A Study of People Perspectives towards Water Purification Technologies using GRA Method of Multiple Attribute Decision Making in Intuitionistic Fuzzy Environment

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Abstract
The incitement of this paper is to analyze water purifying technologies by utilizing grey relational analysis (GRA) method for multiple attribute decision making problems. The weight vectors are determined by using single-objective programming model. Water is an elixir of our life and it has several unique characteristics. Due to various unhealthy issues, drinking water still cause people sick or even destroy them, because it encompasses critical diseases-making pathogens. Water purifier will comfort and cherish the customers opposed to poisonous waterborne infections. To dumbfound this current situation GRA technique is used to solve the complication. Grey analysis is a technique which provides an agreeable explanation for existing world problems. Finally the best one is chosen by utilizing the relative relational degree.

Keywords: Decision making, Intuitionistic fuzzy sets, GRA method, Incomplete weight information, Water purifying technologies.

Introduction

Decision making is excessively intuitive for single criterion problems. The most important alternative is chosen which is based on preference rating among all the alternatives. Multiple Attribute Decision Making (MADM) stands for executing preference decisions (e.g., evaluation, prioritization, selection) concluded the available alternatives which are appropriate to multiple, consistently conflicting, attributes. In 1982, Professor Deng initiated a method called GRA which is an essential part of grey system theory. In business circumstances, the focal influence of Grey relational analysis are predicted on genuine data, simple calculations and actuality straight forward. In 1983, Krassimir Atanassov originated the perception of intuitionistic fuzzy set [1, 2]. Intuitionistic fuzzy sets are immensely helpful to compromise with imprecision. So, it can be used extensively in many areas to compromise with imprecision. This paper is systematized as follows: section 2 collects the basic definitions of intuitionistic fuzzy sets, section 3 encompasses about the GRA method and its procedure, section 4 provides the explanation of the problem and proposed GRA method with an example of water purifying technology and the final section holds the collection of the paper.

2. Preliminaries

Let X be a universal set. Let A be a fuzzy set defined on X, given by

$$A = \{\langle x, \mu_A(x)\rangle / x \in X\},$$

which is described by a membership function $\mu_A : X \to [0,1]$, where $\mu_A(x)$ represents the degree of membership of the element x to the set A. [1]

Let A be an intuitionistic fuzzy set in X which is given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\},$$

where $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$, satisfies the condition: $0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X$. Here $\mu_A(x)$ and $\nu_A(x)$ denotes respectively, the degree of membership and the degree of non-membership of the element x to the set A. [14]

An Intuitionistic fuzzy number is given by, $\tilde{a} = (\mu, \nu)$. Then the accuracy function H of an intuitionistic fuzzy number is defined as $H(\tilde{a}) = \mu + \nu$, $H(\tilde{a}) \in [0,1]$ which is used to assess the degree of accuracy of the intuitionistic fuzzy number $\tilde{a} = (\mu, \nu)$ where the function $H(\tilde{a}) \in [0,1]$. The superior the value of $H(\tilde{a})$, the more the degree of accuracy of the intuitionistic fuzzy number [14].

The Hamming distance between $\tilde{a}_1 = (\mu_1, \nu_1)$ and $\tilde{a}_2 = (\mu_2, \nu_2)$ is described as $d(\tilde{a}_1, \tilde{a}_2) = |\mu_1 - \mu_2| + |\nu_1 - \nu_2|$, where $\tilde{a}_1 = (\mu_1, \nu_1)$ and $\tilde{a}_2 = (\mu_2, \nu_2)$ are two intuitionistic fuzzy numbers [14].

3. Grey Relational Analysis

GRA method is massively convenient to decide the best alternative in the selection problem with intuitionistic fuzzy information in a facile way. Firstly, convert all the alternatives into a comparability sequence. In agreement with this sequences ideal target sequence and grey relational coefficient are computed. Depended on the grey relational coefficient, grey relational degree has been calculated. Lastly, the alternative which has highest grey relational degree that one is premium decision. Let us take $A = \{A_1, A_2, ...A_m\}$ be the discrete set of alternatives. $G = \{G_1, G_2, ...G_n\}$ is the set of attributes and $W = \{W_1, W_2, ...W_n\}$ is the weight vector of the attribute $G_j\,(j = 1, 2, ...n)$, where $W_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$. Let H be a set of the known weight information, that can be formulated as below for $i \neq j$ [7-10]:

Type 1: A weak ranking: $w_i \geq w_j$.

Type 2: A strict ranking: $w_i - w_j \geq \alpha_i, \alpha_i > 0$.

Type 3: A ranking of differences: $w_i - w_j \geq w_k - w_l$, for $j \neq k \neq l$.

Type 4: A ranking with multiples: $w_i \geq \beta_i w_j, 0 \leq \beta_i \leq 1$.

Type 5: An interval type: $\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i, 0 \leq \alpha_i < \alpha_i + \varepsilon_i \leq 1$.

Let us assume that $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = (\mu_{ij}, \nu_{ij})_{m\times n}$ be the intuitionistic fuzzy decision matrix, point μ_{ij} implies the degree that the alternative A_i which satisfies the attribute G_j fixed by the decision maker, ν_{ij} implies the degree that the alternative A_i which does not satisfy the attribute G_j given by the decision maker $\mu_{ij} \subset [0,1]$, $\nu_{ij} \subset [0,1]$, $\mu_{ij} + \nu_{ij} \leq 1$, $i = 1, 2, ...m$, $j = 1, 2, ...n$. To determine an intuitionistic fuzzy MADM with incompletely known weight information by using GRA method [14].

Step: 1

Find out the positive-ideal and negative-ideal solution depended on intuitionistic fuzzy numbers.

$$\tilde{r}^+ = ((\mu_1^+, \nu_1^+), (\mu_2^+, \nu_2^+).....(\mu_n^+, \nu_n^+)), \quad (1)$$

$$\tilde{r}^- = ((\mu_1^-, \nu_1^-), (\mu_2^-, \nu_2^-).....(\mu_n^-, \nu_n^-)), \quad (2)$$

where

$$\tilde{r}_j^+ = (\mu_j^+, \nu_j^+) = (\max_i \mu_{ij}, \min_i \nu_{ij}), \qquad j \in 1, 2, ...n,$$

$$\tilde{r}_j^- = (\mu_j^-, \nu_j^-) = (\min_i \mu_{ij}, \max \nu_{ij}), \qquad j \in 1, 2, ...n.$$

Step: 2

Applying the following equation, the grey relational coefficient of every single alternative from PIS and NIS are computed respectively. The grey relational coefficient of every single alternative from PIS is given by,

$$\xi_{ij}^+ = \frac{\min\limits_{1\leq i\leq m}\min\limits_{1\leq j\leq n} d(\tilde{r}_{ij}, \tilde{r}_j^+) + \rho \max\limits_{1\leq i\leq m}\max\limits_{1\leq j\leq n} d(\tilde{r}_{ij}, \tilde{r}_j^+)}{d(\tilde{r}_{ij}, \tilde{r}_j^+) + \rho \max\limits_{1\leq i\leq m}\max\limits_{1\leq j\leq n} d(\tilde{r}_{ij}, \tilde{r}_j^+)}$$

$i = 1, 2, ...m,\ j \in 1, 2, ...n.$ (3)

Correspondingly, the grey relational coefficient of every single alternative from NIS is given by,

$$\xi_{ij}^- = \frac{\min\limits_{1\leq i\leq m}\min\limits_{1\leq j\leq n} d(\tilde{r}_{ij}, \tilde{r}_j^-) + \rho \max\limits_{1\leq i\leq m}\max\limits_{1\leq j\leq n} d(\tilde{r}_{ij}, \tilde{r}_j^-)}{d(\tilde{r}_{ij}, \tilde{r}_j^-) + \rho \max\limits_{1\leq i\leq m}\max\limits_{1\leq j\leq n} d(\tilde{r}_{ij}, \tilde{r}_j^-)}$$

$i = 1, 2, ...m, \ j \in 1, 2, ...n.$ (4)
where the identification coefficient ρ=0.5.
Step: 3

By applying the following equation, the degree of grey relational coefficient of every single alternative from PIS and NIS are computed respectively:

$$\xi_i^+ = \sum_{j=1}^{n} w_j \xi_{ij}^+, \qquad i = 1, 2, ...m, \tag{5}$$

$$\xi_i^- = \sum_{j=1}^{n} w_j \xi_{ij}^-, \qquad i = 1, 2, ...m. \tag{6}$$

The main assumption of the GRA method is that the selected alternative must have the "largest degree of grey relation" from the positive -ideal solution and the "smallest degree of grey relation" from the negative-ideal solution. Clearly, the weight vector is given, the smaller ξ_i^- and the larger ξ_i^+, the preferable alternative A_i is. On the other hand, the information about attribute weights is inadequately known. So that by finding the ξ_i^+ and ξ_i^- initially, then the weight information is computed. The multiple objective optimization models is to compute the weight information:

$$\text{(M-1)} \quad \begin{cases} \min \xi_i^- = \sum_{j=1}^{n} w_j \xi_{ij}^-, \quad i = 1, ...m \\ \max \xi_i^+ = \sum_{j=1}^{n} w_j \xi_{ij}^+, \quad i = 1, ...m \\ \text{Subject to: } w \in H. \end{cases}$$

Until now every single alternatives non-inferior, then there exists no desire relation on the all the alternatives. Therefore, the above multiple objective optimization models with equal weights into the single-objective optimization model.
The optimal solution is w=($w_1, w_2, ...w_n$),

$$\begin{cases} \min \xi = \sum_{i=1}^{m} \sum_{j=1}^{n} (\xi_{ij}^- - \xi_{ij}^+) w_j, \\ \end{cases}$$

(M-2) Subject to: $w \in H$.
By determining the model (M-2) that can be used as the weight vector of attributes. Therefore $\xi_i^+ = (i = 1.....m)$ and $\xi_i^- = (i = 1.....m)$ are finding out by equations (5), (6) respectively.
Step: 4

Computing the relative relational degree of every single alternatives from PIS utilizing the following equation.

$$\xi_i = \xi_i^+ / (\xi_i^- + \xi_i^+), \qquad i = 1, 2, ...m. \tag{7}$$

Step: 5

Ranking all the alternatives $A_i (i = 1, 2, ...m)$ and chosen the highest one(s) in correspondence with $\xi_i (i = 1, 2, ...m)$. Finally the alternative which has the ultimate ξ_i value, consequently it is the best alternative.

4. Problem Description

Making a selection of water purification technologies is very essential one and also bewilderment in current situation. Nowadays, contamination of water is seen all over the world. People are not able to get purified water from underground directly. So, the people need to get pure healthy water. Thus, the people prefer the water purifiers based on certain setting such as cost, duration, quality, health and environmental impacts which is user friendly and also lowers the speculation of allergies. There are so many water purifiers that exist in water purifying technologies and here is an inquisition of four innovative water purification technologies based on case study which are listed below that is based on public's perception.

Speculate that the customer wants to purchase a water purifier. So, that here occurs four alternatives which are given as follows Personal Purification Straw, Tiny UV Water Purifier, Tata Swach, Photocatalytic Water Purification Technology. Among the four alternatives the customer need to take a decision to select one alternative based on the certain desirable attributes like Human health, Cost, Quantity of water purified, Durability, Easier operation, Environmental benefits. The membership and non-membership of every single alternatives A_i ($i = 1, 2, ...m$) along with the attributes G_j ($j = 1, 2, ...n$) are taken in the form of intuitionistic fuzzy decision matrix. The attribute weights which are incompletely known are also given by the decision maker. Based on the collections of data by using the GRA method for multiple attribute decision making with intuitionistic fuzzy information the ranking order of all the alternatives are find and pick up the most acceptable one.

4.1 Methodology

The intuitionistic fuzzy decision matrix given by the decision maker, the values are taken in the form of every single alternative A_i satisfies the attribute G_j .

Alternatives:

$A_1 \rightarrow$ Personal purification straw

$A_2 \rightarrow$ Tiny UV water purifier

$A_3 \rightarrow$ Tata swach

$A_4 \rightarrow$ Photocatalytic water purification technology

Attributes:

$G_1 \rightarrow$ Human health

$G_2 \rightarrow$ Cost

$G_3 \rightarrow$ Quantity of water purified

$G_4 \rightarrow$ Durability

$G_5 \rightarrow$ Easier operation

$G_6 \rightarrow$ Environmental benefits

$$R = \begin{pmatrix} (0.6,0.4) & (0.8,0.3) & (0.4,0.6) & (0.4,0.6) & (0.7,0.2) & (0.8,0.3) \\ (0.8,0.3) & (0.8,0.3) & (0.5,0.4) & (0.5,0.4) & (0.5,0.4) & (0.6,0.4) \\ (0.3,0.2) & (0.5,0.4) & (0.7,0.2) & (0.6,0.4) & (0.6,0.4) & (0.5,0.4) \\ (0.6,0.4) & (0.5,0.4) & (0.8,0.3) & (0.8,0.3) & (0.8,0.3) & (0.6,0.4) \end{pmatrix}$$

The attribute weights are given by the decision maker which are incompletely known as follows:

H = 0.15 $\leq w_1 \leq$ 0.20, 0.2 $\leq w_2 \leq$ 0.30, 0.30 $\leq w_3 \leq$ 0.40, $w_4 \geq 0.3 * w_2$, $w_5 \geq w_1$, $w_6 \geq$ 0.1+ w_2

Step: 1
The positive-ideal and negative-ideal solution are calculated

$\tilde{r}^{+}$ = ((0.8,0.2) (0.8,0.3) (0.8,0.2) (0.8,0.3) (0.8,0.2) (0.8,0.3))

$\tilde{r}^{-}$ = ((0.3,0.4) (0.5,0.4) (0.4,0.6) (0.4,0.6) (0.5,0.4) (0.5,0.4))

Step: 2
By using PIS and NIS, the grey relational coefficient of every single alternative are computed

$$\xi^{+} = (\xi_{ij}^{+})_{m\times n} = \begin{pmatrix} 0.500 & 1.000 & 0.333 & 0.364 & 0.800 & 1.000 \\ 0.800 & 1.000 & 0.444 & 0.500 & 0.444 & 0.571 \\ 0.444 & 0.500 & 0.800 & 0.571 & 0.500 & 0.500 \\ 0.500 & 0.500 & 0.800 & 1.000 & 0.800 & 0.571 \end{pmatrix}$$

$$\xi^{-} = ((\xi_{ij}^{-})_{m\times n} = \begin{pmatrix} 0.538 & 0.467 & 1.000 & 1.000 & 0.467 & 0.467 \\ 0.368 & 0.467 & 0.538 & 0.538 & 1.000 & 0.778 \\ 0.636 & 1.000 & 0.333 & 0.467 & 0.778 & 1.000 \\ 0.538 & 1.000 & 0.333 & 0.333 & 0.467 & 0.778 \end{pmatrix}$$

Step: 3
To find out the following single-objective programming model by make use of the model (M-2):

$$\{\min \xi(w) = -0.164w_1 - 0.066w_2 - 0.173w_3 - 0.097w_4 + 0.168w_5 + 0.381w_6$$

Subject to: $w \in H$

The weight vectors of attributes are find out by solving this model:

w = (0.150, 0.200, 0.300, 0.060, 0.150, 0.140)

The degree of grey relational coefficient of every single alternative from PIS and NIS are given below:

ξ_1^{+} =0.6567, ξ_2^{+} =0.6297, ξ_3^{+} =0.5858, ξ_4^{+} =0.6749.

ξ_1^{-} =0.6695, ξ_2^{-} =0.6012, ξ_3^{-} =0.6800, ξ_4^{-} =0.5795.

Step: 4
For every single alternative from PIS, the relative relational degree is computed.

ξ_1 = 0.4952, ξ_2 =0.5116, ξ_3 =0.4628, ξ_4 =0.5380.

Step: 5
On the basis of the relative relational degree, the four alternatives are arranged by the ranking order is given by: $A_4 > A_2 > A_1 > A_3$.Therefore A_4 is the most preferable alternative.

Conclusion
Today especially in Tamil Nadu, water scarcity is the most important problem. In addition to this the whole environment is dominated by contamination of water. To reduce and balance our sustainability of region and our own health there is need to adopt certain water purifying technologies in the current situation. GRA method is very facile and also effective to deal with multiple attribute decision making with intuitionistic fuzzy setting. Finally, based on the overall interpretation of this paper 'Photocatalytic Water Purifying Technology' (A_4) is the most preferable alternative.

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