Research Article

# Bayes estimators of Weibull -Lindley Rayleigh distribution parameters using Lindley's approximation

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**Abstract.**
For The Weibull Lindley Rayleigh distribution (WLRD) we have obtained the Bayes Estimators of scale and shape parameters using Lindley's approximation (L-approximation) under square error loss function. The proposed estimator have been compared with the corresponding MLE for their risks based on corresponding simulated samples.

**Keywords**: Bayesian estimation, Lindley's approximation, Maximum likelihood estimates, The Weibull Lindley Rayleigh distribution, Monte Carlo simulation

## 1. INTRODACTION

The fundamental reason for parametric statistical modeling is to identify the most appropriate model that adequately describes a data set obtained from experiments, observational studies, surveys, and so on. Most of these modeling techniques are based on finding the most suitable probability distribution that explains the underlying structure of the given data set. However, there is no single probability distribution that is suitable for different data sets. Thus, this has triggered the need to extend the existing classical distributions or develop new ones. A barrage of methods for defining new families of distributions has been proposed in the literature for extending or generalizing the existing classical distributions in recent times. In this paper , A great deal of research has been done on estimating the parameters of this model distribution by using both classical and Bayesian techniques, and a very good summary of this work can be found in Johnson et al. [7]. Recently, Hossain and Zimmer in [4] have discussed some comparisons of estimation methods for Weibull parameters using complete and censored samples. In this paper, we will estimate and compare the parameters of which is the Weibull -Lindley Rayleigh distribution in the complete and censored data . The three-parameters Weibull -Lindley Rayleigh distribution is defined by the distribution function:

$$h_{WLR}(x) = \frac{2\theta^2 x}{\lambda+1}\left[\lambda\beta\left(1-\exp\left(-\theta^2x^2\right)\right)+\beta(\lambda+1)+\lambda^2\left(2-\exp\left(-\theta^2x^2\right)\right)\right] \times \exp\left(-(\lambda+\beta)(1-\exp(-\theta^2x^2))\right) \tag{1}$$

and cumulative distribution function

$$H_{WLR}(x) = 1-\frac{1+\lambda+\lambda(1-\exp(-\theta^2x^2))}{\lambda+1}\exp\left(-(\lambda+\beta)(1-\exp(-\theta^2x^2))\right) \tag{2}$$

Here $\theta$ is the scale parameter, and $\beta$ and $\lambda$ are the shape parameters. It is remarkable that most of the Bayesian inference procedures have been developed with the usual squared-error loss function, which is symmetrical and associates equal importance to the losses due to overestimation and underestimation of equal magnitude. However, such a restriction may be impractical in most situations of practical importance. For example, in the estimation of reliability and failure rate functions, an overestimation is usually much more serious than an underestimation. In this case, the use of symmetrical loss function might be inappropriate as also emphasized by Basu and Ebrahimi in [1].

Nevertheless, it is difficult to find Bayes 'estimate by analytical methods Therefore, one has to use numerical quadrature techniques or certain approximation methods for the solutions. Lindley's approximation technique is one of the methods suitable for solving such problems. Thus, our aim in this paper is to propose the Bayes estimators of the parameters of Weibull -Lindley Rayleigh distribution under the squared error loss function using Lindley's approximation technique. In Sections 2 and 3, we discuss the estimation of parameters. In Section 4 numerical results are presented, and Section 5 contains the conclusion.

## 2 MAXIMUM LIKELIHOOD ESTIMATION FOR CENSORED DATA OF THE PARAMETERS

For a random sample $t=(t_1,t_2,\dots,t_n)$ of size $n$ form (1) the likelihood function for censored data is

$$L(t,\theta,\beta,\lambda)=\frac{n!}{(n-r)!}\left[\frac{2^r\theta^{2r}\prod_{i=1}^{r}t_i}{(\lambda+1)^r}\right]\left[\prod_{i=1}^{r}\left(\lambda\beta\left(1-\exp\left(-\theta^2t_i^2\right)\right)+\beta(\lambda+1)\right.\right.$$
$$\left.\left.+\lambda^2\left(\left(2-\exp\left(-\theta^2t_i^2\right)\right)\right)\right)\right]\times\exp\left(-(\lambda+\beta)\sum_{i=1}^{r}\left(1-\exp\left(-\theta^2t_i^2\right)\right)-\theta^2t_i^2\right)$$
$$\times\left[\frac{1+\lambda+\lambda\left(\left(1-\exp\left(-\theta^2t_0^2\right)\right)\right)}{\lambda+1}\exp\left(-(\lambda+\beta)\left(1-\exp\left(-\theta^2t_0^2\right)\right)\right)\right]^{n-r}\qquad(3)$$

and taking the logarithm we get

$$l(t,\theta,\beta,\lambda)=\log\frac{n!}{(n-r)!}+r\log 2+2r\log\theta+\sum_{i=1}^{r}\log t_i-r\log(\lambda+1)$$
$$+\sum_{i=1}^{r}\log\left[\lambda\beta\left(1-exp\left(-\theta^2t_i^2\right)\right)+\beta(\lambda+1)+\lambda^2\left(\left(2-exp\left(-\theta^2t_i^2\right)\right)\right)\right]\qquad(4)$$
$$-(\lambda+\beta)\sum_{i=1}^{r}\left(1-\exp\left(-\theta^2t_i^2\right)\right)-\theta^2\sum_{i=1}^{r}t_i^2$$
$$+(n-r)\left[\log\left(1+\lambda+\lambda\left(1-\exp\left(-\theta^2t_0^2\right)\right)\right)-\log(\lambda+1)-(\lambda+\beta)\left(1-\exp\left(-\theta^2t_0^2\right)\right)\right]$$

The maximum likelihood estimates of parameters of the Weibull -Lindley Rayleigh distribution are given as solutions of equations

$$\frac{2r}{\theta}+\sum_{i=1}^{r}\frac{2\theta\lambda\beta t_i^2\exp\left(-\theta^2t_i^2\right)+2\theta\lambda^2t_i^2\exp\left(-\theta^2t_i^2\right)}{\left[\lambda\beta\left(1-\exp\left(-\theta^2t_i^2\right)\right)+\beta(\lambda+1)+\lambda^2\left(2-\exp\left(-\theta^2t_i^2\right)\right)\right]}$$
$$-(\lambda+\beta)\sum_{i=1}^{r}t_i^2\exp\left(-\theta^2t_i^2\right)-2\theta\sum_{i=1}^{r}t_i^2\qquad=0$$
$$-(n-r)\left[\frac{2\theta\lambda t_0^2\exp\left(-\theta^2t_0^2\right)}{1+\lambda+\lambda\left(1-\exp\left(-\theta^2t_0^2\right)\right)}-2\theta(\lambda+\beta)t_0^2\exp\left(-\theta^2t_0^2\right)\right]$$

$$\sum_{i=1}^{r}\frac{\lambda\left(1-\exp\left(-\theta^2t_i^2\right)\right)+(\lambda+1)}{\lambda\beta\left(1-\exp\left(-\theta^2t_i^2\right)\right)+\beta(\lambda+1)+\lambda^2\left(2-\exp\left(-\theta^2t_i^2\right)\right)}-\sum_{i=1}^{r}\left(1-\exp\left(-\theta^2t_i^2\right)\right)=0$$
$$-(n-r)(1-\exp(-\theta^2t_0^2)) \quad (5)$$

$$U_\lambda=-\frac{r}{(\lambda-1)}+\sum_{i=1}^{r}\frac{\beta\left(1-\exp\left(-\theta^2t_i^2\right)\right)+\beta+2\lambda\left(2-\exp\left(-\theta^2t_i^2\right)\right)}{\left[\lambda\beta\left(1-\exp\left(-\theta^2t_i^2\right)\right)+\beta(\lambda+1)+\lambda^2\left(2-\exp\left(-\theta^2t_i^2\right)\right)\right]}$$
$$-\sum_{i=1}^{r}\left(1-\exp\left(-\theta^2t_i^2\right)\right)+(n-r)\left[\frac{\left(2-\exp\left(-\theta^2t_0^2\right)\right)}{1+\lambda+\lambda\left(1-\exp\left(-\theta^2t_0^2\right)\right)}-\frac{1}{\lambda+1}-\left(1-\exp\left(-\theta^2t_0^2\right)\right)\right]$$

which maybe solve using a iteration scheme. We propose here to use a bisection or Newton-Raphson method for solving the above-mentioned normal equations.

## 3 BAYESIAN ESTIMATION OF THE PARAMETERS

In Bayesian estimation, we consider the squared error type of loss functions. This function rises approximately exponentially on one side of zero and approximately linearly on the other side. This more general version allows different shapes of the loss function.

The squared error loss (SEL) function is as follows

$$L_{BS}(\phi^*,\phi)\propto(\phi^*-\phi)^2 \quad (6)$$

For Bayesian estimation, we need prior distribution for the parameters θ,β, and λ. The gamma prior may be taken as a prior distribution for the scale parameter of the Weibull Lindley Rayleigh distribution. It is needless to mention that under the above-mentioned situation, a prior is a conjugate prior. On the other hand, if all the parameters are unknown, a joint conjugate prior for the parameters does not exist. In such a situation, there are a number of ways to choose the priors. For all the parameters we consider the piecewise independent priors, namely a non-informative prior for the shape parameters and a natural conjugate prior for the scale parameter (under the assumption that the shape parameter is known). Thus the proposed priors for parameters θ,β, and λ may be taken as

$$\pi_1(\theta)=\frac{b_1^{a_1}\theta^{a_1-1}e^{-b_1\theta}}{\Gamma(a_1)},\theta>0,a_1,b_1>0 \quad (12)$$

$$\pi_2(\beta)=\frac{b_2^{a_2}\beta^{a_2-1}e^{-b_2\beta}}{\Gamma(a_2)},\beta>0,a_2,b_2>0 \quad (13)$$

and

$$\pi_3(\lambda)=\frac{b_3^{a_3}\lambda^{a_3-1}e^{-b_3\lambda}}{\Gamma(a_3)},\lambda>0,a_3,b_3>0 \quad (14)$$

respectively where $\Gamma(\cdot)$ is the gamma function. Thus the joint prior distribution for λ, β and θ is

$$\pi(\theta,\beta,\lambda)=\frac{b_1^{a_1}b_2^{a_2}\theta^{a_1-1}b_3^{a_3}\beta^{a_2-1}\lambda^{a_3-1}e^{-b_1\theta-b_2\beta-b_3\lambda}}{\Gamma(a_1)\Gamma(a_2)\Gamma(a_3)}\theta,\beta,\lambda>0,a,b>0 \quad (15)$$

Substituting $L(\theta,\beta,\lambda)$ and $\pi(\theta,\beta,\lambda)$ from (3) and (15) respectively we get the correspond joint posterior $P(\theta,\beta,\lambda)$ as

Substituting $L(\theta,\beta,\lambda)$ and $\pi(\theta,\beta,\lambda)$ from (3) and (15) respectively we get the correspond joint posterior $P(\theta,\beta,\lambda)$ as

$$P(\theta,\beta,\lambda \mid x) = K\frac{\theta^{n+a-1}e^{\lambda\sum_{i=1}^{n} x_i-\theta\left(b+\sum_{i=1}^{n} x_i^{\beta}e^{\lambda x_i}\right)}}{\beta\lambda}\cdot\prod_{i=1}^{n}\left[x_i^{\beta-1}(\beta+\lambda x_i)\right]$$

where

$$K^{-1} = \int_0^\infty\int_0^\infty\int_0^\infty \frac{\theta^{n+a-1}e^{\lambda\sum_{i=1}^{n} x_i-\theta\left(b+\sum_{i=1}^{n} x_i^{\beta}e^{\lambda x_i}\right)}}{\beta\lambda}\cdot\prod_{i=1}^{n}\left[x_i^{\beta-1}(\beta+\lambda x_i)\right]d\lambda d\beta d\theta$$

It may be noted here that the posterior distribution of ($\theta$, $\beta$, $\lambda$) takes a ratio form that involves integration in the denominator and cannot be reduced to a closed form. Hence, the evaluation of the posterior expectation for obtaining the Bayes estimator of $\theta$, $\beta$ and $\lambda$ will be tedious. Among the various methods suggested to approximate the ratio of integrals of the above form, perhaps the simplest one is Lindley's in [8] approximation method, which approaches the ratio of the integrals as a whole and produces a single numerical result.

Thus, we propose the use of Lindley's in [8] approximation for obtaining the Bayes estimator of $\theta$, $\beta$, and $\lambda$. Many authors have used this approximation for obtaining the Bayes estimators for some lifetime distributions; see among others, Basu and Ebrahimi in [1], Calabria and Pulcini [2], Green et al [3], Hossain and Zimmer [4] , Howlader and Hossain [5], Jaheen in [6], Nassar and Eissa [9], Parsian N. Sanjari [10] , Soliman et al [12], Zellner [13] and Preda, Vasile [12 ],

In this paper we calculate $E(\theta_i \mid x)$ and $E\left(\theta_i^2 \mid x\right)$ in order to find the posterior variance estimates given by

$$\mathrm{Var}\,(\theta_i \mid x) = E\left(\theta_i^2 \mid x\right) - \left(E(\theta_i \mid x)\right)^2$$

$i = 1,2,3$, where $\theta_1 = \theta, \theta_2 = \beta, \theta_3 = \lambda$

If n is sufficiently large, according to Lindley | in [13], any ratio of the integral of the form

$$I(x) = E[u(\theta_1,\theta_2,\theta_3)] == \frac{\int_{(\theta_1,\theta_2,\theta_3)} u(\theta_1,\theta_2,\theta_3)e^{L(\theta_1,\theta_2,\theta_3)+G(\theta_1,\theta_2,\theta_3)}d(\theta_1,\theta_2,\theta_3)}{\int_{(\theta_1,\theta_2,\theta_3)} e^{L(\theta_1,\theta_2,\theta_3)+G(\theta_1,\theta_2,\theta_3)}d(\theta_1,\theta_2,\theta_3)}$$

where $u(\theta) = u(\theta_1,\theta_2,\theta_3)$ is a function of $\theta_1,\theta_2$ or $\theta_3$ only $L(\theta_1,\theta_2,\theta_3)$ is log of likelihood $G(\theta_1,\theta_2,\theta_3)$ is log joint prior of $\theta_1,\theta_2$ and $\theta_3$.

can be evaluated as

$$I(x) = u\left(\hat\theta_1,\hat\theta_2,\hat\theta_3\right) + (u_1\alpha_1 + u_2\alpha_2 + u_3\alpha_3 + \alpha_4 + \alpha_5) + \frac{1}{2}[AA(u_1\sigma_{11} + u_2\sigma_{12} + u_3\sigma_{13})$$

$$+BB(u_1\sigma_{21} + u_2\sigma_{22} + u_3\sigma_{23})+CC(u_1\sigma_{31} + u_2\sigma_{32} + u_3\sigma_{33})]$$

where $\hat{\theta}_1, \hat{\theta}_2$ and $\hat{\theta}_3$ are the MLE of $\theta_1, \theta_2$ and $\theta_3$ respectively

$$\alpha_i = \rho_1\sigma_{i1} + \rho_2\sigma_{i2} + \rho_3\sigma_{i3}, i = 1,2,3$$

$$\alpha_4 = u_{12}\sigma_{12} + u_{13}\sigma_{13} + u_{23}\sigma_{23}$$

$$\alpha_5 = \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22} + u_{33}\sigma_{33})$$

$$AA = \sigma_{11}L_{111} + 2\sigma_{12}L_{121} + 2\sigma_{13}L_{131} + 2\sigma_{23}L_{231} + \sigma_{22}L_{221} + \sigma_{33}L_{331}$$

$$BB = \sigma_{11}L_{112} + 2\sigma_{12}L_{122} + 2\sigma_{13}L_{132} + 2\sigma_{23}L_{232} + \sigma_{22}L_{222} + \sigma_{33}L_{332}$$

$$CC = \sigma_{11}L_{113} + 2\sigma_{12}L_{123} + 2\sigma_{13}L_{133} + 2\sigma_{23}L_{233} + \sigma_{22}L_{223} + \sigma_{33}L_{333}$$

and subscripts 1,2,3 on the rigth-hand sides refer to $\theta_1, \theta_2, \theta_3$ respectively and

$$\rho_i = \frac{\partial \rho}{\partial \theta_i}, u_i = \frac{\partial u(\theta, \beta, \lambda)}{\partial \theta_i}, i = 1,2,3$$

$$u_{ij} = \frac{\partial^2 u(\theta, \beta, \lambda)}{\partial \theta_i \, \partial \theta_j}, i, j = 1,2,3$$

$$L_{ij} = \frac{\partial^2 L(\theta, \beta, \lambda)}{\partial \theta_i \, \partial \theta_j}, i, j = 1,2,3$$

$$L_{ijk} = \frac{\partial^3 L(\theta, \beta, \lambda)}{\partial \theta_i \, \partial \theta_j \, \partial \theta_k}, i, j, k = 1,2,3$$

Where $\theta_1 = \theta, \theta_2 = \beta, \theta_3 = \lambda$ and $\sigma_{ij}$ is the $(i,j) -$ th element of the inverse of the matrix $\{L_{ij}\}$, all evaluated at the MLE of parameters in complet and censored deta. For the prior distribution (3.2.1) we have

$$\begin{aligned} \rho = \quad & (a_1 - 1)\log\, \theta + (a_2 - 1)\log\, \beta + (a_3 - 1)\log\, \lambda - (\theta b_1 + \beta b_2 + \lambda b_3) \\ & + a_1 \log\, b_1 + a_2 \log\, b_2 + a_3 \log\, b_3 - \log\, \Gamma(a_1) - \log\, \Gamma(a_2) - \log\, \Gamma(a_3) \end{aligned}$$

and then we get

$$\rho_1 = \frac{a_1 - 1}{\theta} - b_1$$

$$\rho_2 = \frac{a_2 - 1}{\beta} - b_2$$

$$\rho_3 = \frac{a_3 - 1}{\lambda} - b_3$$

Also, the values of $L_{ij}$ can be obtained as follows for $i, j = 1,2,3$ and let

$A=(n-r)(\mathrm{l\,n}(1+\lambda+\lambda(1-\exp(-\theta^2 t_0^2))-\ln(\lambda+1)-(\lambda+\beta)(1-\exp(-\theta^2 t_0^2))$

and find

$A_1=\frac{dA}{d\theta}, A_{11}=\frac{d^2A}{d\theta^2}, A_{12}=\frac{d^2A}{d\theta d\beta}, A_{13}=\frac{d^2A}{d\theta d\lambda}, A_{111}=\frac{d^3A}{d\theta^3}, A_{112}=\frac{d^3A}{d\theta^2 d\beta}, A_{113}=\frac{d^3A}{d\theta^2 d\lambda}, A_{123}=\frac{d^3A}{d\theta d\beta d\lambda}$ $A_2=\frac{dA}{d\beta}, A_{22}=\frac{d^2A}{d\beta^2}, A_{23}=\frac{d^2A}{d\beta d\lambda}, A_{221}=\frac{d^3A}{d\beta^2 d\theta}, A_{222}=\frac{d^3A}{d\beta^3}, A_{223}=\frac{d^3A}{d\beta^2 d\lambda},$

$A_3=\frac{dA}{d\lambda}, A_{33}=\frac{d^2A}{d\lambda^2}, A_{331}=\frac{d^3A}{d\lambda^2 d\theta}, A_{332}=\frac{d^3A}{d\lambda^2 d\beta}, A_{333}=\frac{d^3A}{d\lambda^3}$

Assume that

$$B=\left(\beta\lambda\left(1-\exp(-\theta^2x^2)\right)+\beta(1+\lambda)+\lambda^2\left(2-\exp(-\theta^2x^2)\right)\right)$$

and find

$B_1=\frac{dB}{d\theta}, B_{11}=\frac{d^2B}{d\theta^2}, B_{12}=\frac{d^2B}{d\theta d\beta}, B_{13}=\frac{d^2B}{d\theta d\lambda}, B_{111}=\frac{d^3B}{d\theta^3}, B_{112}=\frac{d^3B}{d\theta^2 d\beta}, B_{113}=\frac{d^3B}{d\theta^2 d\lambda}, B_{123}=\frac{d^3B}{d\theta d\beta d\lambda}, B_2=\frac{dB}{d\beta}, B_{22}=\frac{d^2B}{d\beta^2}, B_{23}=\frac{d^2B}{d\beta d\lambda}, B_{221}=\frac{d^3B}{d\beta^2 d\theta}, B_{222}=\frac{d^3B}{d\beta^3}, B_{223}=\frac{d^3B}{d\beta^2 d\lambda},$

$B_3=\frac{dB}{d\lambda}, B_{33}=\frac{d^2B}{d\lambda^2}, B_{331}=\frac{d^3B}{d\lambda^2 d\theta}, B_{332}=\frac{d^3B}{d\lambda^2 d\beta}, B_{333}=\frac{d^3B}{d\lambda^3}$

Suppose that

$$C=\left(-(\lambda+\beta)\left(1-\exp(-\theta^2x^2)\right)\right)$$

and find

$C_1=\frac{d}{d\theta}, C_{11}=\frac{d^2C}{d\theta^2}, C_{12}=\frac{d^2C}{d\theta d\beta}, C_{13}=\frac{d^2C}{d\theta d\lambda}, C_{111}=\frac{d^3C}{d\theta^3}, C_{112}=\frac{d^3C}{d\theta^2 d\beta}, C_{113}=\frac{d^3C}{d\theta^2 d\lambda}, C_{123}=\frac{d^3C}{d\theta d\beta d\lambda}, C_2=\frac{dC}{d\beta}, C_{22}=\frac{d^2C}{d\beta^2}, C_{23}=\frac{d^2C}{d\beta d\lambda}, C_{221}=\frac{d^3C}{d\beta^2 d\theta}, C_{222}=\frac{d^3C}{d\beta^3}, C_{223}=\frac{d^3C}{d\beta^2 d\lambda},$

$C_3=\frac{dC}{d\lambda}, C_{33}=\frac{d^2C}{d\lambda^2}, C_{331}=\frac{d^3C}{d\lambda^2 d\theta}, C_{332}=\frac{d^3C}{d\lambda^2 d\beta}, C_{333}=\frac{d^3C}{d\lambda^3}$

$$L_{11}=U_{\theta\theta}, L_{12}=U_{\theta\beta}=L_{21}, L_{13}=U_{\theta\lambda}=L_{31}$$
$$L_{22}=U_{\beta\beta}, L_{23}=U_{\beta\lambda}=L_{32}, L_{33}=U_{\lambda\lambda}$$

and the values of $\mathrm{L_{ijk}}$ for $\mathrm{i,j,k}=1,2,3$

$$\mathrm{L_{111}}=\frac{4\mathrm{r}}{\theta^3}+\sum_1^{\mathrm{r}}\frac{\mathrm{B^2(B_{111}B+B_1B_{11}-2B_1B_{11})-2(BB_{11}-B_1^2)BB_1}}{\mathrm{B^4}}+\mathrm{C_{111}}+\mathrm{A_{111}}$$

$$\mathrm{L_{112}=L_{121}=L_{211}}=\sum_1^{\mathrm{r}}\frac{\mathrm{B^2(B_{112}B+B_2B_{11}-2B_1B_{12})-2(BB_{11}-B_1^2)BB_2}}{\mathrm{B^4}}+\mathrm{C_{112}}+\mathrm{A_{112}}$$

$$\mathrm{L_{113}=L_{131}=L_{311}}=\sum_1^{\mathrm{r}}\frac{\mathrm{B^2(B_{113}B+B_3B_{11}-2B_1B_{13})-2(BB_{11}-B_1^2)BB_3}}{\mathrm{B^4}}+\mathrm{C_{113}}+\mathrm{A_{113}}$$

$$L_{122} = L_{221} = \sum_1^r \frac{B^2(B_{221}B + B_1B_{22} - 2B_2B_{21}) - 2(BB_{22} - B_2^2)BB_1}{B^4} + C_{221} + A_{221}$$

$$L_{123} = L_{132} = L_{213} = L_{231} = L_{312} = L_{321}$$

$$= \sum_1^r \frac{B^2(B_{123}B + B_3B_{12} - B_1B_{23} - B_2B_{13}) - 2(BB_{12} - B_1B_2)BB_3}{B^4} + C_{123} + A_{123}$$

$$L_{133} = L_{313} = L_{331} = \sum_1^r \frac{B^2(B_{331}B + B_1B_3 - 2B_3B_{31}) - 2(BB_{33} - B_3^2)BB_1}{B^4} + C_{331} + A_{331}$$

$$L_{222} = \sum_1^r \frac{B^2(B_{222}B + B_2B_{22} - 2B_2B_{22}) - 2(BB_{22} - B_2^2)BB_2}{B^4} + C_{222} + A_{222}$$

$$L_{333} = \sum_1^r \frac{B^2(B_{333}B + B_3B_{33} - 2B_3B_{33}) - 2(BB_{33} - B_3^2)BB_3}{B^4} + C_{333} + A_{333}$$

Now we can obtain the values of the Bayes estimates of various parameters in complete data we used the above equations in hypothesis B and C but in censored data, we used the above equations in hypothesis A, B and C. In case of the squared error loss function

i) If $u(\hat{\theta},\hat{\beta},\hat{\lambda}) = \hat{\theta}$ then

$$\hat{\theta}_{BS} = \hat{\theta} \quad + \frac{a_1 - 1 - b_1\hat{\theta}}{\hat{\theta}}\sigma_{11} + \frac{a_2 - 1 - b_2\hat{\beta}}{\hat{\beta}}\sigma_{12} + \frac{a_2 - 1 - b_2\hat{\lambda}}{\hat{\lambda}}\sigma_{13} + \frac{1}{2}(AA\sigma_{11} + BB\sigma_{21} + CC\sigma_{31})$$

ii) If $u(\hat{\theta},\hat{\beta},\hat{\lambda}) = \hat{\beta}$ then

$$\hat{\beta}_{BS} = \hat{\beta} + \frac{a_1 - 1 - b_1\hat{\theta}}{\hat{\theta}}\sigma_{21} + \frac{a_2 - 1 - b_2\hat{\beta}}{\hat{\beta}}\sigma_{22} + \frac{a_2 - 1 - b_2\hat{\lambda}}{\hat{\lambda}}\sigma_{23}$$

$$+ \frac{1}{2}(AA\sigma_{12} + BB\sigma_{22} + CC\sigma_{32})$$

iii) If $u(\hat{\theta},\hat{\beta},\hat{\lambda}) = \hat{\lambda}$ then

$$\hat{\lambda}_{BS} = \hat{\lambda} + \frac{a_1 - 1 - b_1\hat{\theta}}{\hat{\theta}}\sigma_{31} + \frac{a_2 - 1 - b_2\hat{\beta}}{\hat{\beta}}\sigma_{32} + \frac{a_2 - 1 - b_2\hat{\lambda}}{\hat{\lambda}}\sigma_{33}$$

$$+ \frac{a_1 - 1 - b_1\hat{\theta}}{\hat{\theta}}\sigma_{31} + \frac{a_2 - 1 - b_2\hat{\beta}}{\hat{\beta}}\sigma_{32} + \frac{a_2 - 1 - b_2\hat{\lambda}}{\hat{\lambda}}\sigma_{33}$$

$$+ \frac{1}{2}(AA\sigma_{13} + BB\sigma_{23} + CC\sigma_{33})$$

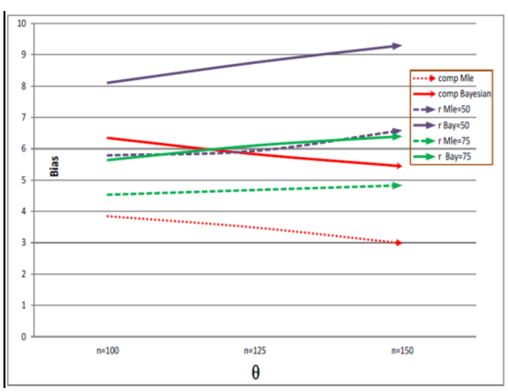
## 4 NUMERICAL FINDINGS

The estimators $\hat{\theta}, \hat{\beta}$, and $\hat{\lambda}$ are maximum likelihood estimators of the parameters in complete and censored data of the WLR distribution; whereas $\hat{\theta}_{BS}, \hat{\beta}_{BS}$, and $\hat{\lambda}_{BS}$, are Bayes estimators obtained by using the L-approximation for squared error loss function respectively. As mentioned earlier, the maximum likelihood estimators and hence risks of the estimators cannot be put in a convenient closed form. Therefore, risks of the estimators are empirically evaluated based on a Monte-Carlo simulation study of samples. A number of values of unknown parameters are considered. Sample size is varied to observe the effect of small and large samples on the estimators. Changes in the estimators and their risks have been determined when changing the shape parameter of loss functions while keeping the sample size fixed. Different combinations of prior parameters $\theta, \beta$ and $\lambda$ are considered in studying the change in the estimators and their risks. The results are summarized in Figures $(A) - (F)$.

It is easy to notice that the risk of the estimators will be a function of sample size, population parameters, parameters of the prior distribution (hyper parameters), and corresponding loss function parameters. In order to consider a wide variety of values, we have obtained the simulated risks for sample sizes $N = 100,125$ and 150.

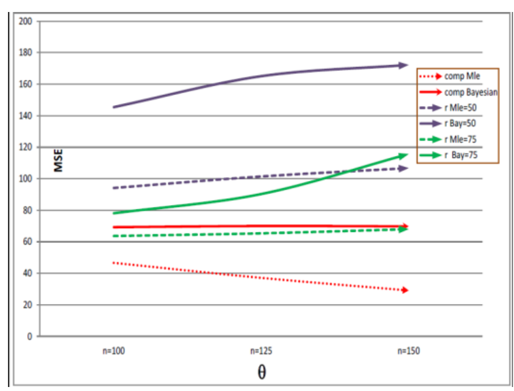
The various values of parameters of the distribution considered are for:
The parameter $\theta = 200$, the parameters $\beta = 200$ and $\lambda = 200$ Prior parameters $a_i = 0$

and $b_i = 0$ with $i = 1,2,3$ are arbitrarily taken as 1 respectively 2 . After an extensive study of the results thus obtained, conclusions are drawn regarding the behavior of the estimators, which are summarized below.



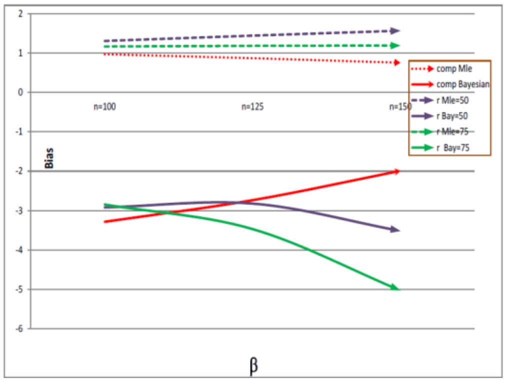

(A)

**FIGURE(A)** The graph of bias for $\theta$



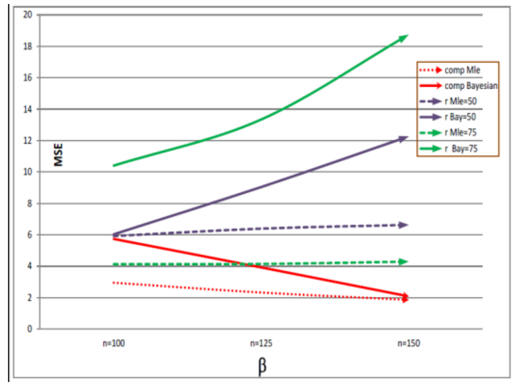

(B)

**FIGURE(B)** The graph of mean square error for $\theta$



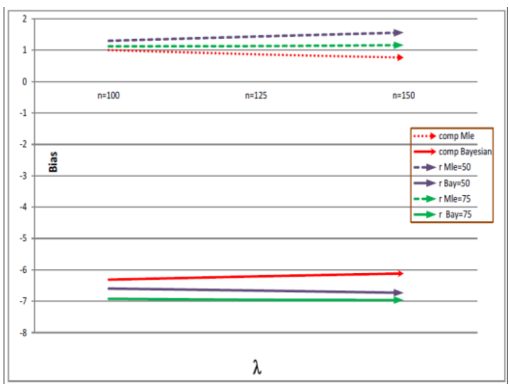

(C)

**FIGURE (C**) The graph of bias for β



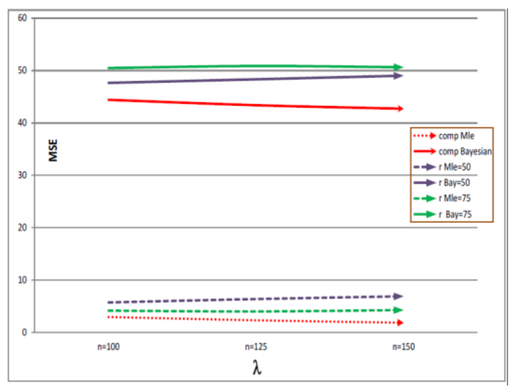

(D)

**FIGURE(D)** The mean square error for β




(E)

**FIGURE(E)** The graph of bias for λ




(F)

**FIGURE(F)** The mean square error for λ

## 5 CONCLUSION

The performance of the proposed Bayesian estimators has been compared to the maximum likelihood estimator in complete and censored data for samples of deferent values and also for samples censored at deferent values of the censor. the maximum-likelihood estimator and better than the corresponding Bayes estimators SEL in case the complete and censored data.

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