Comparison of Doubly and Intelligent Threshold Geometric Stochastic Process in the Study of Covid-19 Virus Infection

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Abstract: During the outbreak of a particular epidemic disease, Sometimes, the number of daily cases of a given epidemic shows several trends: a monotonous increase during the epidemic's development or outbreak period, accompanied by the stabilized stage of the number of daily cases of infection (stabilization), that is, controlling the epidemic to eradicate it, and then decreasing during the epidemic's decline. A comparison of the doubly geometric stochastic process and the intelligent threshold geometric stochastic process was performed in this paper using the chicken swarm optimization algorithm to determine the optimal stratigraphic boundaries for modelling data on the daily numbers of Coronavirus infections (Covid-19) on the three Iraqi governorates (Baghdad, Erbil, and Basra). To that end, a comparison of the two proposed models was performed to determine which model best fitted the data under study. It was discovered that the intelligent threshold geometric stochastic process model outperformed the doubly geometric stochastic process model in modelling epidemic data in the Baghdad governorate by (11.1%), while the supremacy in epidemic data for Basra and Erbil governorates was (3.98%) and (13.90%), respectively. this demonstrates the relevance of the theoretical postulates discussed on the theoretical side.

Keywords: Doubly Geometric Stochastic Process, Intelligent Threshold Geometric Stochastic Process, Parameter Estimation, Chicken Swarm Optimization Algorithm, Multiple Monotone Trends, Root Mean Square Method.

1. Introduction:

During the outbreak of a particular epidemic disease, for example, the severe acute respiratory syndrome Coronavirus (SARS-CoV-2), the number of daily cases of a particular epidemic often shows multiple trends: a monotonic increase during the growing stage or the outbreak of the epidemic, stationary in the number of daily cases called in Some sources are the stabilization stage, that is, controlling the epidemic to eliminate it, and then decreasing during the declining stage. In recent years, The Doubly Geometric Stochastic Process (**DGSP**) has begun to be used as a model to fit data from a series of repetitive events, Since it provides a more flexible application model than the geometric stochastic process (**GSP**).

The geometric stochastic process (GSP) has gotten a lot of attention since it was first introduced by **(Lam,1988)**. The GSP has been used in system reliability analysis **(Jain & Gupta, 2013),** maintenance policy optimization (**Liu & Huang, 2010; Wang, 2011; Zhang, Xie, &Gaudoin, 2013; Zhang, Yam, &Zuo, 2002**), and infectious disease modeling (**Chan et al., 2006**). Meanwhile, some authors (**Braun, Li, & Zhao, 2005; Finkelstein, 1993; Wang & Pham, 1996; Wu & Clements-Croome, 2006**) suggest expanded models address the GSP's limitations. the Doubly geometric stochastic process (DGSP) was introduced and investigated by (**Wu,2018**), it's an extension of the geometric process and which overcome the two limitations of (GSP).

On the other hand, GSP models focus on a single trend data set, are simpler to implement using non-parametric methods with a smaller set of parameters, allow for the identification of transmission patterns, and can be generalized to account for the data's stochastic nature. This distinction makes the GSP more appealing for application because it can model the failure process of aging or deteriorating systems, which may have shorter failure times.

Although the GSP is an important model that has been commonly used in various research areas to solve problems, its scope is still limited and does not meet the needs of various empirical studies. To begin with, this model is not appropriate for a stochastic process in which the inter-arrival times may need to be modeled using distributions with varying shape parameters. Second, it can only characterize stochastic processes that are increasing or decreasing stochastically.

This paper aims to compare the intelligent threshold geometric process with the doubly geometric stochastic process to model covid-19 data with multiple trends. This is the first time these models have been applied to epidemiological evidence. Consequently, GSP models focus on the number of infected cases, with transmissions through regions not taken into account. The following is how the paper will be presented. Section 2 defines the monotone stochastic process which includes GSP and Intelligent Threshold Geometric Stochastic Process (**ITGSP**). Section 3 introduces using the chicken swarm optimization algorithm (**CSO**) and illustrates the steps of the proposed chicken swarm optimization algorithm for finding the optimum stratigraphic boundaries for (**ITGSP**). The covid-19 infection data for three regions in 2020 which motivate this study are described in Section 4. In Section 5, each datum is fitted by using these two proposed models and numerical results are reported with comments. Section 6. Discuss the Results. Finally, a concludes the paper and proposes some directions for future works is given in Section 7.

2. Monotone Stochastic Process:

In this section, the GSP is a stochastic process defined as follows (**Lam,1988**): a sequence $\{X_k, k = 1,2,\dots\}$ of nonnegative random variables which represent the inter-arrival times of the process. The GSP $\{X_k, k = 1,2,\dots\}$ is said to be a GSP with the ratio (a) if there exists a real number $(a > 0)$ such that $a^{k-1}X_k$ for $k = 1,2,...$ form a Renewal Stochastic Process (RSP) with a common distribution function F where the distribution F is the distribution function of the first inter-arrival time X_1.(**Cheng & Li,2011)**

As can be seen, the distinction between the GSP and the RSP lies in the fact that the inter-arrival times of the RSP have the same distribution F(t) over k's and the inter-arrival times of the GSP have a cumulative distribution function (CDF) $F(a^{k-1}t)$, which changes over k's. In some scenarios such as reliability mathematics, this distinction makes the GSP more attractive in the application as it can model the failure process of aging or deteriorating systems, which may have decreasing working times between failures. The monotone stochastic process can be classified as follow: (**Lam,2007a**)

2.1 Doubly Geometric Stochastic Process model (DGSP):

Many authors have worked hard in recent years to develop new methods for modelling data from a series of events; for more details, see (**Wu and Wang,2018**) and (**Wu, 2018**). In this paper, the doubly geometric stochastic process (DGSP), which is proposed by (**Wu, 2018**) as an extension of the GSP and can overcome the limitations described above, is considered. This process's description and some theoretical properties are as follows:

Definition 1: The stochastic process $\{X_k, k = 1,2,\dots\}$ is said to be a DGSP with the parameter a if there exists a real number $(a > 0)$ such that $\left\{a^{k-1}X_k^{h(k)}\right\}k = 1,2,...$ form a RSP with a common distribution function F where $\{h(k) > 0\}$ is a function of k with $\{h(1) = 1\}$ for $k = 1,2,...$ and the common distribution function F is the distribution of the first inter-arrival time X_1 (**Wu ,2018**).

Definition 2: Given a sequence of non-negative random variables $\{X_k, k = 1,2,\dots\}$, If they are independent and the cumulative distribution function of X_k is given by $\left\{F\left(a^{k-1}X_k^{h(k)}\right) \text{for } k = 1,2,...\right\}$ where a is a positive constant, h(k) is taken as $(1 + \log(k))^b$ where b is a real number and log is the logarithm with base 10, then $\{X_k, k = 1,2,\dots\}$ is called a DGSP. (**Wu & Clements-Croome, 2006**)

Since the process can involve two geometric processes, we refer to it as a doubly geometric stochastic process. As an example, consider the following:

- The geometric series$\{a^{k-1}, k = 1,2,\dots\}$ is refer as the scale impact factor.

- The geometric series$\{h(k), k = 1, 2, \ldots\}$ is refer as the shape impact factor.

From Definition 2, we can find the following results:

- If $\{0 < a < 1\}$,$\{b < 0\}$ and $\{P(X_1 > 1) = 1\}$**or** if $\{0 < a < 1\}$,$\{0 < b < 4.898226\}$ and $\{P(0 < X_1 < 1$=1, then {Xk , k = 1, 2, . . . } is stochastically increasing.
- If $\{a > 1\}$,$\{b < 0\}$ and $\{P(0 < X_1 < 1) = 1\}$**or** if $\{a > 1\}$,$\{0 < b < 4.898226\}$ and $\{P(X_1 > 1) = 1\}$, then $\{X_k , k = 1, 2, \ldots\}$ is stochastically decreasing.
- If $(1 + \log(k + 1))^{-b}(\log(y) - k\log(a)) + (1 + \log(k))^{-b}\big((k - 1)\log(a) - \log(y)\big)$, the DGSP differs between negative and positive values, then it is not stochastically monotonous over k′s, where y denotes all possible values on $\{X_k , k = 1, 2, \ldots\}$.(**pekalp et.al,2020**)
- If h(k) = 1, then $\{X_k, k = 1, 2, \cdots\}$ reduces to the geometric stochastic process (GSP).
- The expectation and variance of the doubly geometric stochastic process can be calculated as follows:

μ

μ

Since then, the probability density function has been defined and is given as:

as well as assuming: (**pekalp et.al,2020**)

As a result, finding both the expectation and variance of the geometric stochastic process of the random variable (X_k) is simple as:(**Cheng & Li,2011**)

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V

2.2Intelligent Threshold Geometric Stochastic Process (ITGSP):

we will use the (ITGSP) Model by dividing the (GSP) into (k) layers, as it is one of the best methods for processing multiple trend data (MT). (**Chan et al., 2006**)

As a result, there is a pressing need for this subject because it is involved in the treatment and detection of epidemics that occur in our daily lives. As a result, the monotone stochastic process with a single-trend has been studied, and any attempt to extend it to account for the random nature of the data, which is multiple trends, has been made. Its application to epidemiological data prompts us to look for a method by which the single-trend can be generalized into multiple monotone trends. The monotonic stochastic model is used to achieve this. If the geometric stochastic process model is generalized to the threshold geometric stochastic process model, the process is divided into (k) layer as follows: (**Chan et al., 2006**)

G

To each (n_k)observational pattern that starts with a turning point (T_k), where:

Then the model can be represented as the renewal stochastic process (RSP) as follows :

where $(a_k > 0)$ is called the ratio of the k^{th} geometric stochastic process, which measures the trend and strength of the trend of the process, while the expected value and variance of the threshold geometric stochastic process can be given as follows:

Thus $\{a_k, \mu_k, \& \sigma_k^2\}$ represent the important parameters of the threshold geometric stochastic process which completely determine both the mean and variance of $\left(X_{T_k+t}\right)$ in the k^{th} geometric stochastic process. (**Lam,2007a**)

We propose in this paper to use the chicken swarm optimization algorithm with the moving window technique to estimate the turning points $\{T_k\ ; k = 1,2, \dots, K\}$, which represent the time when the trend changes its direction. the threshold geometric stochastic process models are applied successively to a subset of data of fixed length (L) starting from time $\{s = 1;\ s = 2\}$ and so on up to $\{s = n - L\}$. Since the ratio $(a_s > 0)$ of a (GSP) will change with each (s)as the window moves, turning point s (T_s) can be located by a change of (a_s) from less than one to greater than one and vice versa which occur at $s = T_s;\ k = 1,2, \dots, K$. Then the GSP model are given by:

As a result, depending on the nature of the data, we can detect more than one turning point. In some experiments, the ratio parameter (a_k) can repeatedly shift about one, resulting in short trends and unstable parameter estimates. To prevent these "noisy" shifts, short trends are coupled with the next trends and some constraints are imposed on them so that they are not short and unpredictable. One of these constraints is that the number of observations in each point cannot be less than $\{h = 7\}$. Furthermore, the shift in the geometric stochastic process ratio (a) for each point exceeds the value of (0.001), indicating that:(**Chan et al., 2006**)

Furthermore, different values of h and d will be measured before reasonable results of high accuracy are obtained. Since different window widths (L) offer a different set of turning points (T_i), the window widths L is very important in deciding the degree of accuracy of the results. If the window width (L) is small, the change is measured more precisely. Meanwhile, noise can be captured in a variety of ways. On the other hand, if the window width (L) is very wide or large, then the average changes will be calculated very largely, which leads to the loss of a lot of information and changes in trends because the increase at certain times may be offset by the decrease in other periods, so this will lead to a bias. Inaccuracy in the results. If the window length is equal to the total sample size (n), then only one trend can be identified, meaning that there is one threshold and there are no turning points (T_s).

To analyse the data set, different values were assumed for the basic parameters, as the length or width of the window (L) was set within a range of values (20-30) and for each window, there is a set of corresponding trends as well as a set of parameters for each trend or turning point.

(3) Using CSO Algorithm to Find ITGSP :

The CSO algorithm is one of the very effective intelligence optimization algorithms, which has good performance in solving global optimization problems (**GOPs**). The swarm intelligent optimization algorithm, such as genetic algorithm (**GA**), particle swarm optimization (**PSO**), bat algorithm (**BA**), artificial bee colony

(**ABC**) algorithm et al., It is an optimization algorithm constructed by simulating the swarm behaviour of natural organisms. These algorithms search for the optimal solution to an optimization problem by simulating the physical laws of natural phenomena, the living habits, and behavioral characteristics of various biological populations in nature. The swarmoptimization intelligent algorithms provide a new way to solve global optimization problems in the fields of computational science, mathematical science, and so on. The swarm intelligent optimization algorithms have become a research hotspot and are particularly important. The chicken swarm optimization (**CSO**) algorithm is a stochastic search method based on chicken swarm search behavior, which was proposed by (**Meng, et al.,2014**). In CSO, the whole chicken swarm is divided into several groups, each of which includes a rooster, a couple of hens, and several chicks. Different chickens follow different laws of motion. There exist mutual learning and competitions between different chickens, and the hierarchy of the chicken group is updated again after several generations of evolution. The CSO algorithm has great research potential because of its good convergence speed and convergence accuracy. However, like other swarm intelligent optimization algorithms, the basic chicken swarm optimization algorithm has the disadvantages of premature convergence, whose iteration is easy to fall into a local minimum, in solving the large-scale optimization problem with more complexity. Also, the algorithm for chicken swarm optimization can be illustrated as follows: (**Liang et al., 2020**)

The unconstrained continuous optimization problems can be expressed as follows:

$$\min f(X) \qquad X \in R^D$$

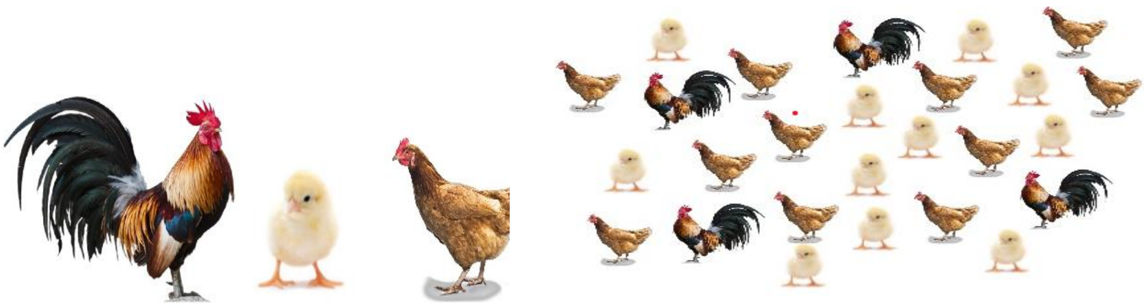
$$\text{if } X^* \in R^D \text{ satisfies that: } f(X^*) \le f(X) \quad \forall X \in R^D \qquad \dots \quad (15)$$

$X^*$is called the global minimum point of $f(X)$in the whole space R^D.

The CSO algorithm is based on the hierarchical order and actions of a swarm of chickens looking for food. Different chickens obey different laws of motion depending on their hierarchical order. To idealize chicken action, the CSO algorithm employs the following four rules: (**Liang et al., 2020**)

1) The chicken swarm is divided into many subgroups. A dominant rooster, a couple of hens, and chicks make up each subgroup.
2) The fitness values of the chickens themselves decide how to split the whole chicken swarm into many classes and how to determine the species of chicken. Several individuals with the best fitness values are marked as roosters in the whole chicken swarm; the chickens with the worst fitness values are acted as chicks, and the rest are hens. The hen selects its subgroup at random, as does the mother-child relationship between the hen and the chick.
3) Until the roles are reassigned, the hierarchical order, dominance relationship, and mother-child relationship in a group remain unchanged for many generations.
4) The hens pursue their rooster-mate in their quest for food, while the chicks look for food in the vicinity of their mothers. When it comes to finding food, the dominant individuals have an advantage.

Three types of chickens make up the whole swarm of chickens. When the CSO algorithm solves the optimization problem (1), each chicken represents a possible solution, and different chickens use different optimization strategies. The number of chickens in the CSO algorithm is assumed to be N, and the chickens are ordered in ascending order based on their fitness values. The NR chickens in the front are roosters, the NC chickens in the back are chicks, and the ($NH = N - NR - NC$) chickens in the middle are hens. Let's take a look at Chicken Movement**.**



The Roosters with higher fitness values have preference over those with lower fitness values when it comes to food. For the sake of convenience, this condition may be simulated by cocks with higher fitness values being able to forage in a wider variety of locations than cocks with lower fitness values. This can be expressed as follows:

$$x_{i,j}^{t+1} = x_{i,j}^{t} * (1 + \mathrm{Randn}(0, \sigma^2)) \quad \ldots \quad (16)$$

$$\sigma^2 = \begin{cases} 1 & ; \quad \text{if } f_i \le f_k \\ \exp\left(\frac{(f_k - f_i)}{|f_i| + \varepsilon}\right) & ; \quad \text{otherwise} \end{cases} \quad k \in [1, N], k \ne i \quad \ldots \quad (17)$$

Where Randn $(0,\sigma^2)$ represents a Normal distribution with zero mean and variance (σ^2).(ε), which is used to avoid zero-division error, is the smallest constant in the computer. A rooster's index (k) is chosen at random from the rooster's group, and the fitness value of the corresponding x is (f). The hens, on the other hand, will hunt for food alongside their roosters. Furthermore, they would steal good food found by other chickens at random, while being repressed by the other chickens. The dominant hens will have an advantage over the submissive hens when vying for food. These phenomena can be mathematically expressed as follows:

$$x_{i,j}^{t+1} = x_{i,j}^{t} + S_1 * \mathrm{Rand} * \left(x_{r1,j}^{t} - x_{i,j}^{t}\right) + S_2 * \mathrm{Rand} * \left(x_{r2,j}^{t} - x_{i,j}^{t}\right) \quad \ldots \quad (18)$$

$$S_1 = \frac{\exp(f_i - f_{r1})}{(\mathrm{abs}(f_i) + \varepsilon)} \quad \ldots. \quad (19)$$

$$S_2 = \exp\big((f_{r2} - f_i)\big) \quad \ldots. \quad (20)$$

Rand is a uniform random number between 0 and 1. $\{r1 \in [1, \ldots, N]\}$is the index of the rooster, which is the ith hen's group-mate, while $\{r2 \in [1, \ldots, N]\}$is the index of the chicken (rooster or hen), which is selected at random from the swarm. $(r1 \ne r2)$.

Obviously, $(f_i > f_{r1})$,$(f_i > f_{r2})$, resulting in S_2<1<S_1. If $S_1 = 0$, then the ith hen will be the first to forage for food, followed by other chickens. The greater the disparity in fitness values between the two chickens', the smaller S_2, and the greater the distance between their positions. As a result, the hens will be less likely to snatch food from other chickens. The reason that the formula form of S_1 differs from that of S_2 is that there exist competitions in a group. the fitness values of the chickens about therooster's fitness value are simulated as the competitions between chickens in a group. If $S_2 = 0$, then the ith hen will look for food within its territory. the fitness rooster's value is unique and it's special in this category. As a result, the smaller the ith hen's fitness value, the closer S_1 approximates to 1, and the smaller the difference between theith hen's location and that of its group-mate rooster. Hence the more dominant hens would be more likely than the more submissive ones to eat the food.

To forage for food, the chicks move around their mother. This is how it's put together:

$$x_{i,j}^{t+1} = x_{i,j}^{t} + FL * \left(x_{m,j}^{t} - x_{i,j}^{t}\right) \quad \ldots \quad (21)$$

The location of the ithchick's mother($m \in [1, N]$) is defined by $x_{m,j}^{t}$. FL $\big(FL \in (0,2)\big)$ is a parameter, which means that the chick would follow its mother to forage for food. Taking into account individual variations, each chick's FL will fluctuate between 0 and 2.

The chicken swarm optimization algorithm consists of several simple steps that are interconnected with one another, and the algorithm cannot be applied to any problem unless all of these steps are followed; otherwise, the algorithm loses its value and usefulness in finding or improving the solution. The proposed chicken swarm optimization algorithm for determining the best stratigraphic boundaries for a threshold geometric stochastic process is as follows:

1. **Parameters for the CSO Algorithm:**
   The CSO algorithm has six basic parameters that will be discussed. Because humans primarily preserve chickens as a source of food, it is also possible to rely on chickens to provide the required eggs, which can be another source of food; thus, raising chickens is more beneficial to humans than raising roosters. As a result, (HN) will be greater than (RN), and because not all hens hatch their eggs at the same time, (HN) is also greater than (MN). Although each chicken can raise more than one chick, we assume that the number of adult hens will be greater than the number of chicks (CN) as for (G),It should be set to a value that is suitable for the problem. If (G) is very high, the algorithm may not be able to rapidly converge to the optimal global level; if (G) is very small, the algorithm might be similar to the optimal local algorithm. It may be inside (G [2,20]). Furthermore, if both (RN) and (MN) are set to zero, the formula for chick movement can be applied to the corresponding section in (DE), as well as through experiments. It is possible to calculate the value of (DE = 2). If the parameter value (FL) is chosen within the range FL [0.4,1], the algorithm performs well. (**Xin-She Yang, 2008**)
2. **Initial Data:** This is a reading of the data values for the problem under study.
3. **Initialize data:** At first, sudden fluctuations in the data are eliminated by using the moving mean for stochastic series, and then the Moving Window Technology is used, which works by taking a set of data with a specific size (30 observations, for example) that represents displaying that window and finding the parameters for that group, then leaving the first value from the group and taking the moving mean. Following that, the differences in the geometric stochastic process parameter (a_k) ratio between each group and the next group were determined using equation No (14),If the difference is greater than the defined threshold value, the specified point is regarded as a turning point.
4. **Determine Inversion Points:** This is one of the most important steps in the proposed procedure, and it is focused on locating inflection points in the data and identifying the points as increasing, decreasing, or stable based on the geometric stochastic process parameter ratio (a).
5. **Individual Representation:** The structure and shape of the individual in the issue of stratification is divided into two parts: the first part represents the inflection points chosen in stage (3) and at which the data is divided, and the second part represents the geometric stochastic process parameters that have been evaluated for the various stratifications.
6. **Generation of the Initial Population**: The size of the algorithm's community (the number of particles) and the method for generating the primary population have a major effect on the algorithm's output and the quality of the results. Each person must have some sites equal to $((L - 1) + L * 2)$, where L is the number of classes the community is to be centered on. In general, an optimal group should ensure as much diversity of individuals (solutions) as possible in the search Space.
7. **Objective Function**: We determine the objective function for the problem and each particle in the community in this step, and the objective function in the stratification problem is a criterion of Root Mean Square Error (**RMSE**). One of the most critical metrics used in comparing models and calculating model accuracy, since this criterion is used in the case of one-way data as well as in the case of multiple trends, implying that there is a limit to the threshold between the data, and it is determined using the following formula: (**Lam,2007a**)

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \hat{\mu}_k \hat{a}_K{}^{-(i-T_K)}\right)^2} \text{ for } T_k \le i \le T_{k+1} \qquad \dots (22)$$

8. **Applying chicken swarm optimization algorithm procedures (Applying CSO Procedures):**
   After creating an elementary community of size (N_{pop})individuals, the procedures of the chicken swarm optimization algorithm will be applied according to the equations shown on (16) to (21).

(4) Covid-19Virus infection Case :

Coronavirus disease (**Covid-19**) is one of the most serious global health threats due to the ease of transmission and the long incubation period of the epidemic, making it one of the most widespread epidemics, as the first case of the epidemic was recorded in Wuhan, China, on December 31, 2019, and began the spread of a global epidemic. Beginning on February 24, 2020, in the city of Najaf, Iraq, when a sample of an Iranian religious student was examined and the result was positive for his infection with Coronavirus disease associated with severe acute respiratory syndrome type II (**SARS-CoV-2**), other cases infected with an epidemic were detected. Corona (Covid-19) in various regions of Iraq. As of 12/31/2020, the total number of confirmed total cases in Iraq was (**595291**), while the total number of deaths in Iraq was (**12813**).

Our investigation focuses on modelling data on daily infection numbers in Iraq as a result of the Corona epidemic, as well as for the three governorates (Baghdad, Erbil, and Basra). The number of daily cases of the Coronavirus epidemic in Iraq and the Baghdad Governorate is depicted in Figure No. 1. Being increasing in one period (the period of the epidemic's onset) and stable in another (the period of taking the necessary measures to control the epidemic), as well as decreasing in others(periods of epidemic decline before the disease fades away).

The covid-19 data for the three regions include information on daily infected cases, daily death cases, daily recovery cases, and daily cases in care. Our research focuses on modelling the number of regular infected cases for each of the three regions separately. Table 1 contains a description of the data.

Table.1. The basic details for the three region's covid-19 data**.**

Governorate	**Basra**	**Erbil**	**Baghdad**
Start date	2020\3\15	2020\3\15	2020\3\15
End date	2020\12\31	2020\12\31	2020\12\31
n = number of data	214	214	214
Number of cases in total Sn	38980	35987	177628
Number of cases per day$\left(Sn/n\right)$	182.149	168.163	830.037

(5) Parameter estimation using the two stochastic models:

In this section, a comparison was made between the proposed model represented by the intelligent threshold geometric stochastic process using the chicken swarm optimization algorithm to determine the stratigraphic boundaries as well as estimate the parameters and the doubly geometric stochastic process model to arrive at the best model representing the data under study.If a proposal is also made to estimate the coefficients of the doubly geometric stochastic process model using the chicken swarm optimization algorithm. Table.2. shows the parameters of the intelligent threshold geometric stochastic process model estimated using the chicken swarm algorithm, as well as the best stratification based on the best window width (h) based on the root mean squares error (RMSE) as a criterion for models comparison.

Table 2: The parameter estimates for the intelligent threshold geometric stochastic process model.

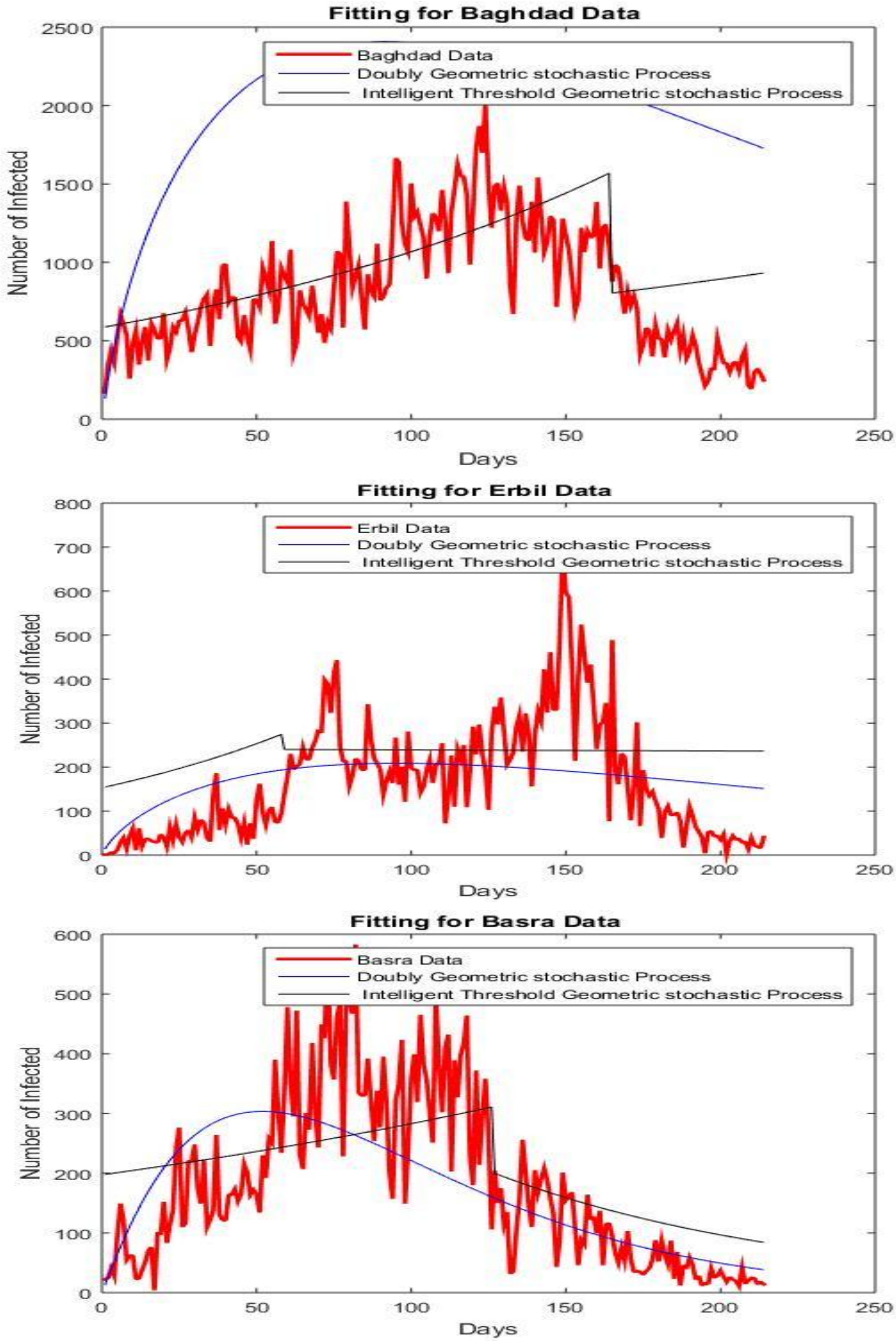
governorates	Baghdad	Erbil	Basra
Number of layers(L)	3	3	3
width Window (h)	30	20	20
Class boundaries	164,173	58,186	126,194
	[0.994,0.997,1.186]	[0.99,1.0001,1.145]	[0.9964,1.01,1.2]
	[588.16,491.45, 566.59]	[154.66,241.39,689.44]	[197.99,700,233.28]
	297.94	99.01	99.18

While Table 3 shows the parameters of the doubly geometric stochastic process estimated using the chicken swarm algorithm under the assumption of related parameter value equal to {G = 10, RN = 0.2N, HN = 0.7N, MN=0.5HN, CN=N−RN−HN and FL∈0.4,1based on the RMSE, with the following results:

Table .3. Using the chicken swarm algorithm, estimates of the parameters of a doubly geometric stochastic process for the daily infection numbers of the Coronavirus epidemic in the three governorates.

governorates				RMSE
Baghdad				335.3904
Erbil				114.9962
Basra				103.2879

Figure .1. The extent of fitting data in the three governorates and to the two stochastic models.



(6) Discuss the Results:

Table.2. displays the estimates of the intelligent threshold geometric stochastic process parameters, as well as the stratified limits. The results revealed that the data are divided into three layers, and these layers are the same that reflect the actions of any epidemic in terms of spread at the beginning of the time, as indicated by the value of ($a < 1$) as the process increases, while the second layer indicates the process's stability (i.e. the initiation). By monitoring the epidemic, we can conclude that the Renewal Stochastic Process is the best model for this period (RSP),Because of the value of the parameter ($a \cong 1$), it is eventually possible to see that the data's behavior begins to decrease and fade, and this is due to the value of the ratio parameter of Geometric stochastic process, which is ($a > 1$).

When the proposed model was compared to the doubly geometric stochastic process model, the results revealed that the proposed model had a higher prevalence because it works on studying each period separately and estimating its parameters, resulting in accurate estimates and high fitting of the original data, while the doubly geometric stochastic process model has less accuracy by using (RMSE).

(7) Conclusion:

In this study, a comparison of doubly and intelligent threshold GSP, a novel model in stochastic processes, is considered when using CSO to determine the optimal stratigraphic boundaries for intelligent threshold GSP and estimate the parameters of DGSP. The following conclusions were reached: First, the study discovered that the spread of infected people in the population is the main cause of coronavirus disease outbreaks, as it is the direct cause of many cluster infections in hospitals and local communities in (Baghdad, Erbil, and Basra). Second, the results of this experiment indicate that the spread of the disease may be affected by different environmental factors such as temperature, humidity, and other external factors such as precautionary measures that may change over time. Third, we also note from the theoretical side the superiority of the proposed model in estimating the parameters as well as fitting the model over the doubly geometric stochastic process model, as the results showed the superiority of the proposed model by (11.17%), in modelling the epidemic data in Baghdad governorate, while the superiority in the epidemic data was for the governorates of Erbil and Basra. with respective rates of (13.90%) and (3.98%). This illustrates the significance of the theoretical postulates discussed on the theoretical side.

Since DGSP and ITGSP are new concepts, many questions must be discussed. For example, what are the differences in DGSP application reliability between one DGSP and the other models? How do we know if a dataset is in agreement with DGSP before fitting it with DGSP? Other intelligent algorithms are used to decide the best ITGSP class boundaries. We will work to answer these questions in the future.

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