Research Article

A NUMRICAL SOLUTION FOR MAGNETOHYDRODYNAMIC STAGNATIONPOINT FLOW TOWARDS A STRETCHING SHEET

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Article History: Received: 11 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 10 May 2021

Abstract

Steady two-dimensional stagnation-point flow of an incompressible viscous electrically con- ducting fluid over a flat deformable sheet is investigated when the sheet is stretched in its own plane with a velocity proportional to the distance from the stagnation-point. Using similarity variables, the governing partial differential equations are transformed into a set of non-dimensional ordinary differential equations. These equations are then solved numerically using Spline collocation method. In the present reported work the effect of magnetic field parameter on flow have been discussed.

Keywords: MHD, Stagnation-point, Heat transfer, Nonlinear Differential equations, Spline collocation.

1. Introduction

Flow of an incompressible viscous fluid over a stretching surface has an important bearing on several technological processes. For example in the extrusion of a polymer in a melt-spinning process, the extrudate from the die is generally drawn and simultaneoulsy stretched into a sheet which is then solidified through quenching or gradual cooling by direct contact with water. Some examples are in the glass blowing, the cooling and/or drying of papers and textiles, the extrusion of a polymer in a melt-spinning process, metals and plastics, continuous casting and spinning of fibers, etc. Crane [1] was the first who studied the two-dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate and obtained an exact solution in closed analyticalform. Since then, many authors have studied various aspects of this problem, such as Chiam [2], Mahapatra and Gupta [3], Ishak et al. [4,5], etc., who have studied the flow behaviors due to a stretching sheet in the presence of magnetic field.

2. Mathematical formulation

Consider the two-dimensional steady flow of an incompressible viscous electrically conduct- ing fluid (with electrical conductivity □) near a stagnation-point at a surface coinciding with the plane y = 0, the flow being confined to y > 0. Two equal and opposite forces are intro- duced along the zaxis (Fig. 1) so that the wall is stretched keeping the origin fixed, and a uni- form magnetic field B_0 is imposed along y-axis.

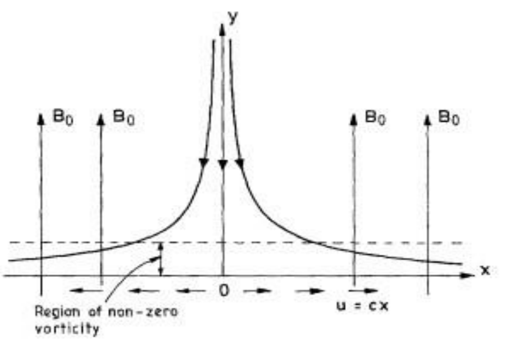

Fig. 1 : A sketch of physical problem

The MHD equations for steady two-dimensional stagnation-point flow in the boundary layer over the stretching surface are, in the usual notation,

∂u ∂v

$\_\_ + = 0$ (1)

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\sigma B_0^2}{\rho}(U-u) \qquad (2)$$

where the induced magnetic field is neglected which is justified for MHD flow at small magnetic Reynolds numbers [6]). It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible. In (2), U(x) stands for the stagnation-point velocity in the inviscid free stream.

The appropriate boundary conditions are

$$u=cx,\ v=0 \text{ at } y=0 \qquad (3)$$

$$u\to U(x)=ax \text{ as } y\to\infty$$

where c and a are constants with c > 0 and a > 0. It may be noted that the constant a is proportional to the free stream velocity far away from the stretching surface. A little inspection shows that Eqs. (1) and (2) along with the boundary conditions (3) and (4) admit of similarity solution of the form.

$$u(x,y)=cxf'(\eta)$$

$$v(x,y)=-(c\nu)^{\frac{1}{2}}f(\eta) \qquad (5)$$

where $\eta=\left(\frac{c}{\nu}\right)^{\frac{1}{2}}y$ and the primedenotes differentiation wrt η Clearly with (5), Eq. (1) is

identically satisfied. Substituting (5) in (2), we get

$$f'''(\eta)+f(\eta)f''-f'^2(\eta)-Mf'(\eta)+M\frac{a}{c}+\frac{a^2}{c^2}=0 \qquad (6)$$

where M is the Hartmann number given by

$$M=B_0\left(\frac{\sigma}{\rho c}\right)^{\frac{1}{2}} \qquad (7)$$

The boundary conditions for (6) follow from (3), (4) and (5) as

$$f(0)=0,\ f'(0)=1,\ f'(\infty)=\frac{a}{c} \qquad (8)$$

Here analyse flow behavior for different values for $\frac{a}{c}$.

3. Quartic Spline Blue method

For three points boundary value problems are

Let $s(x_i)$ be quartic spline in $[x_{i-1},x_i]$

Conditions for natural splines are $s(x_i)$ Almost quartic in each subinterval $[x_{i-1},x_i]$ $s(x_i)=y_i$, for i = 0,1,2,........,n. $s(x_i)$, $s_i'(x_i)$, $s_i''(x_i)$, $s'''(x_i)$ are continuous in $[x_0, x_n]$.

$s'''(x_0) = s'''(x_n) = 0.$

Here spline third derivative must be linear in $[x_{i-1}, x_i]$

$$s'''_i(x) = \frac{1}{h_i}\left[(x_i - x)\,y'''_{i-1} + (x - x_{i-1})\,y'''_i\right] \qquad (3.1)$$

Where $h_i = x_i - x_{i-1}$ and $s'''(x_i) = y'''_i$

Integrate (3.1), twice with respect to x .

$$s'_i(x) = \frac{1}{h_i}\left[\frac{(x_i - x)^3}{6}\,y'''_{i-1} + \frac{(x - x_{i-1})^3}{6}\,y'''_i\right] + c_i(x_i - x) + d_i(x - x_{i-1}).$$

Where use $s'_i(x_{i-1}) = y'_{i-1}$ and $s'_i(x_i) = y'_i$ constants

c_i and d_i as follows

$$c_i = \frac{1}{h_i}\left[y'_{i-1} - \frac{h_i^2}{6}\,y'''_{i-1}\right] \text{ and } d_i = \frac{1}{h_i}\left[y'_i - \frac{h_i^2}{6}\,y'''_i\right]$$

So

$$s'_i(x) = \frac{1}{h_i}\left[\frac{(x_i - x)^3}{6}\,y'''_{i-1} + \frac{(x - x_{i-1})^3}{6}\,y'''_i\right] + \frac{1}{h_i}\left[y'_{i-1} - \frac{h_i^2}{6}\,y'''_{i-1}\right](x_i - x) + \frac{1}{h_i}\left[y'_i - \frac{h_i^2}{6}\,y'''_i\right](x - x_{i-1}). \qquad (3.2)$$

Integrate (3.2), once with respect to x.

$$s_i(x) = \frac{1}{h_i}\left[-\frac{(x_i - x)^4}{24}\,y'''_{i-1} + \frac{(x - x_{i-1})^4}{24}\,y'''_i\right] - \frac{1}{h_i}\left[y'_{i-1} - \frac{h_i^2}{6}\,y'''_{i-1}\right]\frac{(x_i - x)^2}{2} + \frac{1}{h_i}\left[y'_i - \frac{h_i^2}{6}\,y'''_i\right]\frac{(x - x_{i-1})^2}{2} + e_i. \qquad (3.3)$$

Take $s_i(x_{i-1}) = y_{i-1}$, we get constants e_i

Where $e_i = y_{i-1} + \frac{-h_i^3}{8}\,y'''_{i-1} + \frac{h_i}{2}\,y'_{i-1}.$

Substitute e_i in (3.3),

$$s_i(x) = \frac{1}{h_i}\left[-\frac{(x_i - x)^4}{24}\,y'''_{i-1} + \frac{(x - x_{i-1})^4}{24}\,y'''_i\right] - \frac{1}{h_i}\left[y'_{i-1} - \frac{h_i^2}{6}\,y'''_{i-1}\right]\frac{(x_i - x)^2}{2} + \qquad (3.4)$$

1 h_{i2}

$+\hbar_i$ $y_i - 6\, y_i'''$ $(x-2x_{i-1})_2$

$+y_{i-1} - \hbar^3 \; - ''' \; \hbar\, y_{i-1}'.$

$y_{i\,1} +$

8 2

For $s\, x_i''(\,_{i-}) = s_{i+1}''(x_{i+})$

$$y_{i+1}' - 2y_i' + y_{i-1}' = \frac{h^2}{6}(y_{i+1}''' + 4y_i''' + y_{i-1}''') \quad (3.5)$$

For $s\, x_i(\,_{i-}) = s_{i+1}(x_{i+})$ $$y_{i-1} - = -y_i \frac{h}{2}\left(y_i' + y_{i-1}'\right) + \frac{h^3}{24}(3y_{i-1}''' - y_i''') \quad (3.6)$$

Case (i) $\frac{a}{c}$ 1

To obtain the spline solution, we begin with a assume function $f(\,) = -0.25^2 +$ which satisfy given boundary conditions (8). To find the solution of equation (6) along with boundary conditions (8). First we use $f(\,) = -0.25\;^2 +$ and (6) in (3.5) and $h = 0.1$, we gate different values of y_i' for $i = 1,2,3,4$.

To find the final solution we use (3.6) for different values of $i = 1,2,3,4$ respectively, equations as follows

$$y_0 - = -y_1 \frac{h}{2}[y_1' + y_0'] + \frac{h^3}{24}[- + y_1''' \, 3y_0''']$$

$$y\, y_1 - = -_2 \frac{h}{2}[y_2' + + y_1'] \frac{h^3}{24}[- + y_2''' \, 3y_1'''] \quad (3.7)$$

$$y_2 - = -y_3 \frac{h}{2}[y_3 + y_2'] + \frac{h^3}{24}[- + y_3''' \, 3y_2''']$$

$$y\, y_3 - = -_4 \frac{h}{2}[y_4 + y_3'] + \frac{h^3}{24}[- + y_4''' \, 3y_3''']$$

To substitute y_i' and y_i''' for $h = 0.1$in (6). We have four unknown and four equations. Solve those equations using Matlab. We get solution graphs as follows:

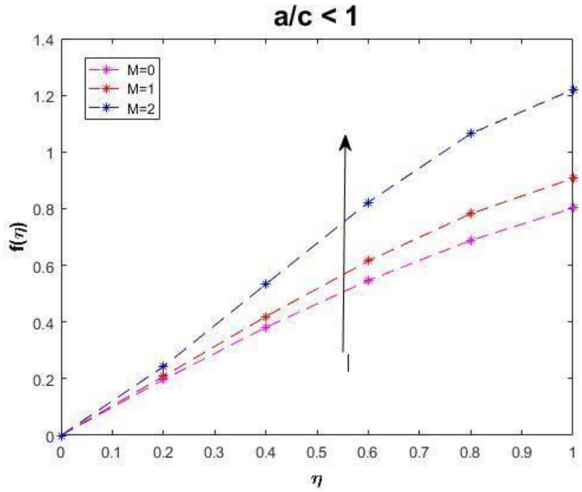

Fig. 2: Normal velocity profile for various values of M

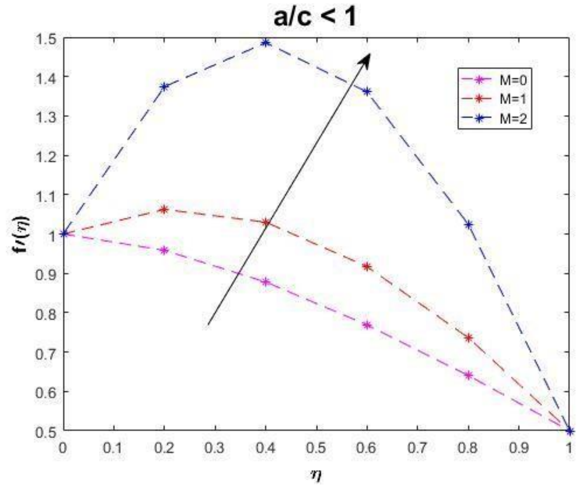

Fig. 3: Horizontal velocity profile for various values of M

Case (ii & iii) $\frac{a}{c}\square 1$ and $\frac{a}{c}=1$

To obtain the spline solution, begin with a assume function $f(\ )\square\ \square\ \square= 0.25\ ^2 +$ and $f(\ )\square\ \square=$ which satisfy given boundary conditions (8).

We get solution graphs as follows:

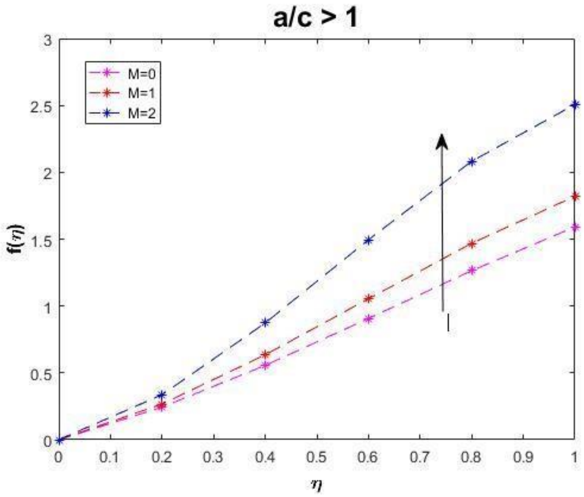

Fig. 4: Normal velocity profile for various values of M

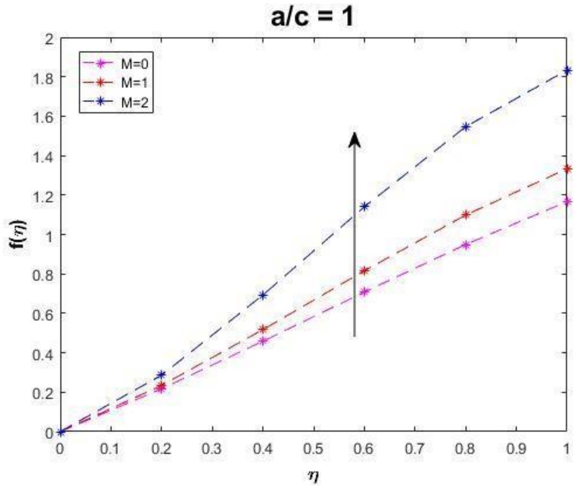

Fig. 5: Normal velocity profile for various values of M

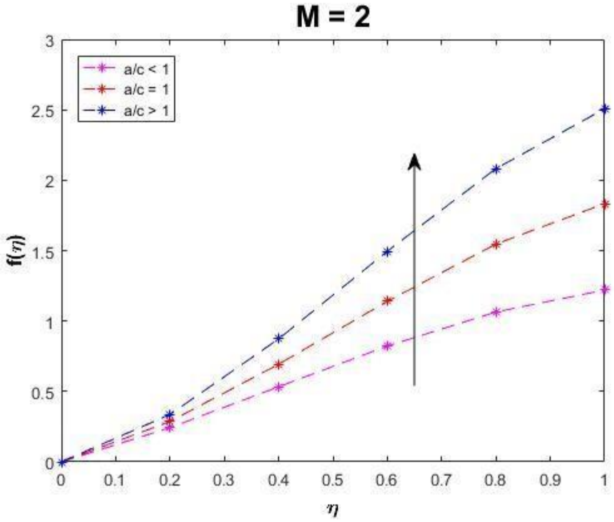

Fig. 6: Variation of $f(\eta)$ with η for various cases of $\frac{a}{c}$ with fix M.

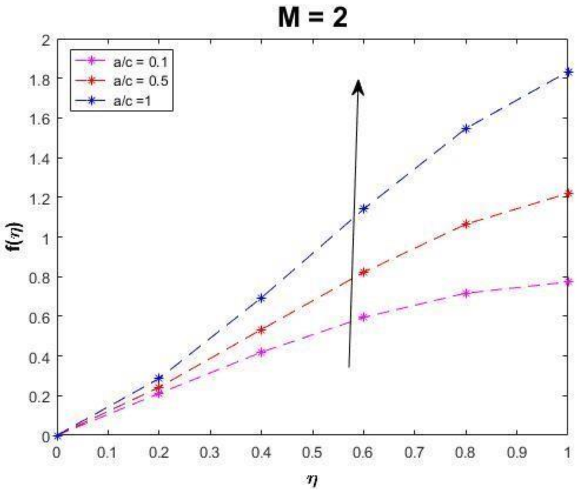

Fig. 7: Variation of $f(\eta)$ with η for various cases of $\frac{a}{c}<1$ with fix M.

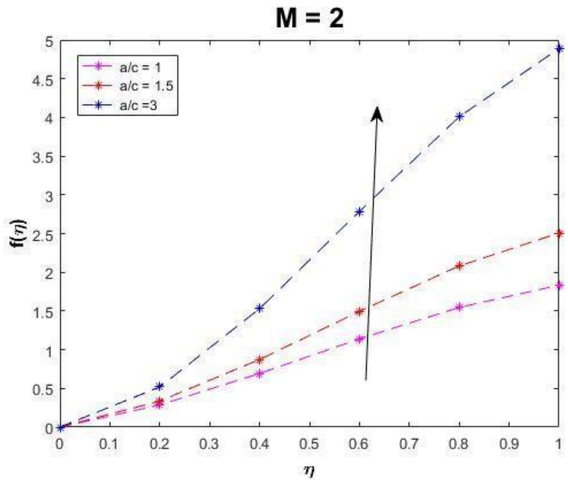

Fig. 8: Variation of $f(\eta)$ with η for various cases of $\frac{a}{c}>1$ with fix M.

4. Result and Discussion:

From figures (1) to (5), it shows that, there is a significant impact of magnetic field on the displacement profile of the flow. In all the cases of a/c, we can see here that Normal and Horizontal velocity profile increase as increase in Magnetic parameter. Similarly from figure (6) to figure (8), velocity profile also increase as increase the value of a/c. Comparison of velocity profile in all cases given in figures (6-8).

5. Conclusion

We find the generalization of blue method for third order problem and solved the problem using blue technique. The beauty of this method is no need to convert nonlinear problem into linear form, we can solve directly in nonlinear form. Thus researcher are able to solve such type of problems using blue method without convert nonlinear problem into linear form. In given problem, it can be conclude that magnetic parameter is directly proportional to velocity and displacement. Velocity profile is also directly proportionl to the values of a/c.

6. References:

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