Channel Capacity Enhancement of MIMO System using Water-Filling Algorithm

Harsha Gurdasani¹, Dr. A G Ananth², Dr. Thangadurai N³

¹Research Scholar, Department of Electronics and Communication Engineering, JAIN (Deemed -to -be University), Bangalore, India

²Dr.A G Ananth, Professor, Electronics & Communication Engineering Department, NMAMIT, Nitte, Mangalore, India

³Dr. Thangadurai N, Professor & Head, Department of Electronics and Communication Engineering, JAIN (Deemed–to–be University), Bangalore, India.

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Abstract – The demand for higher speeds continues to grow today. In many circumstances, wireless systems are an effective solution, providing high speeds at a lower cost than cable-based technologies. Bandwidth and power limitations in today's communications systems make MIMO technology indispensable today. Diversity in MIMO applications tends to upgrade the architecture of system for compensation with upgraded hardware and software requirements. This paper work presents an analysis of bit error rate (BER) performance of STBC, Orthogonal-STBC and channel capacity enhancement framework of MIMO system using water-filling algorithm with ML, ZF and MMSE equalization techniques.

1. Introduction

Wireless communications have become an integral part of our daily life. This technology has contributed to the increase of production and the strengthening of safety at work, in different industries. It would therefore be unfair not to take advantage of these advantages in the mining industry. In fact, a wireless communication system will ensure smooth management of work in mines, communication from inside the mine to the outside for day-to-day operations, as well as for rapid disaster rescue operations.

Most of the communications systems currently in existence, which provide communications in underground mines, are of the wired type [1]. This type of system is reliable, however, it lacks flexibility and is difficult to deploy in inaccessible settings. For these reasons, a good alternative would be the use of wireless communications systems. Still in the same context, the mining environment, by its risky nature, requires the most reliable and secure communication system possible. For this, we propose the incorporation of the MIMO-OFDM solution in the millimeter bands and the evaluation of its performance in a mining environment; an effort that is part of the integration of wireless communications in underground mines. Therefore, the goal is to provide a flexible, reliable wireless communication link that delivers very high transmission rates in underground mines.

New cellular communication systems such as fourth generation cellular telephones require high data rates to provide various services such as the Internet, multimedia, video on demand, and so on. Consequently, methods are needed to increase the system capacity. A MIMO (Multiple Input Multiple Output) radio system is characterized by the presence of multiple antennas at the transmitter and receiver. With such a system, higher bandwidth can be obtained than traditional SISO radio systems (one input, one output).

In order to take full advantage of the capabilities of the MIMO system, various transmission methods have been proposed that provide the advantages of coding and diversity. Authors of paper [2] proposes space-time grid coding based on the general concepts of coding, modulation and channel diversity during transmission and reception. Although this encoding provides high performance, the complexity of its analysis (as measured by the number of grid states in the decoder) can become very high as it increases exponentially with increasing diversity and throughput. To simplify implementation, Alamouti [3] proposed a transmit diversity scheme using two transmit antennas and one receive antenna, which is easily expandable to use multiple receive antennas. This approach can significantly reduce the complexity of the space-time code implementation.

One of the main disadvantages of using multiple antennas in transmission and / or reception is the increase in cost and circuit complexity. Antenna selection techniques allow these variables to be reduced by using multiple antennas with only one or a few RF stages in narrowband systems, sending information from the receiver back to the transmitter that is, closing the loop between transmission and reception. In the literature, several antenna selection systems have been proposed [4-15]. In [4], only one antenna out of all available is selected for transmission, while in [5], [6], [7], a subset of all available antennas are selected and a block space-time code. However, the performance of these techniques is strongly affected by how often the feedback channel is used

compared to the channel's Doppler frequency. In [8], [9], [10] the advantages and performance of mixed selection / maximum ratio combination techniques were analyzed. In [11], [12], [13], [14] and [15] the achievable capacity of MIMO systems in the presence of antenna selection techniques was studied. However, the advantages of antenna selection systems with respect to cost and simplicity are not directly extrapolated to broadband systems. The advantages offered by antenna selection systems would vanish if applied by subcarrier or group of subcarriers in multi-carrier systems.

In this paper, a transmit diversity technique is proposed with stepped selection of space-time code in which the transmitter uses predetermined time intervals for transmission with each code, with the latter varying the number of transmitting antennas. The intervals in which each code is used are found offline through simulations. The technique is proposed for narrowband systems, but the authors estimate that it is possible to extend its application to multicarrier systems with few modifications, replicating the processing per subcarrier or per group of subcarriers. This article is organized as follows. The section II describes the system model and the proposed technique. Subsequently, the performance of the channel capacity enhancement using-Water-filling scheme is compared with respect to the two antenna selection schemes and with respect to space-time coding by transmitting the orthogonal code for four transmitting antennas. Section III shows the simulation results and finally, section IV shows the conclusions.

2. Proposed Methodology

A. Space-Time Block Code (STBC)

This first family only requires knowledge of the channel on reception (Rx-CSI) and the transmitter does not know the channel. We will see different solutions using the principle of space-time block codes (STBC) [16], the best known of which is the Alamouti code for two transmitting antennas. The code proposed by Alamouti [3] is based on a succession of transmissions over two symbol periods and on two antennas: it then appears in both spatial and temporal dimensions. The signals transmitted are based on complex symbols resulting from modulation (conjugate or opposite). The code is defined by:

$$C_2 = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix} \tag{1}$$

Where s_1 and s_2 are the symbols to be transmitted. This code has the particularity of being orthogonal: $C_2C_2^* = C_2^*C_2 = (|s_1|^2 + |s_2|^2)I_2$ (2)

And on average:

$$E[C_2C_2^*] = E[C_2^*C_2] = 2I_2 \qquad \text{with } E[|s_i|^2] = 1 \tag{3}$$

In order to respect the constraint of average total power emitted P_0 on all the antennas during a symbol time, it

is necessary to multiply the code by the constant $\sqrt{\frac{P_0}{2}}$. The input-output relationship in matrix form is:

$$Y = \sqrt{\frac{P_0}{2}} H C_2 + N \tag{4}$$

Where Y, $[n_R \times 2]$, is the matrix of the samples received and N, $[n_R \times 2]$ is the matrix of noise. The receiver estimates the elements of the channel matrix and recombines the samples received. It then forms two particular signals defined by:

$$\begin{cases} \tilde{y}_{1} = \frac{\sum_{j=1}^{n_{R}} h_{1,j}^{*} y_{1}^{j} + h_{2,j} (y_{2}^{j})^{*}}{\|H\|_{F}} \\ \tilde{y}_{2} = \frac{\sum_{j=1}^{n_{R}} -h_{1,j} (y_{2}^{j})^{*} + h_{2,j}^{*} y_{1}^{j}}{\|H\|_{F}} \end{cases}$$
(5)

Where $h_{i,j}$ represents the complex gain between the transmitting antenna *i* and the receiving antenna *j*, y_t^j is the useful signal of the receiving antenna *j* at the symbol period *t*. In addition, $||H||_F$ represents the Frobenius norm of the matrix *H* defined by:

$$||H||_{F}^{2} = \operatorname{trace}(HH^{*}) = \operatorname{trace}(H^{*}H) = \sum_{i}^{n_{T}} \sum_{j}^{n_{R}} |h_{i,j}|^{2}$$
(6)

The useful signal is defined by:

$$\begin{cases} y_1^j = \sqrt{\frac{P_0}{2}} (h_{1,j} s_1 + h_{2,j} s_2) + n_1^j \\ y_2^j = \sqrt{\frac{P_0}{2}} (-h_{1,j} s_2^* + h_{2,j} s_1^*) + n_2^j \end{cases}$$
(7)

Where n_t^j is the additive noise of the receiving antenna *j* at the symbol period *t*. The expressions of equation (5) can be expanded and give the following result:

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$$\begin{cases} \tilde{y}_{1} = \sqrt{\frac{P_{0}}{2}} \frac{\sum_{j=1}^{n_{R}} \|h_{1,j}\|^{2} + \|h_{2,j}\|^{2}}{\|H\|_{F}} s_{1} + \frac{\sum_{j=1}^{n_{R}} h_{1,j}^{*} n_{1}^{j} + h_{2,j} (n_{2}^{j})^{*}}{\|H\|_{F}} \\ \tilde{y}_{2} = \sqrt{\frac{P_{0}}{2}} \frac{\sum_{j=1}^{n_{R}} \|h_{1,j}\|^{2} + \|h_{2,j}\|^{2}}{\|H\|_{F}} s_{2} + \frac{\sum_{j=1}^{n_{R}} h_{2,j}^{*} n_{1}^{j} - h_{1,j} (n_{2}^{j})^{*}}{\|H\|_{F}} \end{cases}$$
(8)

The two recombined signals take a very simple form:

$$\begin{cases} \tilde{y}_{1} = \sqrt{\frac{P_{0}}{2}} \|H\|_{F} s_{1} + \tilde{n}_{1} \\ \tilde{y}_{2} = \sqrt{\frac{P_{0}}{2}} \|H\|_{F} s_{2} + \tilde{n}_{2} \end{cases}$$
(9)

where \tilde{n}_1 and \tilde{n}_2 are combinations of the additive noise and are AWGN if N is a AWGN: for a noise iid $\mathcal{N}_{\mathbb{C}}(0, \sigma^2)$ then \tilde{n}_1 and \tilde{n}_2 are independent and of law $\mathcal{N}_{\mathbb{C}}(0, \sigma^2)$. Each signal \tilde{y}_i only depends on a symbol s_i and can thus be used for its estimation. The Alamouti code therefore decouples the symbols and makes it possible to reduce the decision tests. The system can be viewed equivalently as two SISO systems in parallel. In addition,

without knowing the channel a priori, the code ensures the same gain $\sqrt{\frac{P_0}{2}} \|H\|_F$. This feature allows the code to be robust with respect to the channel and also to ensure an order of maximum diversity equal to the product of the antennas on transmission and reception $n_T \times n_R$ [2], here $2 \times n_R$ for a Rayleigh canal.

On the other hand, the system can use another matrix entry:

$$\tilde{y} = G\tilde{H}_2 s + G\bar{n} \tag{10}$$

With the new channel matrix,

If
$$H = (h_1 \quad h_2)$$
 then $\widetilde{H}_2 = \begin{pmatrix} h_1 & h_2 \\ -\operatorname{conj}(h_2) & \operatorname{conj}(h_1) \end{pmatrix}$ (11)

Where conj is the conjugation operation of the elements of the matrix $(conj(x) = (x^*)^T = (x^T)^*)$, and the noise defined by:

If
$$N = (n_1 \quad n_2)$$
 then $\bar{n} = \begin{pmatrix} h_1 & h_2 \\ -\operatorname{conj}(h_2) & \operatorname{conj}(h_1) \end{pmatrix}$ (12)

The new channel matrix thus defined then has the following property:

$$\widetilde{H}_{2}^{*}\widetilde{H}_{2} = \|H\|_{F}^{2}I_{2}$$
(13)

Then just choose G as the suitable filter:

$$G = \frac{\tilde{H}_2^*}{\|H\|_{\mathcal{F}}} \tag{14}$$

The noise $\tilde{n} = G\bar{n}$ is a Gaussian noise of variance σ_n^2 if the noise N is also an additive Gaussian noise of the same variance.

By considering either a linear recombination of the samples received, or a new channel matrix associated with an adapted filtering, the Alamouti code allows to use orthogonality to diagonalize the channel. The two symbols are decoupled and see the same channel, that is to say, the same gain and a noise of the same statistics.

However, using the code has two major drawbacks. The first is loss of flow. Indeed, an Alamouti code has the same bit rate as a SISO system but half as much as a spatial multiplexing [17]. We will note the performance of the \mathcal{R} code as the number of symbols emitted over the number of symbol periods necessary for transmission. We have $\mathcal{R} = 1$ for the Alamouti code and $\mathcal{R} = n_T$ for spatial multiplexing.

In addition, if you want to control the transmission power, the coefficient $\sqrt{\frac{P_0}{2}}$ is necessary. Consequently, a penalizing coefficient of 1/2 or -3dB appears on the power received from the equivalent scheme compared to a SISO system [3].

B. Orthogonal Space-Time Block Code (OSTBC)

The Alamouti code is only suitable for systems with two transmitting antennas and Tarokh et al. [2, 18] generalized the orthogonal STBC (OSTBC) and this whatever n_T . The principle is to consider a train of N_s symbols to be transmitted that the code will transmit by successive bursts over N_p symbol periods. The flow efficiency is noted:

$$\mathcal{R} = \frac{N_s}{N_p} \tag{15}$$

The bursts emitted will be the symbols or their conjugates weighted by 1 or -1 and the vectors emitted must be orthogonal. The generated code is a matrix whose two dimensions are space and time, $[n_T \times N_p]$. Then, the

associated receiver recombines the samples received after estimating the channel and obtains signals depending on only one transmitted symbol. Thus, the symbol estimates are decoupled.



Figure 1: Diagram of the MIMO transmission using OSTBC

The orthogonality of the code allows an independent estimation of each s_i symbol with the signal \tilde{s}_i .

Figure 1 shows the OSTBC synopsis. The proposed principle is simple and very easy to set up: the compromise between performance (BER and throughput) and complexity (CSI at reception only) is interesting. In the best of cases, it is a matter of obtaining a code with the greatest possible return. For example, $\mathcal{R} = 1$ for the Alamouti code. The solutions are limited and are as follows:

- The solution proposed by Alamouti (Equation (1)) which is the only one to obtain $\mathcal{R} = 1$;
- OSTBC for complex modulations and for any value of n_T but with $\mathcal{R} = \frac{1}{2}$ [18];
- Solutions for $n_T = \{3, 4\}$ but with $\mathcal{R} = \frac{3}{4}$,

$$C_{3} = \begin{pmatrix} s_{1} & -s_{2}^{*} & \frac{s_{3}^{*}}{\sqrt{2}} & \frac{s_{3}^{*}}{\sqrt{2}} & \frac{s_{3}^{*}}{\sqrt{2}} \\ s_{2} & s_{1}^{*} & \frac{s_{3}^{*}}{\sqrt{2}} & -\frac{s_{3}^{*}}{\sqrt{2}} \\ \frac{s_{3}}{\sqrt{2}} & \frac{s_{3}}{\sqrt{2}} & \frac{-s_{1}-s_{1}^{*}+s_{2}-s_{2}^{*}}{\sqrt{2}} & \frac{s_{1}-s_{1}^{*}+s_{2}+s_{2}^{*}}{2} \end{pmatrix}$$

$$C_{4} = \begin{pmatrix} s_{1} & -s_{2}^{*} & \frac{s_{3}^{*}}{\sqrt{2}} & \frac{s_{3}^{*}}{\sqrt{2}} & \frac{s_{3}^{*}}{\sqrt{2}} \\ s_{2} & s_{1}^{*} & \frac{s_{3}^{*}}{\sqrt{2}} & -\frac{s_{3}^{*}}{\sqrt{2}} \\ \frac{s_{3}}{\sqrt{2}} & \frac{s_{3}}{\sqrt{2}} & \frac{-s_{1}-s_{1}^{*}+s_{2}-s_{2}^{*}}{2} \\ \frac{s_{3}}{\sqrt{2}} & -\frac{s_{3}}{\sqrt{2}} & \frac{s_{1}-s_{1}^{*}-s_{2}-s_{2}^{*}}{2} & \frac{s_{1}-s_{1}^{*}+s_{2}+s_{2}^{*}}{2} \\ \frac{s_{3}}{\sqrt{2}} & -\frac{s_{3}}{\sqrt{2}} & \frac{s_{1}-s_{1}^{*}-s_{2}-s_{2}^{*}}{2} & \frac{-s_{1}-s_{1}^{*}-s_{2}+s_{2}^{*}}{2} \end{pmatrix}$$

$$(16)$$

The required orthogonality associated with adequate recombination makes it possible to obtain signals \tilde{s}_i which only depend on the symbol s_i . This result can be seen, like the Alamouti code, as a diagonalization of the channel. In addition, the coefficient $\sqrt{\frac{P_0}{n_T}}$ makes it possible to control the power emitted. A system using OSTBC is equivalent to N_s independent SISO transmissions represented by Figure 2.



Figure 2: Equivalent SISO diagram of a system using OSTBC with the same gain for each channel

The strength of the code is to obtain a constant gain proportional to $||H||_F$ without a priori knowledge and to ensure a maximum order of diversity $n_T \times n_R$. The price to pay is a drop in flow with a yield $\mathcal{R} \leq 1$ corresponding to a spectral efficiency of $\mathcal{R} \log_2 M$.

C. Diversity

Diversity means that the system will provide the receiver with several replicas (copies) of the same signal carrying information, the duplicate signals are slightly modified by the discoloration. The three types of diversity technique are:

• Spatial Diversity: This means that the elements of the antenna are sufficiently spaced from each other to achieve independence between transmit and receive signals. The spatial separation must be at least half the wavelength to obtain the desired results of independence.

• Transmit Diversity: Deprived of deteriorating bandwidth. - The maximum information is communicated in various time slots with the time intervals divided by measures of equivalent or superior than the channel coherence time.

• Frequency Diversity: The similar information is conveyed through various frequencies which are divided by the measures of equivalent or superior than the coherence bandwidth of the channel.

The reliability of the network is to improve the use of space and time of diversity, while the transmission rate is the improvement of multiplexing. MIMO can be used with any access or modulation technique, being the most frequently used technique called Orthogonal Frequency Division Multiplexing (OFDM), which can divide the selective frequency channel into some flat fading channels and then apply the MIMO technique to each of these sub channels [19].

D. Channel Capacity Enhancement using Water-Filling

Recall that, setting the transmitted power to P, the ergodic capacity of MIMO system will be:

$$C = \max_{Tr(Q) \le P} E\left\{ \log\left(I_N + \frac{SNR}{M} H Q H^H\right) \right\}$$
(18)

Where *Q* is the covariance matrix of the input signal.

To reach the maximum, we will have to optimize the covariance matrix of the transmitted signal ($Q = VPV^H$), thus defining the transmission strategy based on the information available from the transmitter. We will now analyze two different scenarios based on this information: CSIT, when we know the channel in the transmitter, and CDIT, when we know the statistical distribution of the channel.

As seen in equation (18), the correlation matrix is a parameter that greatly affects capacity. The study of this correlation matrix and its effect in communication systems with multiple antennas can be found in [20, 21] and is fully characterized by the separation between antennas in the transmitter and receiver as well. As for a statistical model that describes the dispersion given by the angular power profile [22, 23].

1. CSIT: Channel State Information at the Transmitter

With CSIT, we assume that the H channel matrix is instantly known at the transmitter. When the transmitter knows this matrix, we assume in the same way that it knows the distribution perfectly (since the distribution can be obtained from instantaneous observations).

In this first case, the matrix V will be formed by the eigenvalues of the correlation in the transmitter $E\{H^H H\}$.

Since the parallel channels in which the channel is broken down are not, as expected, identical but rather have different quality, the allocation of the power level for each channel (the matrix *P* will be made using the Water-Filling algorithm. This algorithm, as its name anticipates, compares the power allocation of the different channels with containers that fill up with water. Thus, this algorithm leads us to:

$$P_i = \left(\mu - \frac{1}{\sigma_i^2}\right)^+, \quad 1 \le i \le R_H \tag{19}$$

Where P_i is the corresponding power \tilde{x}_i , x^+ is defined as max(x, 0), and the "water fill" level μ is chosen such that), $\sum_{i=1}^{R_H} P_i = P$.

The capacity is therefore reached by choosing each component \tilde{x}_i according to an independent Gaussian distribution with power P_i . The covariance that maximizes the expression is $Q = VPV^H$, where the matrix P is a diagonal matrix defined as $P = \text{diag}(P_1, \dots, P_{R_H}, 0, \dots, 0)$. The resulting capacity will be:

$$C = \sum_{i}^{R_H} (\log(\mu \sigma_i^2))^+ \tag{20}$$

For low SNR levels, the Water-Filling algorithm distributes the available power by placing it on the parallel channel with the highest gain. For high levels of SNR, this algorithm manages the power by assigning to each channel approximately the same, leading us to the following approximation of the capacity for high values of SNR: $C \approx R_H \log_2(P) + O(1)$ (21)

Where the constant depends on the singular values of H.

We can draw several conclusions from this transmission strategy. First, the multiplication of V by the input consists of a pre-coding that aligns these inputs with the channel's eigenmodes. This means that, by doing a simple multiplication of the output signals, we can treat them as independent observations. Furthermore, the transmitter applies the Water-Filling strategy making use of the different channel modes, to take advantage of the different quality of the parallel channels.

2. CDIT: Channel Distribution Information at the Transmitter

In a scenario with CDIT, the optimization of the covariance matrix Q, described by $Q = VPV^H$, is usually decomposed separately in the optimization of the eigenvalues P (which define the power distribution) and the eigenvectors V [24] [25] [26].

First, the eigenvectors that maximize (18) are those of the correlation at the transmitter $E\{H^H H\}$. To find the optimal power distribution we will use the algorithm proposed in [26]. This algorithm proposes a power distribution based on the MMSE. This algorithm is valid when the eigenvalues V are known.

We define a rotated version of the channel, $\hat{H} = HV$, whose j^{th} column is denoted as \hat{h}_j . To reach the capacity the matrix *P* must meet the sufficient and necessary condition described below:

$$\frac{1}{N}E\left[Tr\left\{\left(1+MMSE\hat{h}_{j}\hat{h}_{j}^{H}\right)\left(I+\frac{SNR}{M}\widehat{H}P\widehat{H}^{H}\right)^{-1}\right\}\right] \begin{cases} =1 & if \ p_{j} > 0 \\ \leq 1 & if \ p_{j} = 0 \end{cases}$$
(22)

Since the corresponding parallel channels are not orthogonal, this solution does not correspond to a Water-Filling. In equation (22) indicates that when we are below a certain SNR, the power transmitted by some of the channels will be zero.

Thus, the power injected into each channel described in the matrix *P* cannot be found explicitly, so an algorithm is proposed based on the mean square error for which, in the first place, it defines:

$$B_j \triangleq \left(I + \frac{SNR}{M} \widehat{H}_j P_j \widehat{H}_j^H\right)^{-1}$$
(23)

Where \hat{H}_j is the matrix obtained by eliminating the j^{th} column of H and P_j is the diagonal matrix obtained by eliminating the j^{th} row and column. Thus, the MMSE resulting from the linear estimation of the signal s_j transmitted through the j^{th} eigenvector is:

$$MMSE = \frac{1}{1 + p_j \frac{SNR}{M} \hat{h}_j^H B_j \hat{h}_j}$$
(24)

The received power of the j^{th} signal is given by $1 - MMSE_j$ and therefore the SINR is given by the following expression:

$$SINR_j = \frac{1 - MMSE}{MMSE}$$
(25)

The expectation of (24) with respect to \hat{H} is denoted as:

$$\overline{MMSE_j} \triangleq E[MMSE_j] \tag{26}$$

Furthermore, it can be verified that,

$$T_r\left\{\left(1+\frac{SNR}{M}\widehat{H}P\widehat{H}^H\right)^{-1}\right\} = \sum_{l=1}^M MMSE_l + N - M$$
(27)

Using equations (23) and (27), the conditions originally stated in equation (22) can finally be rewritten as:

$$\begin{cases} p_{j} = 0 & \frac{SNR}{M} E[\hat{h}_{j}^{H}B_{j}\hat{h}_{j}] \leq \frac{1}{M} \sum_{l=1}^{M} (1 - \overline{MMSE}_{l}) \\ p_{j} = \frac{1 - \overline{MMSE}_{j}}{\frac{1}{M} \sum_{l=1}^{M} (1 - \overline{MMSE}_{l})} & \text{in all other cases} \end{cases}$$

$$(28)$$

This iterative algorithm quickly converges to the required power values.

E. Equalizer Used

1. Maximum Likelihood (ML)

The ML detector calculates the Euclidean distance between the received signal and the product of the channel with all the possible transmitted signals and finds the one with the minimum distance. Considering *C* as the set of symbols of the constellation used, with symbols $s_l \in C$, the ML rule estimates the received signal s as:

$$\tilde{s}_{ML} = \underset{l}{\operatorname{argmin}} \left\| y - \sqrt{\rho} H s_l \right\|_F^2$$
(29)

If all symbols are equiprobable, the ML rule gives us optimal detection, however its complexity grows exponentially with the order of modulation or the number of transmitting antennas. Although the complexity of

the ML rule is very high even when dealing with a small number of transmitting antennas, the performance results serve as a reference when comparing other suboptimal techniques with reduced complexity.

2. Zero Forcing (ZF) Equalizer

The ZF equalizer falls under the category of linear equalizers. ZF equalizer reverses the frequency response of the channel. The name Zero forcing is because this equalizer brings ISI to zero level in an ideal noise free case. ZF equalizer is useful in situations where ISI is more dominant than noise.

Let H denote the channel matrix and x be the transmitted signal vector, then ZF equaliser can be implemented by multiplication of inverse channel matrix with the received signal to produce the estimate of transmitted signal \tilde{x}

$$\tilde{\mathbf{x}} = \mathbf{H}^{\mathbf{H}}\mathbf{r} = \mathbf{H}^{\mathbf{H}}(\mathbf{H}\mathbf{x}) = \mathbf{x} \tag{30}$$

Where (.)^H denotes the pseudo-inverse.

When noise is also considered then the resulting signal will be:

$$\tilde{\mathbf{x}} = \mathbf{H}^{\mathsf{H}}\mathbf{R} = \mathbf{H}^{\mathsf{H}}(\mathbf{H}\mathbf{x}+\mathbf{n}) = \mathbf{x} + \mathbf{H}^{\mathsf{H}}\mathbf{n}$$
(31)

It is clear that, the estimate signal \tilde{x} from zero forcing equalizer is a decoded signal with addition to inverted channel matrix and unknown noise. Due to this noise amplification property of ZF equalizer, MMSE equalizer was proposed.

3. MMSE Equalizer

Minimum Mean Square Error equalizer otherwise stated as MMSE equalizer reduces the problem of noise amplification by taking Noise power into consideration while designing filtering matrix through MMSE criterion. The estimated symbol vector from MMSE equalizer can be given as:

$$\tilde{\mathbf{x}} = [[(\mathbf{H}^{\mathrm{H}}\mathbf{H} + (\sigma^{2}\mathbf{I})) - 1]\mathbf{H}^{\mathrm{H}}]^{\mathrm{r}}$$
(32)
riv. σ^{2} is noise variance

Where H represents H channel matrix, σ^2 is noise variance.

3. Simulation and Results





Figure 4: BER performance comparison of Rx-1 and Rx2

Figure 5: BER for BPSK modulation with 2×2 MIMO using MMSE equalization in Rayleigh fading channel



Figure 6: BER for BPSK modulation with 2×2 MIMO using ML equalization in Rayleigh fading channel



Figure 7: Symbol error rate for uncoded 8×8 (4-QAM) system



Figure 8: Channel capacity estimation for Nakagami fading channel

4. Conclusion

The first objective of this paper has been the analysis of the capacity in MIMO systems as a function of various parameters such as the signal-to-noise ratio, the distance between transmitting antennas or the angle of dispersion in the transmitter (parameters that ultimately represent the correlation existing in the transmitter). In this analysis of capacity, two asymptotic scenarios have been emphasized: when the SNR is high and when it is low.

We have seen that one of the factors to take into account is the knowledge of the channel that we have in the transmitter and that, depending on it, it will differentiate between different transmissions strategies to achieve system capacity.

Having observed the results obtained, we can conclude that the correlation in the transmitter affects differently depending on the SNR levels with which we are working.

Having observed the results obtained, we can conclude that the correlation in the transmitter affects differently depending on the SNR levels with which we are working. This correlation increases when we decrease the distance between the radiating elements or when we decrease the angular dispersion in the transmitter.

The difference between the CSIT and CDIT scenarios analyzed is very low when we are at high SNR, as conditions are very favorable. However, for a low SNR, this difference is more notable. While the Water-Filling algorithm used in CSIT tends to yield capacity values that stabilize as the correlation decreases, the algorithm implemented for CDIT has a more complex behavior. In general, for low SNR levels the correlation favors capacity.

The second objective of this project is the analysis of the orthogonality of the channel matrices. This aspect is also key, since the orthogonality determines the signal processing and its complexity. Thus, what we are looking for are matrices H as orthogonal as possible. This analysis has been done the same as the capacity analysis that is, varying parameters present in the transmitter.

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