Research Article

Nusselt number correlations for heat transfer analysis around a tube at subcritical Reynolds numbers

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Abstract: Nusselt number around circular tube has been typically correlated by the power law: Nu= C Re^m Pr^n. In the present study, a numerical technique of Matrix least-squares has been utilized to estimate coefficient and power indices of the typical power law through a MATLAB program using experimental data of local Nusselt Number around the tube surface for subcritical range of Reynolds Number in cross flow condition. Correlations of average Nusselt number have been developed for different flow regimes i.e. before and after boundary layer separation, front and rear portions around the tube surface including front and rear stagnation points. Utilizing before and after boundary layer separation developed correlations, a correlation has been proposed for the estimation of overall average Nusselt Number around the tube surface. The developed correlations were compared with the benchmark data of reported literatures and obtained a reasonably good agreement with them.

Keywords: Correlations around tube; Power-Law; Coefficients and Indices; Nusselt number.

1. Introduction

Convective heat transfer from a cylinder in cross-flow has attracted serious attention of the researchers at a global level and a repeated attempt has been made to develop correlation because of its importance in a variety of applications like radiators and condensers, chemical and food industry processes, reactors, recuperators, etc.

Eckert et al. (1952), Krall et al. (1973) and Achenbach (1975) have shown the circumferential Nusselt number (Nu) distributions by experimentally measuring the local heat flux around a circular tube in flow of air and reported that the Nu variation is caused by the change in the flow region from steady to unsteady flow due to presence of vortex shedding.

Sanitjai and Goldstein (2004) have investigated heat transfer through forced convection around a tube in flow of air, water and different mixture of liquids at various Reynolds number (Re) ranging between 2000 and 90000, and Prandtl Number (Pr) between 0.7 and 176. They have reported correlations for average Nu for front stagnation point (fsp), front portion and rear portion as well as for overall surface around a tube.

Ahmed and Talama (2008) from experiments in wind tunnel for a heated circular tube at different Reynolds numbers between 16285 and 65140 had reported higher heat transfer around stagnation point. Kumar et al. (2016) have investigated the effects of Re and Pr in the unsteady regime around a tube for commencement of vortex shedding. They observed that in between Re = 69 and 70, transition occurs from steady to time-periodic flow. Giordano et al. (2012) have shown the influence of aspect ratio and Re on the convective coefficient over the tube wall. Mortean and Mantelli (2019) have carried out theoretical and experimental investigation and developed Nu correlation in transition region for air flow. Holman (2010) considered Knudsen and Katz (1958) analysis and described that average Nu is governed by power-law $Nu = C Re^m Pr^{1/3}$ in air and liquid flow past a circular tube.

From the numerous literatures review it has been identified that very less work has been reported to establish coefficients and power indices of Re for different segments of tube surface as well as front and rear stagnation points with varying Re in subcritical range. It requires attention for closed heat transfer estimations in heat exchanger design, which is still overlooked. Hence, a matrix least square method has been used which was programmed in commercial software MATLAB by giving inputs of local Nu data to derive local coefficient and power index.

2. Methodology

Nusselt number *(Nu)* around a circular tube is typically correlated by a power-law:

$Nu = C\, Re^m\, Pr^n$. (1)

On taking log both the sides we get,

$$Log\left[{}^{Nu}/_{Pr^n}\right] = LogC + mLog(Re) \qquad (2)$$

General representation of Parametric Equation is:

$$y = f(X) + \epsilon \qquad (3)$$

where, y= Predicted output variable, $\epsilon =$ Irreducible error and $f(X)$ =Unknown function represented as

$$f(X) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots\cdots\cdots + \beta_p X_p \qquad (4)$$

Matrix form of Eq. (3) is represented as:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} \beta_0 + & \beta_1 X_{1,1} + & \beta_2 X_{1,2} + & \cdots + & \beta_p X_{1,p} + & \epsilon_1 \\ \beta_0 + & \beta_1 X_{2,1} + & \beta_2 X_{2,2} + & \cdots + & \beta_p X_{2,p+} & \epsilon_2 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \beta_0 + & \beta_1 X_{N,1} + & \beta_2 X_{N,2} + & \cdots + & \beta_p X_{N,p} + & \epsilon_N \end{bmatrix} \qquad (5)$$

Here, multiple regression estimates are:

$$\hat{y} = \widehat{f(X)} = \widehat{\beta_0} + \widehat{\beta_1} X_1 + \widehat{\beta_2} X_2 + \cdots + \widehat{\beta_p} X_p = X\hat{\beta} \qquad (6)$$

Residual error from Eq. (3) & Eq. (6) is:

$$e = (y - \hat{y}) \qquad (7)$$

For matrix multiple regressions, the objective function RSS (sum of squared residuals) is required to be minimized:

$$RSS = \sum_{i=1}^{n} (e_i)^2 = \sum_{i=1}^{n} (y - \hat{y})^2 \qquad (8)$$

$$RSS = y^T y - y^T X\hat{\beta} - \widehat{\beta^T} X^T y + \widehat{\beta^T} X^T X \hat{\beta} \qquad (9)$$

To minimize RSS, differentiating Eq. (9) w.r.to $\hat{\beta}$ and equating to zero provides:

$$\widehat{\beta^T} X^T X = y^T X$$

And applying $(X^T X)^{-1}$ on both sides and transposing we get (OLS) of β:

$$\hat{\beta} = (X^T X)^{-1} X^T y \qquad (10)$$

Comparing each term of Eqns. (3) and (4) with Eq. (2) we get in Matrix form as:

$$y = \begin{bmatrix} Log\left[\frac{Nu_1}{Pr^n}\right] \\ Log\left[\frac{Nu_2}{Pr^n}\right] \\ Log\left[\frac{Nu_3}{Pr^n}\right] \\ \vdots \\ \vdots \\ Log\left[\frac{Nu_n}{Pr^n}\right] \end{bmatrix};\; X = \begin{bmatrix} 1 & Log(Re_1) \\ 1 & Log(Re_2) \\ 1 & Log(Re_3) \\ \vdots & \vdots \\ \vdots & \vdots \\ 1 & Log(Re_N) \end{bmatrix}; \qquad (11)$$

Combining Eqs. (10) and (11) for Nu_i, Re_i (i=1 to N) and Pr=0.71 (air) by considering n=0.33 (for air n varies between ⅓ and ¼), C and m are represented as:

$$\hat{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} LogC \\ m \end{bmatrix} \quad (12)$$

2.1 Estimates of C and m: Nusselt Number distribution around a tube obtained by experiments conducted by Dhiman et. al. (2017) has been utilized to estimate C and m. The estimates of C and m are plotted in Figure 1 that shows the plots at interval of 15° between fsp at 0° and rsp at 180° due to symmetric behaviour across the central plane along the flow direction.

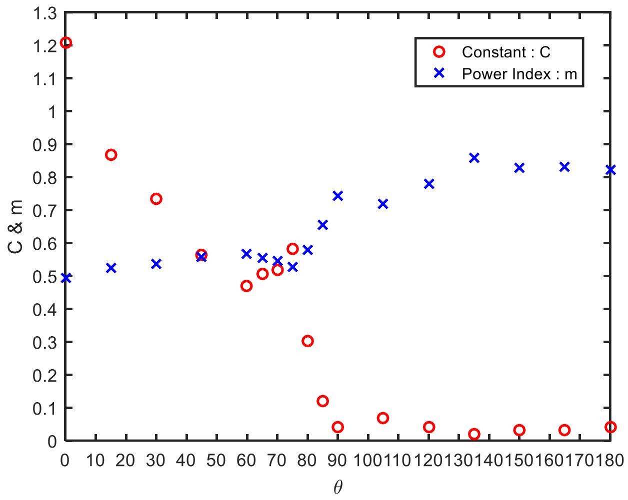

Figure 1. Variations of *C and m v/s θ°* over 11000≤*Re*≤62000.

The estimates of C and m for all the regions have been shown in Table 1.

Table 1. Estimates of C and m for the regions

θ	**C_j**	**m_j**
0°	C_1 =1.2	m_1 = 0.49
0° ≤ θ ≤ 60°	C_2 = 0.768	m_2 = 0.532
60° ≤ θ ≤ 75°	C_3 = 0.52	m_3 = 0.547
75° ≤ θ ≤ 90°	C_4 = 0.261	m_4 = 0.626
90° ≤ θ ≤ 105°	C_5 = 0.054	m_5 = 0.731
105° ≤θ ≤ 135°	C_6 = 0.042	m_6 = 0.786
135°≤ θ ≤180°	C_7 = 0.030	m_7 = 0.83
180°	C_8 = 0.0398	m_8 = 0.82

In Table 2 the proposed correlations of Nusselt number have been represented.

Table 2. A list of present study developed Correlations

θ	**Region**	**Correlation**	**Correlation no.**
0°	θ_{fsp}	$Nu_{fsp} = C_1 (Re)^{m_1} (Pr)^{0.33}$	(13)
180°	θ_{rsp}	$Nu_{rsp} = C_8 (Re)^{m_8} (Pr)^{0.33}$	(14)
0° ≤ θ ≤ 75°	$\delta\theta_{bsf}$	$Nu_{\delta\theta bsf} = 0.5Pr^{0.33} \sum_{j=2}^{3} C_j Re^{m_j}$	(15)
75° ≤ θ ≤ 180°	$\delta\theta_{sf}$	$Nu_{\delta\theta sf} = 0.25Pr^{0.33} \sum_{j=4}^{7} C_j Re^{m_j}$	(16)
0° ≤ θ ≤ 90°	$\delta\theta_{upstream}$	$Nu_{\delta\theta upstream} = 0.33Pr^{0.33} \sum_{j=2}^{4} C_j Re^{m_j}$	(17)
90° ≤ θ ≤ 180°	$\delta\theta_{downstream}$	$Nu_{\delta\theta downstream} = 0.33Pr^{0.33} \sum_{j=5}^{7} C_j Re^{m_j}$	(18)
0° ≤ θ ≤ 180°	$\delta\theta_{overall}$	$Nu_{\delta\theta overall} = 0.16Pr^{0.33} \sum_{j=2}^{7} C_j Re^{m_j}$	(19)

3. Results and Discussion

The Nu distributions normalized with Re and Pr around the circular tube have been shown in Figure 2. The local Nu has been estimated by utilizing the estimated coefficient C and power index m, shown in Figure 1, in the typical power law relation.

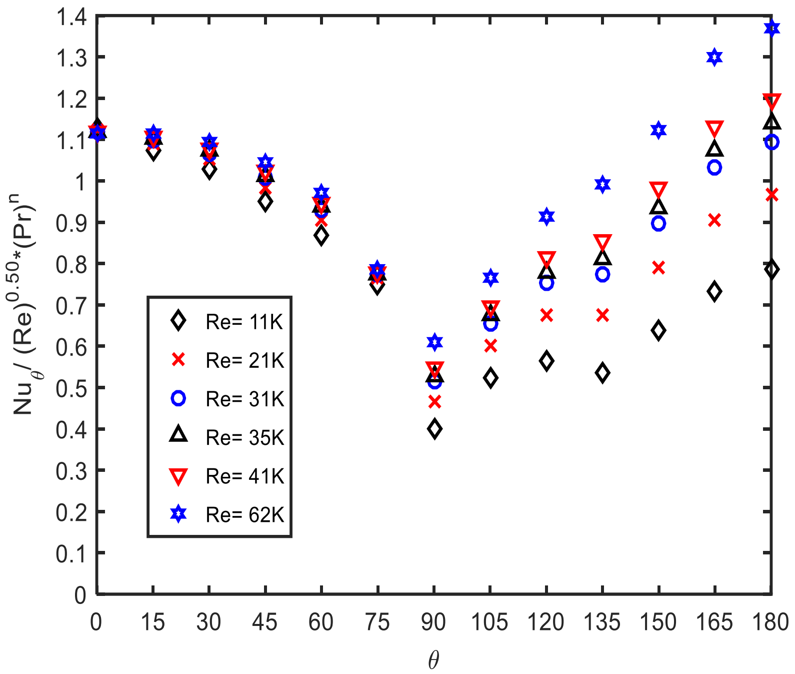

Figure 2. Distribution of normalized Nu_θ *v/s* θ over the cylinder at *Pr=0.71, n=0.33*

The normalized Nu distributions follows the usual phenomena of heat transfer from the tube surface i.e., the declination of heat transfer from fsp till the separation point followed by further decrease in heat transfer up to 90° measured from fsp. Thereafter, it shows an increase in the heat transfer in the separated flow region towards the downstream side of the tube showing a minor bulge up to Re of 40000. In addition, increase in heat transfer with increase in Re has also been obtained.

Figure 3 illustrates the overall average Nusselt Number Nu_{av} obtained from the estimated local Nu which have been compared with that of the experimental data reported in [Igarashi and Hirata (1977)]. The estimates of Nu_{av} shows a reasonable agreement with the reported data.

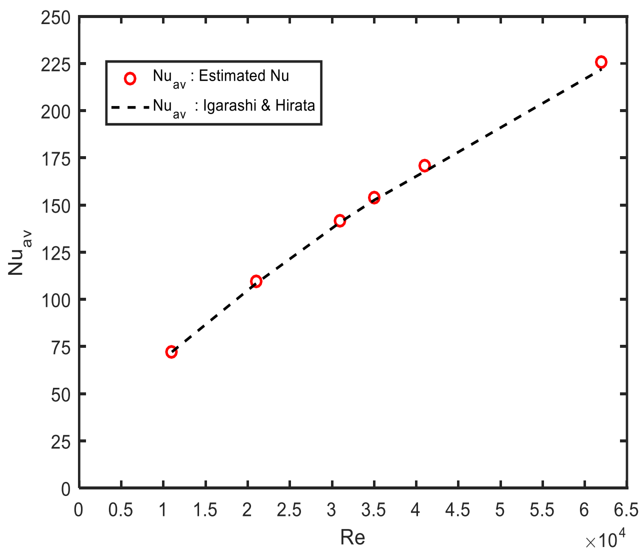

Figure 3. Comparison of estimates of overall Nu_{av} obtained from Estimated local Nu with that of data [Igarashi and Hirata (1977)]

Figure 4 compares the average Nu_{fsp} (correlation 13) with that of Sanitjai and Goldstein and Nu_{rsp} (correlation 14) with that of Igarashi and Hirata (1977). As can be observed from the figure that average Nu increases with Re for both fsp and rsp, however their slopes are different because of which they show a reverse trend of increment across Re≈30000. With this observation one can predict that there is a possibility of discrepancy in heat transfer on the upstream and downstream side of the tube.

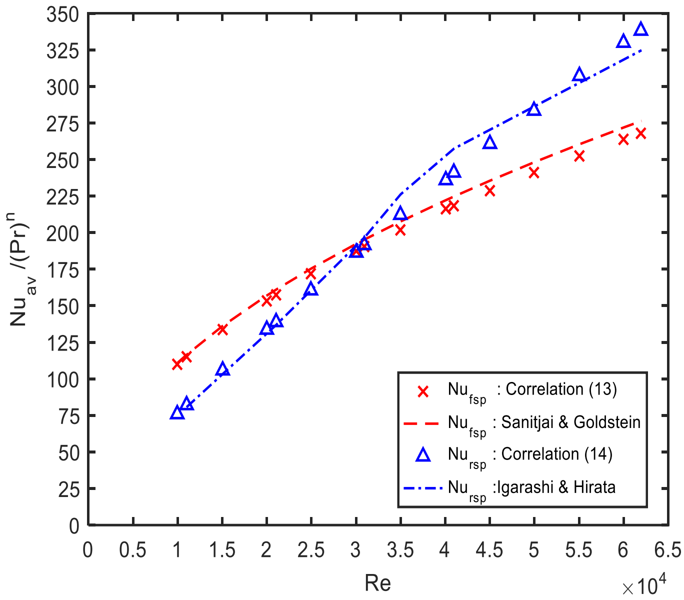

Figure 4. Comparison of average Nu_{fsp} & Nu_{rsp} obtained from correlations (13, 14) with Sanitjai and Goldstein (2004) for fsp and with the data of Igarashi and Hirata (1977) for rsp.

The comparison, shown in Figure 5, illustrates that the percentage deviation of Nu_{fsp} has a minor increment with Re showing a maximum deviation of 3.3% at Re=62000, while Nu_{rsp} has a minor decrement upto Re≈30000 and thereafter a gradual increment showing a maximum deviation of 5.9% at Re=62000. Hence, the correlations (13, 14) show a reasonable agreement with that of the reported data.

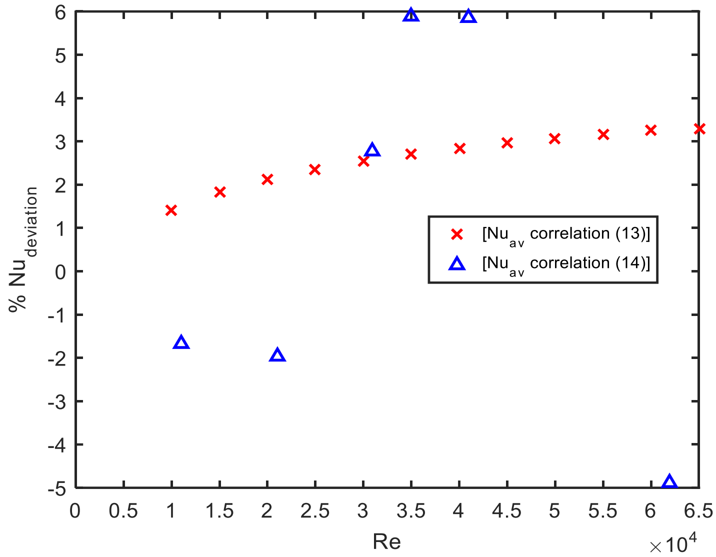

Figure 5. Percentage deviation of estimates of Nu_{fsp} & Nu_{rsp} obtained from correlations (13, 14) with Sanitjai and Goldstein (2004) and data of Igarashi and Hirata (1977) respectively.

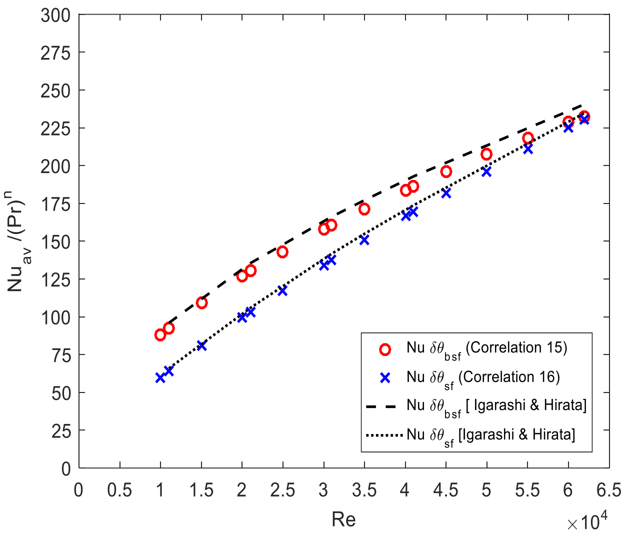

Figure 6. Comparison of estimates of $Nu_{av}/(Pr)^n$ from correlations (15,16) with that of data [Igarashi and Hirata (1977)]

Figure 6 illustrates the comparison of average Nusselt Number normalized with Pr estimated from the correlations 15 and 16 for the segments $\delta\theta_{bsf}$ and $\delta\theta_{sf}$ respectively over the tube surface with that of experimental data reported by [Igarashi and Hirata (1977)].Figure reveals that with increase in Re the Nu_{av} increases in both the segments, however their slopes indicates the different heat transfer which may be governed by the varying flow structure of fluid over these segments [Sanitjai and Goldstein (2004), Bloor (1964), Giedt (1949)]. The increase of Nu_{av} for $\delta\theta_{bsf}$ indicates that the flow over this segment is approximately unchanged with change in Re. In the segment $\delta\theta_{sf}$ there is a steep rise in heat transfer with increase in Re and indicates that the greater heat transfer can be achieved at higher Re. This is attributed due to the fact that there is a high frequency vortex shedding associated with this segment [Desai et.al. (2020)].

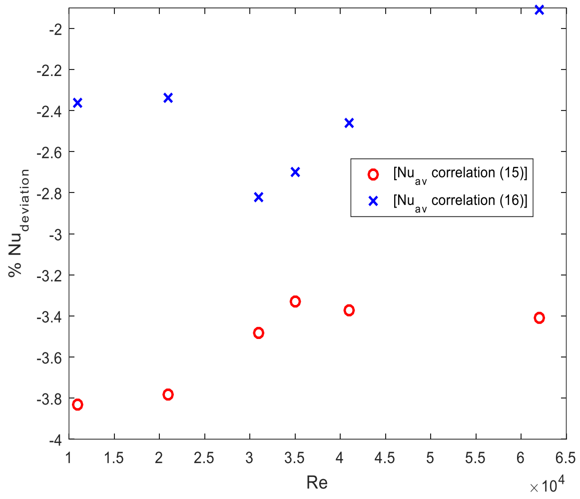

Figure 7. Percentage deviation of estimates of $Nu_{av}/(Pr)^n$ from correlations (15,16) with that of data [Igarashi and Hirata (1977)]

The estimates of both the Nu_{av} from correlations (15 and 16) shows a reasonable agreement with respect to the data reported in the literature by [Igarashi and Hirata (1977)] which can be depicted from the deviation plots shown in Fig.7. The deviations of $\delta\theta_{bsf}$ has fall within the higher limits of 3.8% while that for $\delta\theta_{sf}$ falls within the higher limits of 2.8%.

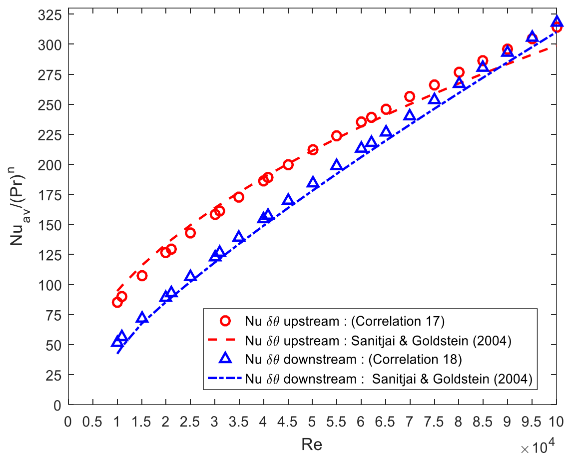

Figure 8. Comparison of estimates of $Nu_{av}/(Pr)^n$ from correlations (17,18) with Sanitjai and Goldstein correlations (2004).

Figure 8 illustrates the Nu_{av} for upstream and downstream portion of the tube from correlations (17, 18) with respect to the Re extended upto 10^5. The flow associated to upstream portion exhibits growth of boundary layer phenomena [Sanitjai and Goldstein (2004), Igarashi and Hirata (1977)] that separates around 75° from fsp followed by separation bubble elongating to about 90°. Consequent upon the boundary layer growth the heat transfer decreases as the flow advances the tube surface from fsp and results in diminishing increase of average heat transfer with raising the Re. Flow associated with downstream portion exhibits two-fold phenomenon of reattachment of shear layer together with high frequency vortex shedding [Sanitjai and Goldstein (2004), Desai et.al. (2020)].

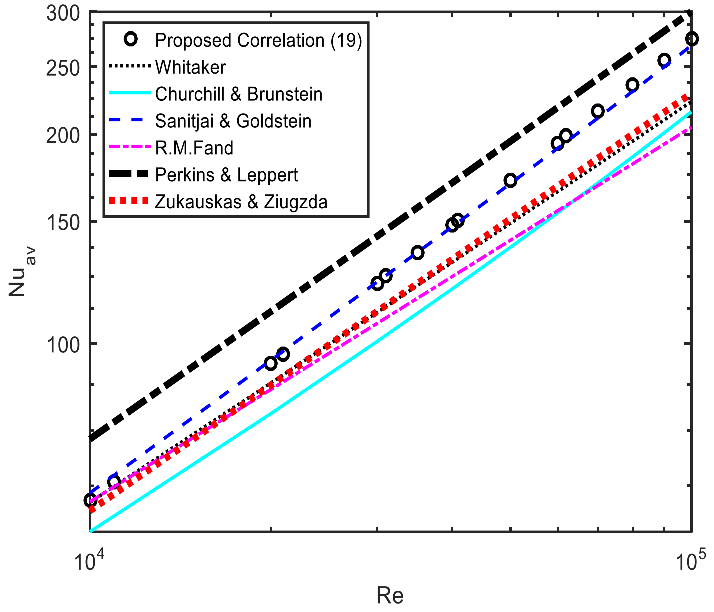

Figure 9. Comparisons of the proposed Correlations (19) for overall average Nu estimates with the reported correlations for $10000 \leqslant Re \leqslant 100000$ at Pr=0.71 and n=0.33.

Figure 9 shows the plots of estimates of overall average Nusselt Number over the entire tube surface from correlations (19) and their comparison with that of the equations proposed by benchmark of various authors. The overall Nu_{av} from the correlations in the present study follows very closely to the latest benchmark work of Sanitjai and Goldstein (2004).

4. Conclusion

In the present study, a Matrix least squares method has been utilized to estimate coefficients (C) and power indices (m) of Re in a typical power law, over a circular tube surface that is placed in cross flow to transfer heat. Local Nu distribution over the tube surface has been utilized to estimate C and m, and different correlations (13) through (19) were developed and compared with the benchmark reported data of other authors. Based on the present analysis, the following conclusions are drawn:

(i) The Matrix Least square method has estimated a reasonably acceptable value of C and m as it validates the benchmark results.

(ii) The distributions of the normalized Nusselt number $(Nu_{local} / (Re)^{0.50} (Pr)^{n})$ around the cylinder in $0^0 \leq \theta \leq 75^0$ region is closely proportional to $Re^{0.50}$ as expected for a laminar boundary layer.

(iii) The Nu data at fsp and rsp has indicated that the heat transfer from upstream and downstream portion of tube has dependency on Re. Although the heat transfer from fsp and rsp has shown contrary characteristics across Re≈30000, the collective Nu data of upstream and downstream portion has shown the contrary characteristics across Re≈$9.0x10^4$ i.e. downstream portion promotes the heat transfer beyond this Re.

(iv) The discrepancy in variation of Nu distribution over the tube surface as well as with change in Re encourages identifying segment-wise correlations. The segment-wise as well as average Nu correlations (13) through (19) have shown considerable conformity with respect to the benchmark reported data.

5. Nomenclature

Re	Reynolds number
Nu_θ	Local Nusselt number
Nu_{av}	Average Nusselt number,
Pr	Prandtl number
Nu_{fsp}	Nusselt number at front stagnation point
Nu_{rsp}	Nusselt number at rear stagnation point
$Nu_{\delta\theta bsf}$	Average Nu for $0°\leq\theta\leq75°$
$Nu_{\delta\theta sf}$	Average Nu for $75°\leq\theta\leq180°$
$Nu_{\delta\theta upstream}$	Average Nu for $0°\leq\theta\leq90°$
$Nu_{\delta\theta downstream}$	Average Nu for $90°\leq\theta\leq180°$
$Nu_{\delta\theta overall}$	Average Nu for $0°\leq\theta\leq180°$
fsp	Front stagnation point
rsp	Rear stagnation point
m	Power index of Re
n	Power index of Pr
C	Constant in equation
θ	Angular position around circular cylinder

6. References

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