Prospective Mathematics-Teacher Students' Reversible Thinking in Solving Math Insurance Problem

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Abstract: This article describes prospective-teacher students' reversible thinking in solving problems. The problem was focused on mathematical insurance. This qualitative research involves prospective-teacher students with high math competence in major of mathematics education who had passed mathematical insurance course. The data was collected by providing two reversible problems and then having an interview. Those two problems were reversible. The first involved inverse, while the second not. The data was analyzed by referring to the procedure problem solving by Polya and the aspects of reversible thinking such as inversion and reciprocity. The result found that the students were not capable to solve either inverse or reciprocal problems. Otherwise, they could only solve problems that had no relation to inverse. Thus, students' reversible thinking in solving mathematical insurance problems was found in trouble.

Keywords: Reversible Thinking, Prospective-Teacher Students, Mathematical Insurance Problem

1. Introduction

Why was prospective mathematics - teacher students' reversible thinking important? First, reversible thinking was a mental ability that required reasoning in two opposite directions [1] so that this ability could help a person solve complex problems. Second, reversible thinking was a part of mathematical abilities that influence student success in solving mathematical problems [2]. While students' ability to solve mathematical problems was one of the goals of mathematics education [3,4,5]. This means that the reversible thinking of prospective mathematics teacher students might be considered and developed so that they could develop students' abilities in solving mathematical problems to the maximum when they were already teachers.Third, thinking reversibly helped building relationships that support the mathematical understanding [2,6]. This was supported by Hiebert & Carpenter [7] who explained that thinking was reversible as a mechanism for developing the type of connection between concepts and procedures studied, this is a characteristic of mathematical understanding. Researchers had also identified reversibility as a key to understanding multiple mathematical principles and formulas [2,8,9,10]. While understanding mathematics was needed by students to get knowledge, and knowledge was needed as basis to solve problems. As argued by Schoenfeld [11] and Silver [12] that a key element in the problem solving process was prior knowledge. So, reversible thinking was important for students and mathematics teachers. The fact, the reversible thinking of prospective mathematics teacher students still there were problem [13,14]. Thus, students were required to have reversible thinking, so the teacher's duty was to think about how to optimize students' reversible thinking. If teachers were required to optimize students 'reversible thinking, then prospective mathematics-teacher students' reversible thinking might also be considered so that they could develop students' reversible thinking optimally. Therefore, the reversible thinking of prospective mathematics teacher students must be considered and optimized as their provision when they become teachers. Thus, the purpose of this research was describing the reversible thinking of prospective mathematics teacher students in solving problems. Focused this problem was mathematics by a consideration that mathematics insurance was tightly related to daily life, and thus, it was expected to bring direct benefits for students. So it's a need of an hour to increase healthy dietary awareness among prospective teachers.

2. Literature Review

Piaget [15] defined the term reversible thinking as a competence to reverse a pertinent operation to its initial state. Furthermore, Slavin [16] argued that it was a competence to do a mental operation and then reverse it back to the initial state. In short, reversibility refers to an individual's mental competence to alter his/her mind back to where it derives from. In addition, Slavin gave an example. A student identified that if $7 + 5 = 12$, then $12 - 5 = 7$. It was due to the fact that $12 - 5 = 7$ was equal to $7 + 5 = 12$. His thinking implied that if he added five items to seven items, and then put five items back to the rack, it would be seven items in his trolley. It is called reversible thinking; reversing what he has done from adding to five items and then putting five items back to the rack.

In relation to reversible thinking, Piaget [15,17] classified two concepts within; negation and reciprocity. Negation involved an understanding that a one-way move could be foregone by a reversal move. In this case, reversibility reveals an idea that every operation has inverse to be used for undergoing the operation. For example, addition is forgone by subtraction; multiplication is forgone by division. It implies that negation of addition is subtraction, and the negation of multiplication is division. In addition, reciprocity deals with compensations or things with equivalent relationships.

Ramful [1] argued that the concept of reversible thinking increasingly developed based one several existing researches. Reversible thinking began to be seen as an issue by many researchers in education field, particularly to mathematics. The finding showed that it was important to have reversible thinking in problem-solving, as what had been previously discussed. In its development, the concept of reversible thinking deals with arithmetic, fraction, ratio, algebra, and some other math cases Ramful& Olive [18]. Following Ramful [1], there are two kinds of reversible thinking in education field; inversion and reciprocity. The example of inversion and reciprocity in math is when students are given an equation $10-\frac{5}{8-x}=9$, it can be solve by involving either inversion and reciprocity. Following Adi [19], if it needs to involve inversion to define x, what the student may think is "what number it should subtract 10 to get 9 as the result", and then "what number it should divide 5 to get 1 as the result," and "what number it should subtract 8 to get 5 as the result," Furthermore, in case of defining x by involving reciprocity, they may multiply the two sides of equation with $8-x$, and thus it results into $80-10x-5=72-9x$.

To identify students' reversible thinking in mathematic, teachers may provide some problems related to inverses or the opposite problems. In this study, the selected problem was mathematics insurance problems designed in such a way to explore prospective-teacher students' reversible thinking. Therefore, this study aimed to figure out prospective-teacher students' reversible thinking in solving mathematics insurance problems. Problem-solving was an activity of seeking solution for particular situation, using the insight previously obtained [20,21,22]. The procedures of problem-solving by Polya [20] involved (1) understanding the problem; (2) designing the plan; (3) implementing the plan; (4) having a reversal move. In this study, thus, solving mathematics insurance problems referred to students' process to find solution for mathematics insurance problems the researcher had provided by using their insights.

To explore students' reversible thinking in solving math insurance problems, the researcher designed two reversing math insurance problems. The first problem dealt with invers, while the second problem had nothing to do with inverse. However, those two problems were in the same context. The sub-material of math insurance problems selected in this study was multiple interests. The illustration was as follow.

> *"If one saved his money in a bank but he did not take the periodical interest of his saving, it would be calculated and considered as new assets and it would also get interests at the subsequent period. The value of the new interest would be bigger than the previous one*."

In addition, those two reversal problems were presented in the following Table 1.

Table 1. First and second problems

First Problem	Second Problem
Adi saved his money in a bank in Indonesia. The bank gave a multiple interest at 10% per year. Given that he never took the interest, and no administrative cost within, if the amount of interest in 3 years was Rp.331.000,00. Identify the amount that Adi saved as the initial asset?	Adi saved Rp.1.000.000,00 in a bank in Indonesia. The Bank gave him a multiple interest at 10% per year. Given that he never took the interest, and no administrative cost within, find the amount of interest for 3 years?

Based on a study of inversion and reciprocity, and two reversing math insurance problems that are deliberately designed by researchers in order to explore students' reversible thinking, the next step is to determine aspects of reversible thinking as the basis for researchers to analyze research data. Furthermore, prospective-teacher students were considered having reversible thought in solving math insurance problems if they met the aspects of reversible thinking. Inversion, if the subject was capable to solve the first problem through a correct process. Reciprocity, if the subject was capable to solve both problems through a correct process. When given that both problems were reversible, and thus, when the students were capable to solve the second problem in correct way, they had conducted an equal process of thinking.

The process of solving problems in this study referred to the procedures developed by Polya [20], as presented in the following Table 2.

Table 2. Procedure of problem solving

Procedure	Explaining
Understanding the problem	It is mentioning or noting things identified and asked on the test sheet
Designing the plan of solution	It is noting and mentioning the ideas to be used for solving problems
Implementing the plant	It is mentioning the ideas of problem solving

Having a reversal move	It is reviewing the result

3. Method

Bogdan and Biklen [23] explained the characteristics of qualitative research. Those were (i) naturalistic in nature, since it was conducted using the real situation as the data source and the researcher as its primary instrument; (ii) descriptive, since the data collected was qualitative, such as a set of words or writing, in case, the data was in the form of subject's work; (iii) inductive, since it did not aim to prove any hypothesis, but merely described a phenomena. As they were in accordance to this present study, thus, this study was a qualitative research. The researcher provided a test to the subject, and conducted an interview after all in order to reveal any uncover stuff on the test result.

The subject of this study was prospective-teacher students of mathematics education major who had passed math insurance course. Among 127 students, the researcher selected one student with high mathematics competence that was capable to communicate his thought in both verbal and writing. He was considered having high mathematics competence if he had score of GPA at least 3.40.

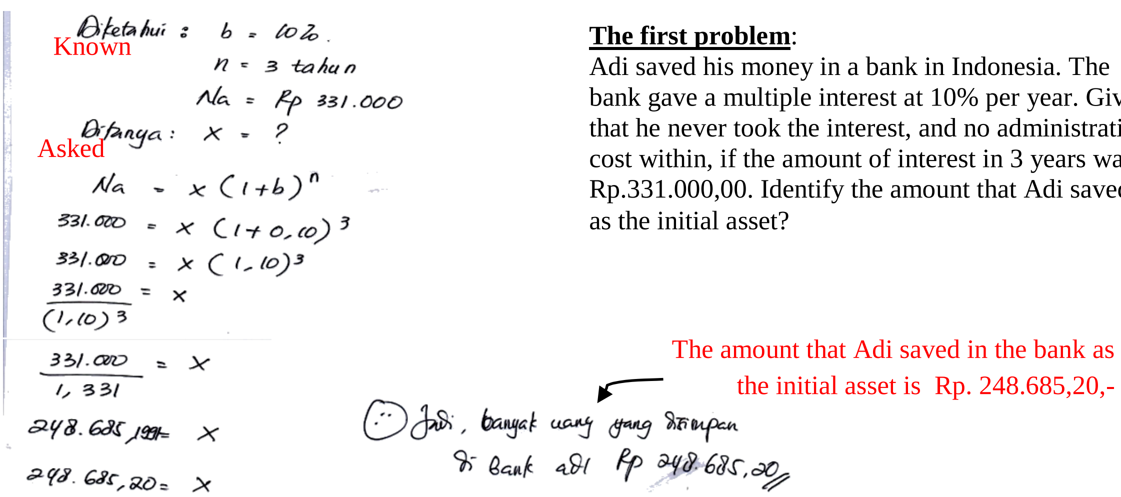
The procedures of data collection involved providing two subsequent validated tests and then an interview. The tests were given in different time. Each of them consisted of a problem on multiple interests. They were revesible, as previously discussed. The validated data was then analyzed based on Polya's procedures as presentend in preliminary and based on the aspects of reversible thinking. Time triangulation technique was used for data validity

4. Result and Discussion

The validated data was then analyzed based on Polya procedures and the aspects of reversible thinking. The result was as follow

4.1. The process of Solving the First Problem by the Subject

The subject's work for the first problem was presented in the following figure.



The first problem:
Adi saved his money in a bank in Indonesia. The bank gave a multiple interest at 10% per year. Given that he never took the interest, and no administrative cost within, if the amount of interest in 3 years was Rp.331.000,00. Identify the amount that Adi saved as the initial asset?

Figure 1. The subject's work for the first problem

The stages of solving the first problem were as follow.

Stage 1 : Understanding the problem

On this stage, the subject noted the identified information and what being asked as presented in Figure 1. Furthermore, the researcher did an interview to clarify the meaning of notation on the subject's work. The following was a part of interview the researcher had with the subject of the study.

Researcher	:	"Please explain the meaning of notation b, n, Na, and x on the stage of *being identified and asked*!"
Subject	:	"Ok, the *b* is the notation for multiple interests, *n* for how long the interest settled (i.e., in this case was 3 years). and *x* is for the amount of money that Adi saved as the initial assets"
Researcher	:	"what about *Na*?"
Subject	:	(keeping silent for a while) "it is the notation for the amount of interest

that settled"

Researcher : "Why does it take long time for you to answer it?"
Subject : "Yes, it felt little bit confused" (smiling)
Researcher : "In which part?"
Subject : "I thought it's like Na was not the amount of the interest. however, if it was not so, where the amount of interest then…."

Based on the interview, it found that b was the notation for the multiple interests per year, n was for how long the interest settled, *Na* was for the amount of interest for 3 years, and x was the amount of money that Adi saved as the initial asset.

Stage 2: Designing the plan of solution

The information collected in this stage was through interview, as follow.

Researcher : "before noting the process of solution, what was in your mind dealing with the strategy or method for solving this problem?"
Subject : "I thought the formula of multiple interests"
Researcher : "What was it?"
Subject : "This" (pointing a formula $N_a = x(1+b)^n$ on his work)

Based on the interview, it found that the idea to be used for solving the problem was applying a formula of final asset N_a of asset x to be lined up with multiple interest at $b\%$ per period for n period.

Stage 3: Implementing the plan of solution

In this stage, the subject applied or implemented the idea thought in the second stage; using a formula $N_a = x(1+b)^n$ to solve the first problem. Figure 1 showed the detail.

Stage 4: Having a reversal move

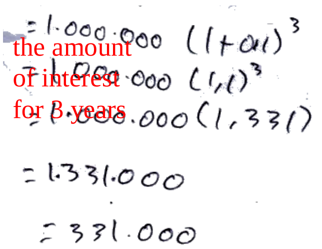
Information for this stage was identified by interview. The following passage was the part of interview with the subject.

Researcher : "Are you sure with your work?"
Subject : "Yeah"
Researcher : "What did make you sure about it?"
Subject : "I have counted it many times"
Researcher : "What did you count?"
Subject : "This is the answer" (while pointing to the formula $N_a = x(1+b)^n$ and up to the final result)

From the passage above, it found that the subject rechecked his work by having recalculation.

4.2. Process of Solving the Second Problem by Subject

The subject's work for the second problem was showed in Figure 2 as follow.



The second problem:
Adi saved Rp.1.000.000,00 in a bank in Indonesia. The Bank gave him a multiple interest at 10% per year. Given that he never took the interest, and no administrative cost within, find the amount of interest for 3 years?

Figure 2. Subject's work for the second problem

The stages of solving the second problem were as follow.

Stage 1: Understanding the problem

In this stage, the subject did not note any data or information to be identified and asked. Therefore, the researcher took an interview for data collection. The following was the result.

Researcher : "Do you understand this second problem?"
Subject : "Yeah"
Researcher : "What is the instruction?"
Subject : "it wanted me to define the amount of interest settled for 3 years"
Researcher : "Would you please mention the identified data of this problem?"
Subject : "Adi's initial saving IDR 1.000.000, the interest was 10 % per year, and the term the interest was settled was 3 years."
Researcher : "What else?"

Subject : "oh, one more thing, it was multiple interests"

Based on the interview, it found that the subject understood the problem. He could mention the identified data of this problem, such as the initial saving at IDR 1.000.000,-, the interest was 10% with multiple interests

Stage 2: Designing the plan of solution

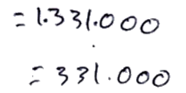
The information in this stage could be identified through interview, and the following was the result.

Researcher : "After understanding the problem, what kind of strategy or method you think it may solve the problem?"
Subject : "I think the formula of multiple interest"
Researcher : "what is the formula?"
Subject : "Na equals to x, and in parentheses, one is added by b with n degree (what the subject meant was $N_a = x(1 + b)^n$)"
Researcher : "What is *Na*?"
Subject : "It is the total amount of the saving"
Researcher : "What about *x, b* and *n*?"
Subject : "x is the amount of the initial saving, b is the interest, and n is the term of how long the interest settles."

Stage 3: Implementing the plan

In this stage, the subject implemented the idea he thought in the previous stage; using a formula $N_a = x(1 + b)^n$ to solve the second problem. However, the researcher did not have any interview to clarify the subject's work. The following was the result.

Researcher : "See your work" while pointing to



Subject : "Yeah"
Researcher : "Where does three hundred and thirty one come from?"
Subject : "It is from one million three hundred and thirty one subtracted by one million"

Stage 4: Having a reversal move

The information in this stage was identified through interview, as follow.

Researcher : "Are you sure with your work??"
Subject : "Sure"
Researcher : "What makes you so sure?"
Subject : "I have counted it many times"
Researcher : "What did you count?"
Subject : "The formula was correct and I have counted it twice"

Based on the interview, it found that the subject rechecked his work by having recalculation. In general, therefore, the reversible thinking of prospective-teacher students with high math competence was as follow.

a) Inversion

The indicator of this aspect was that the student was able to solve the first problem which contained inverse in correct way. However, the result showed that the subject (i.e., prospective-teacher student with high math competence) could not solve the problem in correct way. Hence, if the prospective-teacher student with high math competence feels difficult in solving inverse problem, students with moderate and low competences in math are likely difficult to do so as well, since there is a correlation between one's reversible thinking and his mathematical competence. Reversible thinking is the part of mathematical competence [2]. This finding was consistent to what Maf'ulah, Juniati, and Siswono [23] had found in their study on elementary graders. They suggested that elementary students were still difficult to solve problems that dealt with inverse.

b) Reciprocity

The indicator of this aspect was the subject was able to solve both problems in correct way. If those two reversal problem could be solved correctly, equilibrium between a pair of reversible relation could be found. As discussed in inversion, the student could not solve a problem related to inverse. Therefore, it inferred that reciprocity was not encountered although he could solve the second one. As previously discussed in introduction, those two problems had the same context or plot with reversal information of being identified and asked since the researcher intentionally aimed to figure out what the extent of their reversible thinking in solving mathematical insurance problem. The whole plot of the problems was as follow.

"*Adi saved Rp.1.000.000,- in a bank in Indonesia. The Bank gave him 10% interest rate per year with multiple interests. If the interest was never be withdrawn, and bank required no administrative cost, the amount of the interest settled for 3 years was Rp.331.000,00"*

The statement was classified into two problems with reversible relationship, as presented in the following Table 3.

Table 3. Frame of reversible problems

	Identified	Asked
The first problem	The amount of interest settled for 3 (three) years and the percentage of multiple interests per year	The amount that Adi initially saved
The second problem	The amount that Adi initially saved and the percentage of multiple interests per year.	The amount of interest settled for 3 (three) years

If the first problem turned into a statement, it would be as follow.

"*If the amount of interest settled for 3 (three) years was Rp.331.000,00 with 10% percentage of multiple interests per year, then the amount that Adi initially saved was Rp.1.000.000,-"*

If the second problem turned into a statement, it would be as follow.

"*If Adi's initially saving was Rp.1.000.000,- with 10% percentage of multiple interests per year, then the amount of interest settled for 3 (three) years was Rp.331.000,00*"

Therefore, if those two problems turned into one single statement, those two statements were equivalent. As Adi [17] argued that both reversible thinking and reciprocity referred to equal things. For instance, if $A < B$ then $B > A$ with an understanding that $A < B$ was equivalent to $B > A$. Relating to the result of this study, it was presented as follow.

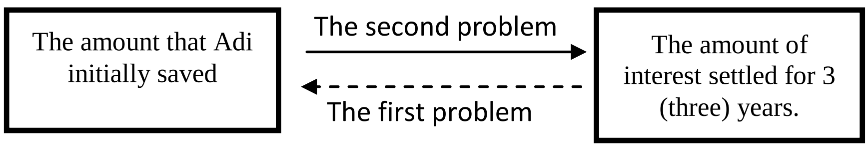


Figure 3. Prospective-Teacher Students' Reversible Thinking in Solving Math Insurance Problem

The continual arrow showed that the students could define the amount of percentage settled for 3 years. The dotted arrow showed that the students could not define the amount of Adi's initial saving. The two opposing problems in this study were in line with those delivered by Ramful [24] that in order to know reversible reasoning could use opposite problems, but Ramful uses the primal problem and dual problem terms. According to Ramful, ".... in the primal source and relation problem is to find the result. While in the dual, the problem and the aim is to find the source. The result found that prospective-teacher students could not yet construct a reversible two - way relationship. Therefore, their reversible thinking was in trouble. The results of this study were in line with the research conducted by Maf'ulah, et. al [25], but the subject was senior high school students, and the problems were given was a problem of functions and graphics. The first problem contained a function and the graph was asked. The second problem, it was known a function graph and asked for its function. The result showed that among senior high school students, only 5 students were capable of constructing a two-way correlation between a function and its graphic. The other 118 students are solely capable of drawing graphics without being able to define the function of the identified graphic.

5. Conclusion

Based on the result of this study, it found that prospective-teacher students' reversible thinking in solving math insurance problem was still in trouble. The analyzed data referred to the aspects of reversible thinking; inversion and reciprocity. It found that the students could not solve either inverse or reciprocal problems correctly. They only could solve problems out of inversion. If the students with high mathematics competence could not solve the problems that dealt with inversion, what about the moderate and low ones? Since theoretically, one's reversible thinking was influenced by their mathematics competence. Based on the results of research by Maf'ulah&Juniati [26], that to overcome the problem of reversible thinking for prospective mathematics teacher students, lecture can implementate the Learning with Reversible Problem-Solving Approach.

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