Bianchi Type III String Cosmological Model In f(R,T) Gravity theory with Bulk Viscous fluid

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Abstract :- we investigated a "Bianchi type III of strings cosmological model in f[R,T] gravity theory bulk viscous"[7]. To get a deterministic model, We suppose that the "barotropic equation of state for bulk viscous pressure and pressure density is proportional to the"[16] energy density i.e. $\bar{p} \propto \rho$ and in this model string do not carry on. Also a special law of variation is used which is given by Berman [1983] is also taken to the for equation of this theory. 'Harko et al. [2011]', was proposed the scalar tensor theory, where "energy momentum tensor is the source for bulk viscous fluid and one dimensional cosmic strings in frame work of f[R,T] gravity also include"[5].
Keywords :- "Bianchi type –III, String Cosmological model, Bulk viscous, f[R,T] gravity"[3].

➢ 1.Introduction:-

Bianchi-III space time plays a major role for study of the cosmological models. These models have an important role to play in conventional cosmology appropriate for expressing the development of the universe in early stage. It becomes interesting when the anisotropic and homogeneous character of Bianchi-III Cosmological models was discus.

General relativity theory of Einstein is successful elucidating gravitational phenomena but it disappoints to settle some of the issuse in cosmology such as "the accelerating expansion of the universe"[24]. Without involving dark energy we have to describe current accelerated expansion for the "f[T] gravity theory is being recommended where T is the scalar known as torsion"[8].

In this paper, we have to determine "a Bianchi type III Bulk viscous string cosmological model in f[R,T] gravity theory"[13] including anisotropic and homogeneous models represent the generalisation of FRW cosmology of the universe and also distress "the large scale structure of the universe"[23]. Our paper derived explicit equation of field equation with reference to "bianchi type III bulk viscous string cosmological model of f[R,T]gravity theory"[3].

'In section 2 and section 3, discuses about the field equation and model. In section 4, some needed properties of our model are also discussed and last part contain conclusion'.

In year(2011) "Harko et al. suggested a new modified theory of general relativity is f[R,T] gravity"[9]. Here in the gravitational Lagrangian, "the trace of the stress energy tensor T is specified by"[19] an "arbitrary function and Ricci scalar R"[3] . Albert-Einstein type variational principles is the source of derivation of gravitational in field equation

$$S = \frac{1}{16\pi}\int f[R,T]\sqrt{-g}\, d^4x + \int L_m\sqrt{-g}d^4x \quad [1]$$

here in equation (1) "f[R,T] is an arbitrary function of the Ricci scalar R and T is the trace of stress-energy tensor of the matter"[2]. Also " T_{ij} and L_m are the matter Lagrangian density and stress-energy tensor T_{ij} of matter is given as"[15]:

$$T_{ij} = \frac{-2}{\sqrt{-g}}\frac{\delta[\sqrt{-g}L_m]}{\delta g^{ij}} \quad [2]$$

and its respective trace is given by T $=g^{ij}T_{ij}$. By presuming that "L_m of matter is depends only upon g_{ij}, the metric tensor components and not on its derivatives"[2], we acquired

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}} \quad [3]$$

Here g^{ij} is represent by gravitational field which is component of metric tensor with varying action S of gravitational field, "we have of f[R,T] gravity as"[25]

$$f(R,T)R_{ij} - \frac{1}{2}f(R,T)\,g_{ij} + \left(g_{ij}\Box - \nabla_i\nabla_j\right)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij} \quad [4]$$

Where $$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\,\partial g^{lm}}$$

[5]

Here, The covariant derivative is $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}\Box = \nabla^i\nabla_i, \nabla_i$ and "T_{ij} is standard matter energy momentum tensor . Evaluated from the Lagrangian L_m"[2]. It is found that the equation [4] yields the "field equation of f[R]gravity at the time when f[R,T] ≡ f[R]"[8]. Here a complicated problem of perfect fluid arise due to energy density ρ, four velocity u^i, pressure p as because of "no unique definition of matter Lagrangian. Nevertheless, here we presume that the stress energy tensor of the matter is"[2]

$$T_{ij} = (\rho + p)u_iu_j - pg_{ij} \quad [6]$$

and the Lagrangian matter is taken as $L_m = -p$ and we have

$$u^i\nabla_j u_i = 0, \qquad u^iu_i = 1 \quad [7]$$

"by using equation [5] the variation of stress-energy of perfect fluid is expressed as"[5]

$$\theta_{ij} = -2T_{ij} - pg_{ij} \quad [8]$$

Normally, the field equation depend on θ_{ij}, i.e. on the physical behavior of the matter field. Hence the f[R,T] gravity theory depends on the properties of the matter source, we obtained many theoretical model corresponds to the choosing of f[R,T], let us first suppose

$$f[R,T] = R + 2f(T) \quad [9]$$

"here f[T] is arbitrary function of matter of the trace of stress-energy tensor. From equation [4] we obtain the field equation of f[R,T] gravity as"[2]

$$R_{ij} - \frac{1}{2} R\, g_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij} \qquad [10]$$

Here, differentiation along with respect to the case is denoted by the prime. Uncertainty a matter source is a perfect fluid,

$$\theta_{ij} = -2T_{ij} - pg_{ij}$$

Then the field equation takes the form

$$R_{ij} - \frac{1}{2} R\, g_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [(2pf'(T) + f(T)]g_{ij} \qquad [11]$$

- **2. Field equation and Metric**

We observed the Bianchi type-III space time whose metric is

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{-2rx}B^2(t)dy^2 - C^2(t)dz^2 \qquad [12]$$

"here the metric potential A ,B and C are functions of time t and r is treated as constant"[5].
The energy momentum tensor which contains one dimensional cosmic strings for a bulk viscous fluid is taken as

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p}g_{ij} - \lambda x_i x_j \qquad [13.]$$

$$\bar{p} = p - 3\zeta H \qquad [.14]$$

here ρ ,is system's rest energy, density, coefficient of bulk viscosity is ζ[t], 3ζH is pressure of bulk viscous, Hubble's parameter is H, which is the four velocity of the fluid is u^i, the direction of the string is represented by x^i and string tension density is represented by λ . Here four-velocity vector is $u^i = \delta^i_4$ which satisfy the equation given below

$$g_{ij} u^i u_j = -x^i x_j = -1, \qquad u^i x_i = 0 \qquad [15]$$

"Here ρ, $\bar{p}$ and λ are time dependent terms only"[4]. "The f[R,T] gravity field equation [11]along with particular choice of the function (Harko et al. 2011)"[2] we consider with moving coordinates and equation 13 and 15.

$$f(T) = \mu T, \qquad \mu, \text{ is a constant} \qquad [16]$$

so equation [12] lessens to the arrangement,

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\gamma^2}{A^2} = -\bar{p}\,(8\pi+7\mu) + \lambda(8\pi+3\mu) + \mu\rho \qquad [17]$$

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-\,\bar{p}\,(8\pi+7\mu)+\lambda\,\mu+\mu\rho \quad [18]$$

$$\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}+\frac{\ddot{A}}{A}=\lambda\,\mu-\bar{p}\,(8\pi+7\mu)+\mu\,\rho \quad [19]$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{\gamma^2}{A^2}=\rho\,(8\pi+7\mu)-5\,\bar{p}\,\mu+\mu\lambda \quad [20]$$

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=0 \quad [21]$$

here differentiation along with respect to t is denoted by an over head dot.

Now for the metric [12], Respective Spatial volume and scale factor are, expresses by

$$V^3=ABC \quad [22]$$

$$a=(ABC)^{1/3} \quad [23]$$

In cosmology, the observational salient physical quota are the expansion scalar θ, "the shear scalar σ^2 and mean anisotropy parameter A_h are represented as"[22]

$$\theta=3H=3\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \quad [24]$$

Here H, the mean Hubble Parameter is represented by

$$3A_h=\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \qquad \Delta H_i=H_i-H, \qquad i=1,2,3 \quad [25]$$

$$2\sigma^2=\sigma^{ik}\sigma_{ik}\sum_{i=1}^{3}H_i^2-3H^2=3A_h-H^2 \quad [26]$$

➢ 3. Solution of field equation and metric

The solution of the field equation [17]–[21] reduces into the independent equation given as

$$\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}\dot{C}}{BC}-\frac{\gamma^2}{A^2}=\lambda\,(8\pi+2\mu) \quad [27]$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{\gamma^2}{A^2}=\rho\,(8\pi+7\mu)-5\,\bar{p}\,\mu+\mu\lambda \quad [28]$$

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{C}}{AC}-\frac{\dot{B}\dot{C}}{BC}=0 \quad [29]$$

$$A=kB \quad [30]$$

here k is integration constants. Which here is can be selected as unity without loss of simplification, from equation [30] we get

$$A=B \quad [31]$$

"So we obtain four independent from equation [27] – [30] which are highly non linear containing six unrevealed, A, B, C, p, ρ, and λ. Now to obtain a definite solution, we use the following possible physical conditions:

[i] The shear scalar σ^2 and scalar expansion θ are proportional.

$$B = C^m \quad [32]$$

Where non –zero constant is m i.e. m≠0.

(i) In year (1983) Berman propounded the variation of Hubble's parameter that provides the sustained deceleration parametermodels of the galaxy granted by

$$q = -a\frac{\ddot{a}}{\dot{a}^2} = \text{constant} \quad [33]$$

From which a mixture is acquired $a = (ct + d)^{\frac{1}{1+q}}$ [34]

(ii) Here for the barotropic fluid , $\bar{p} = \rho$, indicate the united outcome of bulk viscous pressure and proper pressure is permitted even as-

$$\bar{p} = p - 3\zeta H = \varepsilon\rho \quad [35]$$

here $\varepsilon = \varepsilon_0 - \beta$ $(0 \le \varepsilon_0 \le 1)$ $p = \varepsilon_0\rho$ [36]

here ε_0 and β are constants. Here using the equation [23], [31], [32] and [34] we have

$$A = B = (ct + d)^{\frac{3m}{(1+q)(2m+1)}} \quad , \qquad C = (ct + d)^{\frac{3}{(1+q)(2m+1)}} \quad [37]$$

Using equation [32] in equation [29], gives m = 1, since 1+q>0. By appropriate choice of coordinates and constants or by applying this value of m in equation [37] (i.e. grasping d =0 and c=1) the equation [12] can be transformed into

$$ds^2 = dt^2 - t^{\frac{2}{(1+q)}}[dx^2 + e^{-2rx}dy^2 + dz^2] \quad [38]$$

➢ 4. Kinetically property of the model

From the model (38), we have

The spatial volume— $V^3 = t^{\frac{3}{(1a+q)}}$ [39]

The Scalar of expansion- $\theta = \frac{3}{(1+q)\,t}$ [40]

The Mean Hubble parameter- $H = \frac{1}{(1+q)\,t}$ [41]

The Mean anisotropy parameter- $A_l = 0.$ [42]

The Shear scalar $\sigma^2 = 0.$ [43]

The String tension density- $\lambda = 0.$ [44]

The Energy density- $\rho = \frac{1}{8\pi+\mu(7-5\varepsilon)}\left[\frac{3}{[(1+q)t]^2} - \gamma^2 t^{\frac{-2}{(1+q)}}\right]$ [45]

Pressure $p = \frac{\varepsilon_0}{8\pi+\mu(7-5\varepsilon)}\left[\frac{3}{[(1+q)t]^2} - \gamma^2 t^{\frac{-2}{(1+q)}}\right]$ [46]

Coefficient of bulk viscosity $\zeta = \frac{\varepsilon_0-\varepsilon}{3[8\pi+\mu(7-5\varepsilon)]}\left[\frac{3}{(1+q)t} - \gamma^2(1+q)t^{\frac{-(1-q)}{(1+q)}}\right]$ [47]

The new obtained results equation [38] are beneficial to describe f[R,T] gravity. Equation [39] shows expanding model with time t since 1+q>0. . At t =0, the model does not show any initial singularity. Equation [44] shows that a string of the models vanishes. Also at the model becomes shear free and isotropic. These observations [Caldwell et al. 2006] shows the transition phase of decelerated to accelerated phase along with decrees of time we noticed that ζ ,θ , ρ, p and H also decreases and at t →∞, approaches to zero and when t →0, all these parameters i.e. ζ ,θ , ρ, p and H become infinitely large.

5. Conclusions

We have obtained a model which is anisotropic and homogeneous in a scalar tensor theory of gravitation. “Here a bulk viscous fluid containing one dimensional cosmic string is the source of energy momentum tensor”[12]. Also we got that the string density $\lambda = 0$ which concludes that the string vanishes. We investigate that the average anisotropy parameter vanishes at t=0 so that all over the development of the universe, the model does not remain isotropic. As $\sigma = 0$ which shows that the model becomes shear free. Moreover, at the initial epoch t=0 these models are singularity free and expands with time t. also at t →∞, we get a inflationary model as the bulk viscosity decreases with time.

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