} = (\mu_1, \mu_2, \mu_3, \mu_4)$ and $\tilde{c_2} = (c_{21}, c_{22}, c_{23}, c_{24})$

$$\widetilde{TC} = \tilde{c_1} \frac{\tilde{\lambda}}{\tilde{\mu} - \tilde{\lambda}} + \tilde{c_2}\tilde{\mu}$$

which implies that

$$\widetilde{TC} = \left(\tilde{c}_{11} \frac{\tilde{\lambda}_1}{\tilde{\mu}_1 - \tilde{\lambda}_1} + \tilde{c}_{21}\tilde{\mu}_1, \tilde{c}_{12} \frac{\tilde{\lambda}_2}{\tilde{\mu}_2 - \tilde{\lambda}_2} + \tilde{c}_{22}\tilde{\mu}_2, \tilde{c}_{13} \frac{\tilde{\lambda}_3}{\tilde{\mu}_3 - \tilde{\lambda}_3} + \tilde{c}_{23}\tilde{\mu}_3, \tilde{c}_{14} \frac{\tilde{\lambda}_4}{\tilde{\mu}_4 - \tilde{\lambda}_4} + \tilde{c}_{24}\tilde{\mu}_4 \right)$$

which finally turns out to be as

$$\widetilde{T}C = (W, X, Y, Z)$$

Where, $W = \tilde{c}_{11} \frac{\tilde{\lambda}_1}{\tilde{\mu}_1 - \tilde{\lambda}_1} + \tilde{c}_{21}\tilde{\mu}_1$, $X = \tilde{c}_{12} \frac{\tilde{\lambda}_2}{\tilde{\mu}_2 - \tilde{\lambda}_2} + \tilde{c}_{22}\tilde{\mu}_2$, $Y = \tilde{c}_{12} \frac{\tilde{\lambda}_2}{\tilde{\mu}_2 - \tilde{\lambda}_2} + \tilde{c}_{22}\tilde{\mu}_2$

$$Z = \tilde{c}_{14} \frac{\tilde{\lambda}_4}{\tilde{\mu}_4 - \tilde{\lambda}_4} + \tilde{c}_{24}\tilde{\mu}_4$$

Now we define,

$$C_L(\alpha) = W + (X - W)\alpha$$

$$C_L(\alpha) = \left(\tilde{c}_{11} \frac{\tilde{\lambda}_1}{\tilde{\mu}_1 - \tilde{\lambda}_1} + \tilde{c}_{21}\tilde{\mu}_1 \right) + \left[\left\{ \left(\tilde{c}_{12} \frac{\tilde{\lambda}_2}{\tilde{\mu}_2 - \tilde{\lambda}_2} + \tilde{c}_{22}\tilde{\mu}_2 \right) - \left(\tilde{c}_{11} \frac{\tilde{\lambda}_1}{\tilde{\mu}_1 - \tilde{\lambda}_1} + \tilde{c}_{21}\tilde{\mu}_1 \right) \right\} \right] \alpha \quad \text{and}$$

$C_L(\alpha) = \left(\tilde{c}_{11}\,\frac{\tilde{\lambda}_1}{\tilde{\mu}_1-\tilde{\lambda}_1} + \tilde{c}_{21}\tilde{\mu}_1\right) + \left[\tilde{c}_{12}\,\frac{\tilde{\lambda}_2}{\tilde{\mu}_2-\tilde{\lambda}_2} - \tilde{c}_{11}\,\frac{\tilde{\lambda}_1}{\tilde{\mu}_1-\tilde{\lambda}_1} + \tilde{c}_{22}\tilde{\mu}_2 - \tilde{c}_{21}\tilde{\mu}_1\right]\alpha$ and

$C_R(\alpha) = Z - (Z - Y)\,\alpha$

$$C_R(\alpha) = \left(\tilde{c}_{14}\,\frac{\tilde{\lambda}_4}{\tilde{\mu}_4-\tilde{\lambda}_4} + \tilde{c}_{24}\tilde{\mu}_4\right) + \left[\left\{\left(\tilde{c}_{14}\,\frac{\tilde{\lambda}_4}{\tilde{\mu}_4-\tilde{\lambda}_4} + \tilde{c}_{24}\tilde{\mu}_4\right) - \left(\tilde{c}_{12}\,\frac{\tilde{\lambda}_2}{\tilde{\mu}_2-\tilde{\lambda}_2} + \tilde{c}_{22}\tilde{\mu}_2\right)\right\}\right]\alpha$$

$$C_R(\alpha) = \left(\tilde{c}_{14}\,\frac{\tilde{\lambda}_4}{\tilde{\mu}_4-\tilde{\lambda}_4} + \tilde{c}_{24}\tilde{\mu}_4\right) + \left[\tilde{c}_{14}\,\frac{\tilde{\lambda}_4}{\tilde{\mu}_4-\tilde{\lambda}_4} - \tilde{c}_{12}\,\frac{\tilde{\lambda}_2}{\tilde{\mu}_2-\tilde{\lambda}_2} + \tilde{c}_{24}\tilde{\mu}_4 - \tilde{c}_{22}\tilde{\mu}_2\right]\alpha$$

By using signed distance method, the defuzzified value of fuzzy number $\widetilde{TC}$, is given by

$$\widetilde{TC}_{ds} = \frac{1}{2}\int_0^1 \left(C_L(\alpha) + C_R(\alpha)\right)d\alpha$$

$$\widetilde{TC}_{ds} = \frac{1}{2}\left(\tilde{c}_{11}\,\frac{\tilde{\lambda}_1}{\tilde{\mu}_1-\tilde{\lambda}_1} + \tilde{c}_{21}\tilde{\mu}_1 + \tilde{c}_{14}\,\frac{\tilde{\lambda}_4}{\tilde{\mu}_4-\tilde{\lambda}_4} + \tilde{c}_{24}\tilde{\mu}_4\right) + \frac{1}{4}\left(\tilde{c}_{12}\,\frac{\tilde{\lambda}_2}{\tilde{\mu}_2-\tilde{\lambda}_2} - \tilde{c}_{11}\,\frac{\tilde{\lambda}_1}{\tilde{\mu}_1-\tilde{\lambda}_1} + \tilde{c}_{22}\tilde{\mu}_2 - \tilde{c}_{21}\tilde{\mu}_1 + \tilde{c}_{14}\,\frac{\tilde{\lambda}_4}{\tilde{\mu}_4-\tilde{\lambda}_4} - \tilde{c}_{12}\,\frac{\tilde{\lambda}_2}{\tilde{\mu}_2-\tilde{\lambda}_2} + \tilde{c}_{24}\tilde{\mu}_4 - \tilde{c}_{22}\tilde{\mu}_2\right)$$

Computing Algorithm

Following computing flowchart is developed to find out the optimal service rate and total cost of the model.

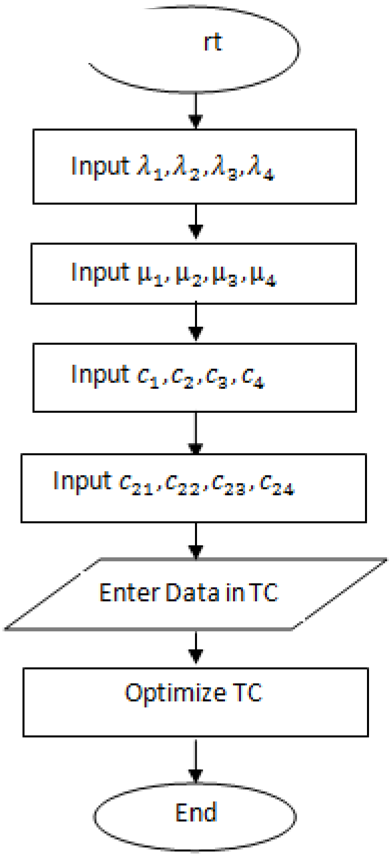


Table 7: Computation table for $\tilde{\lambda}$ and $\widetilde{TC}$

$\tilde{\lambda}$				$\tilde{\mu}$				$\tilde{c_1}$				$\tilde{c_2}$				$\widetilde{TC}$
29	31	33	35	39	41	43	45	2	4	6	8	3	5	7	9	279.6774
31	33	35	37	39	41	43	45	2	4	6	8	3	5	7	9	274.7919
33	35	37	39	39	41	43	45	2	4	6	8	3	5	7	9	269.9064
35	37	39	41	39	41	43	45	2	4	6	8	3	5	7	9	265.0209
37	39	41	43	39	41	43	45	2	4	6	8	3	5	7	9	260.1355

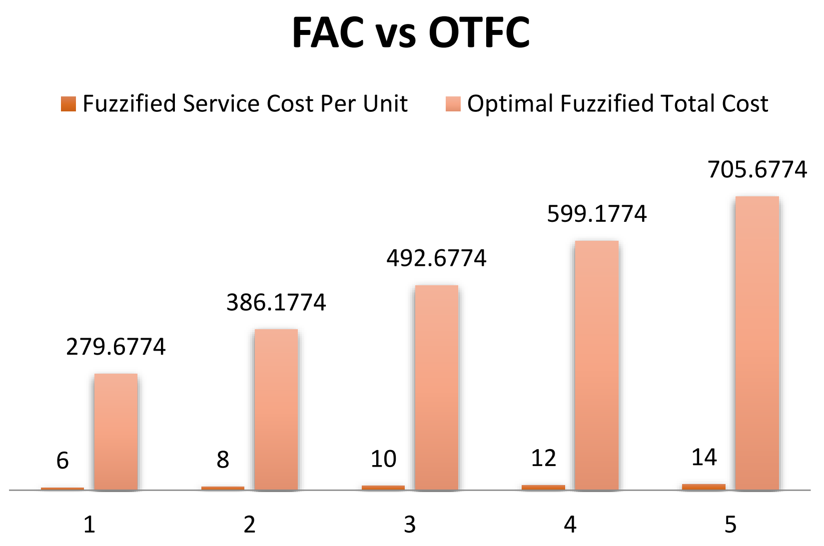


Table 8: Computation table for $\tilde{\mu}$ and $\widetilde{TC}$

$\tilde{\mu}$				$\tilde{\lambda}$				$\tilde{c}_1$				$\tilde{c}_2$				$\widetilde{TC}$
39	41	43	45	29	31	33	35	2	4	6	8	3	5	7	9	279.6774
41	43	45	47	29	31	33	35	2	4	6	8	3	5	7	9	296.093
43	45	47	49	29	31	33	35	2	4	6	8	3	5	7	9	312.5158
45	47	49	51	29	31	33	35	2	4	6	8	3	5	7	9	328.9448
47	49	51	53	29	31	33	35	2	4	6	8	3	5	7	9	345.3793

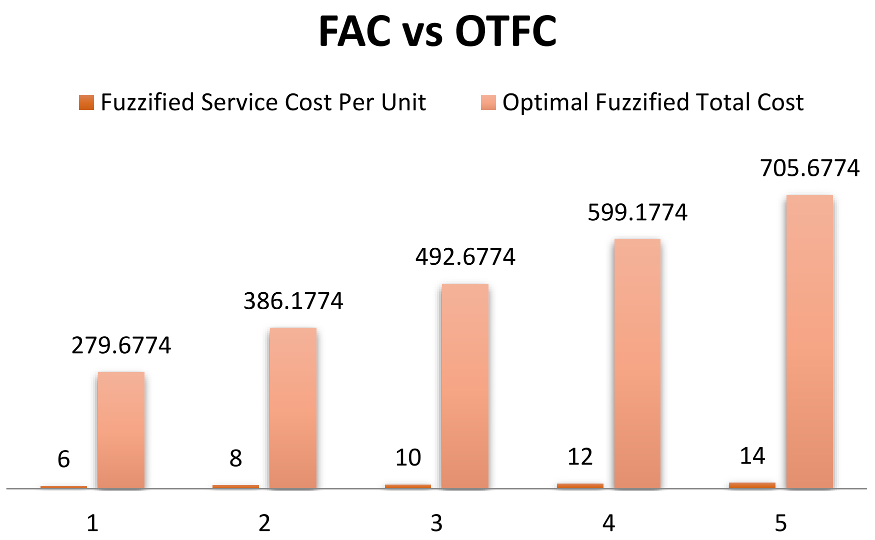


Table 9: Computation table for $\tilde{c}_1$ and $\widetilde{TC}$

$\tilde{c}_1$				$\tilde{\lambda}$				$\tilde{\mu}$				$\tilde{c}_2$				$\widetilde{TC}$
2	4	6	8	29	31	33	35	39	41	43	45	3	5	7	9	279.6774
4	6	8	10	29	31	33	35	39	41	43	45	3	5	7	9	248.438
6	8	10	12	29	31	33	35	39	41	43	45	3	5	7	9	217.1987
8	10	12	14	29	31	33	35	39	41	43	45	3	5	7	9	185.9594
10	12	14	16	29	31	33	35	39	41	43	45	3	5	7	9	154.7201

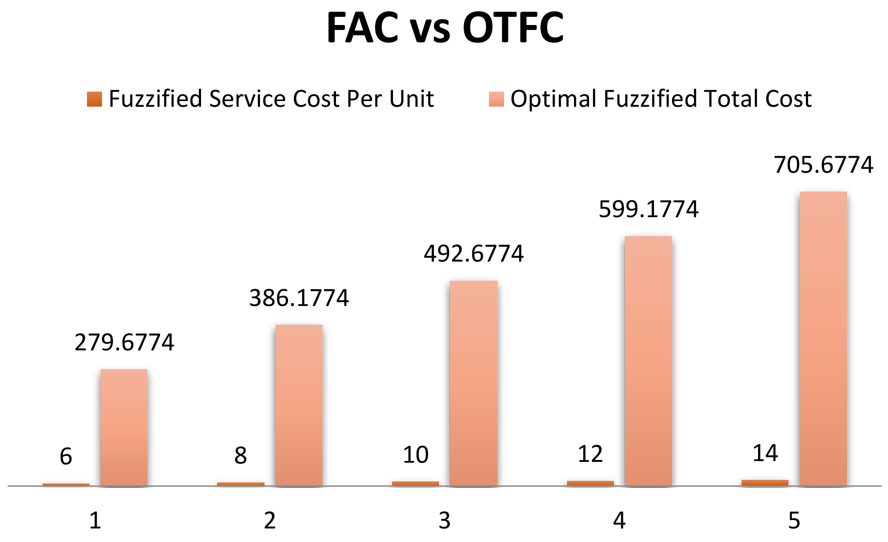
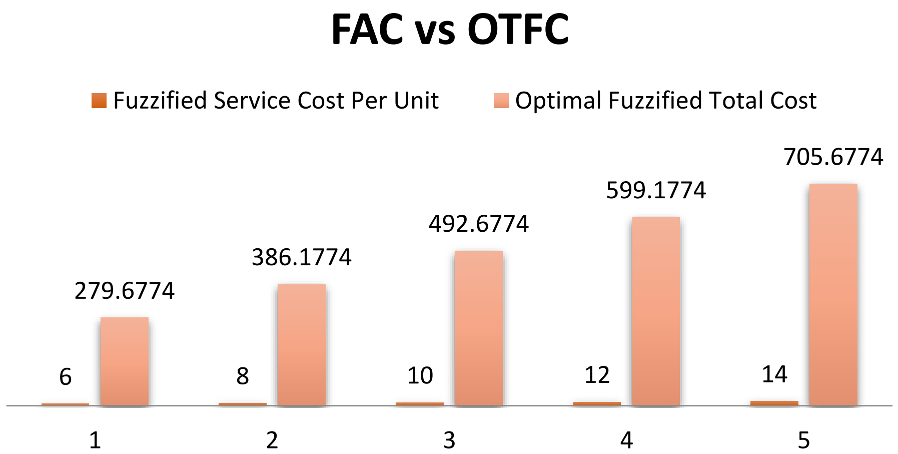


Table 10: Computation table for $\tilde{c}_2$ and $\widetilde{TC}$

$\widetilde{c_2}$				$\tilde{\lambda}$				$\tilde{\mu}$				$\widetilde{c_2}$				$\widetilde{TC}$
3	5	7	9	29	31	33	35	39	41	43	45	2	4	6	8	279.6774
5	7	9	11	29	31	33	35	39	41	43	45	2	4	6	8	386.1774
7	9	11	13	29	31	33	35	39	41	43	45	2	4	6	8	492.6774
9	11	13	15	29	31	33	35	39	41	43	45	2	4	6	8	599.1774
11	13	15	17	29	31	33	35	39	41	43	45	2	4	6	8	705.6774



4. Conclusion

As we know that Markovian queues are fundamental queueing models which speak a volume about its application in various areas. For the fuzzy model, table 7 shows that the increase in fuzzy arrival rate per unit amounts to increase in total optimal fuzzy cost of the model under consideration as shown as Graph-I. Table-8 represents that the fuzzyfied average cost per unit if increases, then total optimal fuzzy cost of the model also increases, which is depicted by the Graph-II. At last in tables 9-10, it may be observed that the fuzzy service cost of customers to the service channel whenever increases, it results increase in total optimal fuzzy cost of the model.

Table 6 gives the result of simulation. Table 5 details the formation and item wise values at various nodes of ANN and figures 2 & 3 present the lucid facets of intended results. Its validation is made by an established rule of coefficient of variation. This problem of research can be believably used in verifying and strengthening the analytical methods used in such problem. This can be served as a guiding framework for the application in various areas including identification, classification and prediction, data processing, image and pattern recognition, marketing, finance and management, bioinformatics, health, medical and robotics etc.

References

1. Altiparmak F., Dengiz B., Bulgak A. A. (2007), Buffer allocation and performance modeling in asynchronous assembly system operations: An artificial neural network metamodeling approach, Applied Soft Computing 7(3), 946-956.
2. Amdeo V. Kalyankar (2009), Network Traffic Management, Journal of Computing, Vol.2, 191-194.
3. Averill Law (2017), Simulation Modeling and Analysis, McGraw Hill Education; 4 edition, ISBN-10: 9780070667334, ISBN-13: 978-0070667334.
4. Badiru A.B., Sieger D. B. (1998), Neural network as a simulation metamodel in economic analysis of risky projects, European Journal of Operational Research 105(1), 130-142.
5. Baskett F, Chandy K M, Muntz R R (1975), Palacios F G Open closed and mixed networks of queues with different classes of customers. J. ACM 22: 248–260.
6. Bernard P. Zeigler, Herbert Praehofer, Tag Gon Kim (2000), Theory of Modeling and Simulation, Academic Press; 2 edition, ISBN-10: 0127784551, ISBN-13: 978-0127784557.
7. Bunday B.D. (1996), An introduction to queueing theory, (Oxford University Press, Oxford, England.
8. Chen H, Yao D D (2001), Fundamentals of queueing networks (New York: Springer)
9. Cortez P., Rio M., Rocha M, Sousa P. (2006), Internet traffic forecasting using neural networks, International Joint Conference on Neural Networks, 2635–2642.
10. Daganzo C F (1997), Fundamentals of transportation and traffic operations (New York: Elsevier Science)
11. Demuth, H., Beale M., Hagan M. (2006), Neural Network Toolbox User’s Guide. MathWorks, Inc., Natick, Mass.
12. Ding Z., Zhao F., Wu Y. (2009). Artificial Neural Network Delay Model for Traffic Assignment Incorporating Intersection Delay Costs. Transportation Research Record: Journal of the Transportation Research Board, 2132(1), 25–32.
13. Ding, W., Zhang, J., & Leung, Y. (2016). Prediction of air pollutant concentration based on sparse response back propagation training feed-forward neural networks. Journal of Environmental Science Pollution Resources, 23, 19481–19494.
14. Ertunc H.M., Hosoz M. (2006), Artificial neural network analysis of a refrigeration system with an evaporative condenser, Applied Thermal Engineering 26, 627-635.
15. Hagan M.T. (1996), Demuth H.B., Beale M., “Neural Network Design”, PWS Publishing Company, Boston.
16. Hensher D.A, Ton T.T. (2000), A comparison of the predictive potential of artificial neural networks and nested logit models for commuter mode choice. Transportation Research Part E Logistics and Transportation Review, 36, 155–172.
17. Hillier F.S and Liebermann G.J., (1995), Introduction to Operations Research, McGraw-Hill, New York.
18. Jain A.K., Mao J. (1996), Mohiuddin KM. Artificial neural networks: a tutorial. Computer 29(3):31–44.
19. Jain R, Smith J MacGregor (1997): Modelling vehicular traffic flow using M/G/C/C state dependent queueing models. Transportation Sci. 31: 324–336
20. Kalogirou S.A. (2000), Application of artificial neural-networks for energy systems, Applied Energy, 67, 17-35.
21. Kelly F P (1976) Networks of queues. Adv. Appl. Prob. 8: 416–432
22. Kendall D.G. (1951), Some Problems in theory of queues, Journal of the Royal Statistical Society(B) 13(2), 151-157 and 184-185.
23. Koenigsberg E (1958), Cyclic queues. Operational Research Quarterly 9: 22–35
24. Mala and S.P. Varma (2016), Minimization of Traffic Congestion by Using Queueing Theory, Journal of Mathematics (IOSR-JM), e-ISSN: 2278-5728, p-ISSN: 2319-765X. Vol. 12, 1, PP 116-122.
25. Miguel M.L.F. (2012), Penna MC, Nievola JC, Pellenz M.E.,New models for long-term Internet traffic forecasting using artificial neural networks and flow based information, IEEE Network Operations and Management Symposium, 1082–1088.
26. Mishra S S, Shukla D C (2009) ‘A computational approach to the cost analysis of machine interference model’, American Journal of Mathematical and Management Sciences, Vol. 29, No. 1&2, pp.277-293.

27. Mishra S. S., Neural Computing Approach to Development of Customer Profile Indicator for Financial Inventory Management, American Journal of Operational Research 2014, 4(1): 17-22 DOI: 10.5923/j.ajor.20140401.03
28. Mishra, S.S. and Yadav, D.K. (2010), Computational Approach to Cost and Profit analysis of clocked Queuing Networks, Contemporary Engineering Sciences, Vol. 3, No. 8, pp.365-370.
29. Modestus O. Okwu, Benjamin Ufuoma Oreko, Stanley Okiy, Austin C. Uzorh and Onyewuchi Oguoma, Artificial neural network model for cost optimization in a dual-source multi-destination outbound system, Cogent Engineering (2018), 5: 1447774
30. Mostafa M. M., El-Masry A. A. (2016). Oil price forecasting using gene expression programming and artificial neural networks. Economic Modelling, 54, 40–53. https://doi.org/10.1016/j.econmod.2015.12.014
31. Nagel K (1996), Particle hopping models and traffic flow theory. Physical Rev. E 53: 4655–4672.
32. Namdeo V. Kalyankar (2009), Network Traffic Management, Journal of Computing, Vol.2, 191-194.
33. Priya, R. S. and Sudhesh, R. (2018), Transient analysis of a discrete-time infinite server queue with system disaster, International Journal of Mathematics in Operational Research, Vol. 12, No. 1, pp.91-101.
34. Rawat, Sandeep, Ali, S. Ahmad, Singh, Abishek, Mishra, S. S. (2020). Artificial Neural Network Approach to Simulation and Prediction of Traffic Congestion for Morkovian Queue with Single Server, IJASS, communicated and in process, Corres. Auth.
35. R Development Core Team (2007a). R: A Language and Environment for Statistical Comput- ing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, URL http://www.R-project.org/.
36. R Development Core Team (2007b). Writing R Extensions.
37. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-11-9, URL http://www.R-project.org/.
38. Raheja Tushar (2010), Modelling traffic congestion using queuing networks, Sdadhana, 35(4), 427–431. © Indian Academy of Sciences.
39. Sharma, P. (2016), Optimal Flow Control of Multi-server Time Sharing Queuing network with priority, International Journal of Mathematics in Operational Research, 9(3), 363-374.
40. Singh, C. J., Jain, M. and Kumar, B. (2016), Analysis of Single Server Finite Queuing Model with reneging', International Journal of Mathematics in Operational Research, 9(1), 15-38.
41. Sivakami Sundari M, Palaniammal S (2019), An Ann Simulation of Single Server with Infinite Capacity Queuing System, International Journal of Innovative Technology and Exploring Engineering (IJITEE), 8(12), 2278-3075.
42. Specht D.F. (1990), Probabilistic neural networks. Neural Networks, 3:109–118.
43. Sundari, M. S., Palaniammal S. (2015), Simulation of M/M/1 Queuing System Using ANN, Malaya Journal of Mathematik, Vol. S, No. 1, pp.279-294.
44. Taha H.A. (1987), Operation research: an introduction (Fourth Edition, Macmillan Publishing: New York.
45. Udagawa K., Nakamura G. (1956), On a certain queuing system, Kodai Mathematical Seminar Reports, 8(3), 117-124.
46. Van Woensel T, Vandaele N (2007), Modelling traffic flows with queueing models: A review. Asia-Pacific J. Operational Res. 24(4): 435–461
47. Vandaele N, van Woensel T, Verbruggen A (2000), A queueing based traffic flow model. Transportation Res. D 5(2): 121–135
48. Vlahogianni E.I., Karlaftis M.G., Golias J.C. (2005), Optimized and meta-optimized neural networks for short-term traffic flow prediction: a genetic approach. Transportation Research Part C Emerging Technologies, 13(3):211–234.
49. Wagner H.M., (1975), Principles of Operations Research, New Jersey..