Optimum Integer Solution of Goal Programming Problem by Modified Gomory Constraint Technique

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Abstract: In this paper, we made an attempt to crack Goal programming problem by using Modified Gomory Constraint Technique. This method will be a new approach and easy to crack goal programming problem. The above method will be a powerful method to find better solution. It will take less iterations and save our precious time by omiting calculations of net evaluation.

Keywords: Goal Programming Problem, Optimal Solution, proposed algorithm. Less iteration, save time.

1. Introduction

Integer programming problem is a unique class of L.P.P. where the entire or some variables are constrained to presume non – negative integer values. This kind of problem is of particular importance in several business and industry whenever distinct nature of the variables is occupied in many conclusion – building situation.

1.1 Gomory's All I.P.P. Method:

The I.P.P. is initial solved ignoring the integer constraint, with any standard technique. After an optimum basic feasible solution has been achieved, if all the variables within the solution contain integer values, the present solution is the desired optimum integer solution; otherwise the measured I.P.P. is modified by inserting a new constraint that eliminates some non – integer solutions, but now any feasible integer one. The optimum solution to this customized I.P.P. is acquired, by any standard algorithm. The optimum integer solution is obtained if every variables within this solution are integers, if not a new constraints is added to the I.P.P. and the process is repeated. The optimum integer solution will be attain finally after sufficient new constraints have been added to pave away all the better non – integer solutions. The production of additional constraints, called Gomory's constraints, is thus essential that it needs special notice.

In 1961, **Charnes and Cooper** studied Management Models and Industrial Applications of Linear Programming. Charnes et al. (1968) discussed a goal programming model for media planning**. Contini** (1968) studied a stochastic approach to goal programming. Dauer And Krueger (1977) developed an Iterative Approach to Goal Programming. **Kornbluth and Steuer** (1981) studied Goal programming with linear fractional criteria. **Moitra and Pal** (2002) discussed a fuzzy goal programming approach for cracking bilevel programming problems**. Pramanik and Kumar** (2006) applied Fuzzy goal programming approach to multi-level programming problems. **Baky** (2009) derived Fuzzy goal programming algorithm for solving decentralized bi-level multi-objective programming problems. **Khobragade; Vaidya and Lamba** (2014) developed an Approximation algorithm designed for optimal solution towards the linear programming problem. **Birla et al.** (2017) developed an Alternative Approach in favour of Solving Bi-Level Programming Problems. **Putta Baburao and Khobragade** (2019) derived Optimum solution of Goal and Fractional Programming Problem.

In this paper, modified gomory constraint technique has been suggested and solved goal programming problem (GPP).

2. PROPOSED MODIFIED GOMORY CONSTRAINT TECHNIQUE FOR GOAL PROGRAMMING PROBLEM

Here we added the following steps of alternative technique to solve Goal Programming Problems.

Step (1). Select $\min \sum x_{ij}$, $x_{ij} \geq 0$, for incoming vector.

Step (2). Select highest coefficient of decision variables.

(a) If highest coefficient is unique, then element corresponding to this row and column turn into pivotal element.
(b) If highest coefficient is not unique, then apply tie breaking method.

Step (3). Neglect corresponding row and column. continue to step 2 for remaining elements and repeat the similar process till an optimal solution is found or there is an indication for absolute solution.

Step (4). If the entire rows and columns are neglected, then optimal solution exists.

Step (5). Investigate whether the optimum solution includes integer values or not.

(a) If all values of the optimum solution are integer values, an optimum basic feasible integer solution has been achieved.
(b) If all values of the optimum solution are not integer values then go on next stage.

Step (6). Check the constraint equations corresponding to the existing optimum solution. Let these equations are

$$\sum_{j=0}^{n'} y'_{ij}\ x_j = b'_i \qquad [i = 0\,1\,2 \ldots m']$$

where n' represents the number of variables and m' the number of equations.

Select the highest fraction of b'_i s i.e. find $\max_i \{b'_i\}_f$.

Let it be $[b'_k]_f$ otherwise note down it simply as f_{k0} .

Step (7). If there is any negative fraction, then state each one of the negative fractions, in the kth row of the optimum simplex table when the addition of a negative integer and a non negative fraction.

Step (8). Find the Gomorian constraint $\sum_{j=0}^{n'} f_{kj} x_j \geq f_{k0}$ and append the equation

$$G_{sla}^{(1)} = -f_{k0} + \sum_{j=0}^{n'} f_{kj} x_j$$

to the present set of equation constraints.

Step (9). Initiating with this latest set of equation constraints, discover the new optimum solution by dual

Step (10). If this latest optimum solution for the modified I.P.P. is an integer solution, it is as well feasible and optimum for the given I.P.P. or else return to step (4) and do the procedure again till an optimum feasible integer solution has been achieved.

3. STATEMENT OF THE PROBLEM

3.1 Solve the following GPP

A production house of leather belts compose three types of belts A, B and C which are processed on three machines M_1, M_2 *and* M_3. Belt A requires 2 hours on machine M_1 and 3 hours on machine M_3 . Belt B requires 3 hours on machine M_1 , 2 hours on machine M_2 and 2 hours on machine M_3 ; and Belt C requires 5 hours on machine M_2 and 4 hours on machine M_3 . There are 8 hours of time per day offered on machine M_1 , 10 hours of time per day offered on machine M_2 and 15 hours of time per day offered on machine M_3 . The profit achieved from belt A, B and C is Rs. 3.00 per unit, Rs. 5.00 per unit and Rs. 4.00 per unit respectively. Formulate the goal programming problem to find the daily production of all type of belts so that the profit is maximum.

3.2 Solution:

Let x_1, x_2 *and* x_3 denotes unit of type A belt, unit of type B belt and unit of type C belt respectively. So, the constraints and goals of the problem can be expressed as follow:

$$Maximize \quad z = 3x_1 + 5x_2 + 4x_3$$

Subject to the constraints: $2x_1 + 3x_2 \le 8,\ 2x_2 + 5x_3 \le 10,\ 3x_1 + 2x_2 + 4x_3 \le 15,\ x_1, x_2, x_3 \ge 0$

Now, the formulation of the given problem as goal programming model is as follows:

$Minimize\ \ Z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints: $2x_1 + 3x_2 + d_1^- - d_1^+ = 8$

$$2x_2 + 5x_3 + d_2^- - d_2^+ = 10$$

$$3x_1 + 2x_2 + 4x_3 + d_3^- - d_3^+ = 15$$

$$x_1,\ x_2,\ x_3,\ d_1^+,\ d_1^-,\ d_2^-,\ d_2^+,\ d_3^-,\ d_3^+ \ge 0$$

$d_1^-,\ d_2^-, d_3^-$ are the amount by which we underachieve our objective.

$d_1^+,\ d_2^+, d_3^+$ are the amount by which we overachieve our target.

Table (1). Initial table:

			3	5	4	0	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+
0	d_1^-	8	2	3	0	1	-1	0	0	0	0
0	d_2^-	10	0	2	5	0	0	1	-1	0	0
0	d_3^-	15	**3**	2	4	0	0	0	0	1	-1
			↑							↓	

Since $\min \sum x_{ij} = 5$

Hence the column vector x_1 enter in the basis and the column vector d_3^- leaves the basis.

Table (2): Introduce x_1 and drop d_3^-

			3	5	4	0	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+
0	d_1^-	-2	0	5/3	-8/3	1	-1	0	0	-2/3	2/3
0	d_2^-	10	0	2	**5**	0	0	1	-1	0	0
3	x_1	5	1	2/3	4/3	0	0	0	0	1/3	-1/3
					↑			↓			

Since $\min \sum x_{ij} = 11/3$

Hence the column vector x_3 enter in the basis and the column vector d_2^- leaves the basis.

Table (3): Introduce x_3 and drop d_2^-

			3	5	4	0	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+
0	d_1^-	10/3	0	41/15	0	1	-1	8/15	-8/15	-2/3	2/3
0	x_3	2	0	2/5	1	0	0	1/5	-1/5	0	0
3	x_1	7/3	1	2/15	0	0	0	-4/15	4/15	1/3	-1/3

As, the optimum solution is not integer valued, we assume simply the fractional parts of

$x_{B1}=\frac{10}{3}=3+\frac{1}{3}$, $x_{B3}=\frac{7}{3}=2+\frac{1}{3}$

Now we select Maximum $\{f_1, f_3\}=\left\{\frac{1}{3},\frac{1}{3}\right\}=\frac{1}{3}$, both f_1, f_3 are equal. So, we arbitrarily select any one of these. Let us choose f_3.

In third row, $\frac{7}{3}=2+\frac{1}{3}=x_1+\frac{2}{15}x_2+0x_3+0x_4+0x_5-\frac{4}{15}x_6+\frac{4}{15}x_7+\frac{1}{3}x_8-\frac{1}{3}x_9$

Now, introducing Gomorian slack variable G_1. Then we write

$$-\frac{2}{15}x_2-\frac{11}{15}x_6-\frac{4}{15}x_7-\frac{1}{3}x_8-\frac{2}{3}x_9+G_1=-\frac{1}{3}$$

Adding this additional constraint into the above optimum simplex table, we get

Table (4): Introduce G_1

			3	5	4	0	0	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	G_1
0	d_1^-	10/3	0	41/15	0	1	-1	8/15	-8/15	-2/3	2/3	0
0	x_3	2	0	2/5	1	0	0	1/5	-1/5	0	0	0
3	x_1	7/3	1	2/15	0	0	0	-4/15	4/15	1/3	-1/3	0
0	G_1	-1/3	0	-2/15	0	0	0	-11/15	-4/15	**-1/3**	-2/3	1
										↑	↓	

Since Maximum $\left\{\frac{(z_j-c_j)}{y_{4j}}, y_{4j}<0\right\}=-3$

Table (5): Introduce d_3^- and drop G_1

			3	5	4	0	0	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	G_1
0	d_1^-	4	0	9/5	0	1	-1	2	0	0	2	-2
0	x_3	2	0	2/5	1	0	0	1/5	-1/5	0	0	0
3	x_1	2	1	0	0	0	0	-1	0	0	-1	1
0	d_3^-	1	0	2/5	0	0	0	33/15	4/5	1	2	-3

The above table confirms that an optimum basic feasible integer solution has been achieved.
Hence, we get required optimum solution as

$$x_1=2, x_3=2,\ d_1^-=4,\ d_3^-=1$$

3.3 Solve the following GPP

A manufacturer company of toys makes two types of toys, say A and B. Processing of these toys is completed on two machines X and Y. Toy A needs 2 hours on machine X and 5 hours on machine Y. Toy B requires 4 hours on machine X and 3 hours on machine Y. There are 7 hours of time per day available on machine X and 15 hours on machine Y. The profit acquired on toy A is Rs. 1 per toy and on toy B is Rs. 4 per toy. Formulate this as a goal programming problem to find the everyday production of each type of two toys so that the profit is maximum.

3.4 Solution

Let x_1 *and* x_2 denotes two types of toys, say A and B respectively.

So, the constraints and goals of the problem can be expressed as follow:

$$Maximize \quad z = x_1 + 4x_2$$

Subject to the constraints: $2x_1 + 4x_2 \le 7,\ 5x_1 + 3x_2 \le 15,\quad x_1, x_2 \ge 0$

Now, the formulation of the given problem as goal programming model is as follows:

$$Minimize\ Z = x_1 + 4x_2 + 0d_1^- + 0d_2^-$$

Subject to the constraints: $x_1 + 4x_2 + d_1^- - d_1^+ = 7$

$$5x_1 + 3x_2 + d_2^- - d_2^+ = 15$$

$$x_1,\ x_2,\ d_1^+,\ d_1^-,\ d_2^-,\ d_2^+ \ge 0$$

$d_1^-,\ d_2^-$ are the amount by which we underachieve our objective.

$d_1^+,\ d_2^+$ are the amount by which we overachieve our target.

Table (1). Initial table:

			1	4	0	0	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+
0	d_1^-	7	2	**4**	1	-1	0	0
0	d_2^-	15	5	3	0	0	1	-1
				↑	↓			

Since $\min \sum x_{ij} = 7$

Hence the column vector x_2 enter in the basis and the column vector d_1^- leaves the basis.

Table (2): Introduce x_2 and drop d_1^-

			1	4	0	0	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+
4	x_2	7/4	1/2	1	1/4	-1/4	0	0
0	d_2^-	39/4	7/2	0	-3/4	3/4	1	-1

As, the optimum solution is not integer valued, we assume simply the fractional parts of

$$x_{B1} = \frac{7}{4} = 1 + \frac{3}{4},\ x_{B2} = \frac{39}{4} = 9 + \frac{3}{4}$$

Now we select Maximum $\{f_1, f_2\} = \left\{\frac{3}{4}, \frac{3}{4}\right\} = \frac{3}{4}$, both f_1, f_2 are equal. So, we arbitrarily select any one of these. Let us choose f_1.

In first row, $\frac{7}{4} = 1 + \frac{3}{4} = \frac{1}{2}x_1 + x_2 + \frac{1}{4}x_3 - \frac{1}{4}x_4 + 0x_5 + 0x_6$

Now, introducing first Gomorian slack variable G_1. Then we write

$$-\frac{1}{2}x_1 - \frac{1}{4}x_3 - \frac{3}{4}x_4 + G_1 = -\frac{3}{4}$$

Adding this additional constraint into the above optimum simplex table, we get

Table (3): Introduce G_1

			1	4	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	G_1
4	x_2	7/4	1/2	1	1/4	-1/4	0	0	0
0	d_2^-	39/4	7/2	0	-3/4	3/4	1	-1	0
0	G_1	-3/4	**-1/2**	0	-1/4	-3/4	0	0	1
			↑						↓

Since Maximum $\left\{\frac{(z_j - c_j)}{y_{3j}}, y_{3j} < 0\right\} = -2$

Table (4): Introduce x_1 and drop G_1

			1	4	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	G_1
4	x_2	1	0	1	0	1/2	0	0	-1
0	d_2^-	9/2	0	0	-5/2	-9/2	1	-1	-7
1	x_1	3/2	1	0	1/2	3/2	0	0	2

As, the optimum solution is not integer valued, we assume simply the fractional parts of

$$x_{B2} = \frac{9}{2} = 4 + \frac{1}{2},\ x_{B3} = \frac{3}{2} = 1 + \frac{1}{2}$$

Now we select Maximum $\{f_2, f_3\} = \left\{\frac{1}{2}, \frac{1}{2}\right\} = \frac{1}{2}$, both f_2, f_3 are equal. So, we arbitrarily select any one of these. Let us choose f_2.

In second row, $\frac{9}{2} = 4 + \frac{1}{2} = 0x_1 + 0x_2 - \frac{5}{3}x_3 - \frac{9}{2}x_4 + x_5 - x_6 - 7x_7$

Now, introducing second Gomorian slack variable G_2. Then we write

$$-\frac{1}{2}x_3 - \frac{1}{2}x_4 + G_2 = -\frac{1}{2}$$

Adding this additional constraint in the above optimum simplex table, we get

Table (5): Introduce G_2

			1	4	0	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	G_1	G_2
4	x_2	1	0	1	0	1/2	0	0	-1	0
0	d_2^-	9/2	0	0	-5/2	-9/2	1	-1	-7	0
1	x_1	3/2	1	0	1/2	3/2	0	0	2	0
0	G_2	-1/2	0	0	**-1/2**	-1/2	0	0	0	1
					↑					↓

Since Maximum $\left\{\frac{(z_j - c_j)}{y_{4j}}, y_{4j} < 0\right\} = -1$

Table (6): Introduce d_1^- and drop G_2

			1	4	0	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	d_1^-	d_1^+	d_2^-	d_2^+	G_1	G_2
4	x_2	1	0	1	0	1/2	0	0	-1	0
0	d_2^-	7	0	0	0	-2	1	-1	7	-5
1	x_1	1	1	0	0	1	0	0	2	1
0	d_1^-	1	0	0	1	1	0	0	0	-2

The above table confirms that an optimum basic feasible integer solution has been achieved.
Hence, we get required optimum solution as

$$x_1 = 1, x_3 = 1, \; d_1^- = 1, d_2^- = 7$$

3.5 Solve the following GPP

A textile mill has three departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suiting, shirting and woollen yielding a profit of Rs. 4. Rs. 5 and Rs. 3 per meter correspondingly. One meter of suiting requires 1 minute in weaving, 1 minute in processing and 2 minutes in packing. Similarly one meter of shirting requires 1 minute in weaving, 1 minute in processing and 3 minutes in packing. One meter of woollen requires 1 minute in weaving, 0 minute in processing and 1 minutes in packing. In a week, total time to be used for each department is 10, 1 and 40 hours for weaving, processing and packing correspondingly. Formulate the goal programming problem to find the product mix to maximize the profit.

3.6 Solution

Let x_1, x_2 *and* x_3 denotes different types of clothes namely suiting, shirting and woollen correspondingly.

So, the constraints and goals of the problem can be expressed as follow:

$$Maximize \quad z = 4x_1 + 5x_2 + 3x_3$$

Subject to the constraints: $x_1 + x_2 + x_3 \le 10, \; x_1 - x_2 \le 1, \; 2x_1 + 3x_2 + x_3 \le 40, \; x_1, x_2, x_3 \ge 0$

Now formulation of the given problem as goal programming model is as follows:

$$Minimize \; Z = 4x_1 + 5x_2 + 3x_3$$

Subject to the constraints: $x_1 + x_2 + x_3 + d_1^- = 10, \; x_1 - x_2 + d_2^- - d_2^+ = 1$

$$2x_1 + 3x_2 + x_3 + d_3^- - d_3^+ = 40, \; x_1, \; x_2, \; x_3, \; d_1^-, \; d_2^-, \; d_2^+, \; d_3^-, \; d_3^+ \ge 0$$

$d_1^-, \; d_2^-, d_3^-$ are the amount by which we underachieve our objective.

d_2^+, d_3^+ are the amount by which we overachieve our target.

Table (1). Initial table:

			4	5	3	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_2^+	d_3^-	d_3^+
0	d_1^-	10	1	1	**1**	1	0	0	0	0
0	d_2^-	1	1	-1	0	0	1	-1	0	0
0	d_3^-	40	2	3	1	0	0	0	1	-1
					↑	↓				

Since $\min \sum x_{ij} = 2$

Hence the column vector x_3 enter in the basis and the column vector d_1^- leaves the basis.

Table (2): Introduce x_3 and drop d_1^-

			4	5	3	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_2^+	d_3^-	d_3^+
3	x_3	10	1	1	1	1	0	0	0	0
0	d_2^-	1	1	-1	0	0	1	-1	0	0
0	d_3^-	30	1	**2**	1	-1	0	0	1	-1
				↑					↓	

Since $\min \sum x_{ij} = 2$

Hence the column vector x_2 enter in the basis and the column vector d_3^- leaves the basis.

Table (3): Introduce x_2 and drop d_3^-

			4	5	3	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_2^+	d_3^-	d_3^+
3	x_3	-6	1/2	0	1	3/2	0	0	-1/2	1/2
0	d_2^-	16	**3/2**	0	0	-1/2	1	-1	1/2	-1/2
5	x_2	15	1/2	1	0	-1/2	0	0	1/2	-1/2
			↑				↓			

Since $\min \sum x_{ij} = 5/2$

Hence the column vector x_1 enter in the basis and the column vector d_2^- leaves the basis.

Table (4): Introduce x_1 and drop d_2^-

			4	5	3	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_2^+	d_3^-	d_3^+
3	x_3	-34/3	0	0	1	5/3	-1/3	1/3	-3/4	3/4
4	x_1	32/3	1	0	0	-1/3	2/3	-2/3	1/3	-1/3
5	x_2	29/3	0	1	0	-1/3	-1/3	1/3	1/3	-1/3

As, the optimum solution is not integer valued, we assume simply the fractional parts of

$$x_{B1} = \frac{-34}{3} = -12 + \frac{2}{3},\ x_{B2} = \frac{32}{3} = 10 + \frac{2}{3},\ \ x_{B3} = \frac{29}{3} = 9 + \frac{2}{3}$$

Now we select Maximum $\{f_1, f_2, f_3\} = \left\{\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right\} = \frac{2}{3}$, both f_1, f_2, f_3 are equal. So, we arbitrarily select any one of these. Let us choose f_3.

In third row, $\frac{29}{3} = 9 + \frac{2}{3} = 0x_1 + x_2 + 0x_3 - \frac{1}{3}x_4 - \frac{1}{3}x_5 + \frac{1}{3}x_6 + \frac{1}{3}x_7 - \frac{1}{3}x_8$

Now, introducing Gomorian slack variable G_1. Then we write

$$-\frac{2}{3}x_4-\frac{2}{3}x_5-\frac{1}{3}x_6-\frac{1}{3}x_7-\frac{2}{3}x_8+G_1=-\frac{2}{3}$$

Adding this additional constraint into the above optimum simplex table, we have

Table (5): Introduce G_1

			4	5	3	0	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_2^+	d_3^-	d_3^+	G_1
3	x_3	-34/3	0	0	1	5/3	-1/3	1/3	-3/4	3/4	0
4	x_1	32/3	1	0	0	-1/3	2/3	-2/3	1/3	-1/3	0
5	x_2	29/3	0	1	0	-1/3	-1/3	1/3	1/3	-1/3	0
0	G_1	-2/3	0	0	0	**-2/3**	-2/3	-1/3	-1/3	-2/3	1
						↑				↓	

Since Maximum $\left\{\frac{(z_j-c_j)}{y_{4j}}, y_{4j}<0\right\}=-1/2$

Table (6): Introduce d_1^- and drop G_1

			4	5	3	0	0	0	0	0	0
c_B	y_B	x_B	x_1	x_2	x_3	d_1^-	d_2^-	d_2^+	d_3^-	d_3^+	G_1
-3	x_3	13	0	0	1	0	-4/3	-1/2	-19/12	-1/4	5/2
4	x_1	11	1	0	0	0	1	-1/2	1/2	0	-1/2
5	x_2	10	0	1	0	0	2/3	1/2	1/2	2/3	-1/2
0	d_1^-	1	0	0	0	1	1	1/2	1/2	1	-3/2

The above table confirms that an optimum basic feasible integer solution has been achieved. Hence, we get required optimum solution as

$$x_1=11, x_2=10, \ x_3=13, \ d_1^-=1$$

4. Conclusion

In this article, modified Gomory Constraint technique for Goal programming problem has been suggested. It is observed that the proposed technique decreases number of iterations, saves valuable time as well as got optimum solutions. Thus, our technique is most powerful method and gives results in lesser time.

5. References

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