Existences of Mean Square Convergence for RL Circuit using Random Fourth Order Runge Kutta Method

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Abstract: In this paper, a random stochastic initial value problem for a RL circuit is considered and the mean square convergent is proved through the random Runge Kutta method and its expectation and variance are computed.

Keywords: Stochastic Initial Value Problem, Runge Kutta Fourth Order, Mean Square Convergence, Numerical Problem.

1. Introduction

Stochastic differential equation (SDEs) plays a vital role in many fields such as science, economics, finance, population dynamics, biology, mechanics etc. Many of the researcher ignored stochastic effects because of the difficulty in solution [1]. A Stochastic Differential Equation is comprised of differential equation that includes at least one of the stochastic process the resulting solution is also stochastic process.

A stochastic initial value problems of the form,

$$\begin{cases} \frac{dY(t)}{dt} = f(Y(t), t), \\ Y(t_0) = Y_0 \end{cases} \quad t \in [t_0, T] \qquad (1.1)$$

Here, the stochastic process f(Y (t), t) defined on the probability space (Ω, F, Q) and Y_0 is a random variable. J.C. Cortes et al., proved that the numerical solution of random Euler method converges under some specific condition even though the exact solution are not satisfied [2]. J.C. Cortes et al., proved that when the approximation are far from the initial condition, the numerical results become worst [3]. Khodabin and Rostami proved that the mean square convergence using random Runge-Kutta method and illustrated numerical examples using different types of methods and obtained more accuracy results using suitable method [4].

2. Preliminaries

Definition 2.1: The density functionf_Z of second order random variables is defined as

$$E[Z^2] = \int_{-\infty}^{\infty} z^2 f_Z(z)dz < \infty$$

where E indicates the expectation and it allows all second order random variable Banach space L_2 with the norm

$$\|Z\| = \sqrt{E[Z^2]}.$$

Definition 2.2: For each t, q (t) is the second order random stochastic process defined on a same probability space (Ω, F, Q). Then, the mean square limit in L_2 takes the form,

$$\dot{q}(t) = \frac{q(t+\Delta t) - q(t)}{\Delta t}, \ as\ \Delta t \to 0.$$

Definition 2.3: The mean square bounded function f: $I \rightarrow L_2$ and h > 0, then the function f is mean square modulus if,

$$\omega(f,h) = Sup_{|t-t*|\leq h} \|f(t) - f(t^*)\|, \ \ t, t* \in I$$

Definition 2.4: The mean square uniformly continuous function f in I, if

$$\lim_{h\to 0} \omega(f,h) = 0$$

Lemma 2.1: Let the sequence $\{X_n\}$ and $\{Y_n\}$ is second order mean square convergent to the two random variable X, Y if

$X_n \rightarrow X$ and $Y_n \rightarrow Y$ as $n\rightarrow\infty$, Then

$$\lim_{n\to\infty} E[X_n] = E[X] \ and \ \lim_{n\to\infty} Var[Y_n] = Var[Y]$$

Theorem 2.1: One Dimensional Ito Formula

Let U_t be an Ito processes given by $dU_t = A\, dt + B\, dW_t$ and $f(t, x) \in C^2([0,\infty)\times R)$, then

$V_t = f(t, X_t)$ is an Ito process then,

$$dV_t = \frac{\partial f}{\partial t}(t,X_t)dt + \frac{\partial f}{\partial x}(t,X_t)dX_t + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(t,X_t)(dB_t)^2 dt$$

where $(dX_t)^2 = (dX_t)(dX_t)$ is determine, according to the rules

$$ds.ds = ds.dB_t = dB_t.ds = 0, \ dB_t.dB_t = ds$$

Theorem 2.2:

Let Y (t) be a second order stochastic process which is mean square differentiable and continuous in I = $[t_0, T]$. Then there exists $\eta \in I$ such that

$$Y(t) - Y(t_0) = \dot{Y}(\eta)(t - t_0)$$

Theorem 2.3:

Let $f(Y(t), t): R \times I \rightarrow L_2$, where R is a bounded set. Then it satisfies the following condition

i. The function f (Y, t) is randomly bounded uniformly continuous
ii. It satisfies the mean square Lipschitz condition, then

$$\| f(Y,t) - f(Z,t) \| \leq k(t) \| Y - Z \|$$

where

$$\int_0^T k(t)dt < \infty.$$

Then (1.1) is mean square convergent of the random fourth order Runge Kutta Scheme.

3. Mean Square Convergence for RL Circuit

By fourth order random Runge-Kutta method

$$X_{n+1} = X_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \quad n = 1,2,3, \dots \dots \qquad (3.1)$$

where

$$k_1 = hf(x_n, t_n)$$
$$k_2 = hf\left(X_n + \frac{k_1}{2}, t_n + \frac{h}{2}\right)$$

$$k_3 = hf\left(X_n + \frac{k_2}{2}, t_n + \frac{h}{2}\right)$$
$$k_4 = hf(X_n + k_3, t_n + h)$$

Let us consider the RL circuit with constant parameters:

$$\begin{cases} L\,\frac{dI(t)}{dt} + RI(t) = V(t) + \alpha(t)W(t)\ t \in [0,2] \\ I(0) = I_0 \end{cases} \tag{3.2}$$

Error: $e_n = I_n - I(t)$ (3.3)

where, equation (3.2) is the solution of the fourth order stochastic process.

From Theorem 2.2,

$$\|e_{n+1}\| \le \|e_n\| + \frac{h}{6}\|f(X_n, t_n) - f(X(t_\xi), t_\xi)\| + \frac{h}{3}\left\|f\left(X_n + \frac{k_1}{2}, t_n + \frac{h}{2}\right) - f(X(t_\xi), t_\xi)\right\| + \frac{h}{3}\left\|f\left(X_n + \frac{k_2}{2}, t_n + \frac{h}{2}\right) - f(X(t_\xi), t_\xi)\right\| + \frac{h}{6}\|f(X_n + k_{3,}\, t_n + h) - f(X(t_\xi), t_\xi)\| \tag{3.4}$$

Using Theorem-2.2 & Theorem 2.3

$$\|f(X_n, t_n) - f(X(t_\xi), t_\xi)\| \le K(t_n)\|e_n\| + K(t_n)Mh + w(h)$$
$$\|f(X_n, t_n) - f(X(t_\xi), t_\xi)\| \le \frac{1}{L}[V(t_n) + \alpha(t_n)W(t_n)]\|e_n\| + \frac{Mh}{L}\{V(t_n) + \alpha(t_n)W(t_n)\} + w(h) \tag{3.5}$$
$$\left\|f\left(X_n + \frac{k_1}{2}, t_n + \frac{h}{2}\right) - f(X(t_\xi), t_\xi)\right\| \le K\left(t_n + \frac{h}{2}\right)\|e_n\| + \frac{3Mh}{2}K\left(t_n + \frac{h}{2}\right) + w(h)$$

$$\left\|f\left(X_n + \frac{k_1}{2}, t_n + \frac{h}{2}\right) - f(X(t_\xi), t_\xi)\right\| \le \frac{1}{L}\left[V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right]\|e_n\| + \frac{3Mh}{2L}\left[V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right] + w(h) \tag{3.6}$$
$$\left\|f\left(X_n + \frac{k_2}{2}, t_n + \frac{h}{2}\right) - f(X(t_\xi), t_\xi)\right\| \le K\left(t_n + \frac{h}{2}\right)\|e_n\| + \frac{3Mh}{2}K\left(t_n + \frac{h}{2}\right) + w(h)$$

$$\left\|f\left(X_n + \frac{k_2}{2}, t_n + \frac{h}{2}\right) - f(X(t_\xi), t_\xi)\right\| \le \frac{1}{L}\left(1 - \frac{h}{2L}\right)\left\{V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right\}\|e_n\| + \frac{3Mh}{2L}\left\{\left(1 - \frac{h}{2L}\right)V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right\} + w(h) \tag{3.7}$$
$$\| f(X_n + k_3, t_n + h) - f(X(t_\xi), t_\xi) \le K(t_n + h)\|e_n\| + 2MhK(t_n + h) + w(h)$$

$$\| f(X_n + k_3, t_n + h) - f(X(t_\xi), t_\xi) \le \frac{1}{L}[V(t_n + h) + \alpha(t_n + h)W(t_n + h)]\|e_n\| + \frac{2Mh}{L}\{V(t_n + h) + \alpha(t_n + h)W(t_n + h)\} + w(h) \tag{3.8}$$

Substitute equation (3.5), (3.6), (3.7) & (3.8) in equation (3.4),

$$\|e_{n+1}\| \le \|e_n\|\left[1 + \frac{h}{6L}(V(t_n) + \alpha(t_n)W(t_n) + \left(\frac{2h}{3L} - \frac{h^2}{6L^2}\right)\left\{V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right\} + \frac{h}{6L}\{V(t_n + h) + \alpha(t_n + h)W(t_n + h)\}\right] + \frac{Mh^2}{6L}\{V(t_n) + \alpha(t_n)W(t_n)\} + \left(\frac{Mh^2}{L} - \frac{Mh^3}{4L^2}\right)\left\{V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right\} + \frac{Mh^2}{3L}\{V(t_n + h) + \alpha(t_n + h)W(t_n + h)\} + hw(h) \tag{3.9}$$

Setting

$$a_n = \left[1 + \frac{h}{6L}(V(t_n) + \alpha(t_n)W(t_n) + \left(\frac{2h}{3L} - \frac{h^2}{6L^2}\right)\left\{V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right\} + \frac{h}{6L}\{V(t_n + h) + \alpha(t_n + h)W(t_n + h)\}\right] \tag{3.10}$$
$$b_n \le \frac{Mh^2}{6L}\{V(t_n) + \alpha(t_n)W(t_n)\} + \left(\frac{Mh^2}{L} - \frac{Mh^3}{4L^2}\right)\left\{V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right\} + \frac{Mh^2}{3L}\{V(t_n + h) + \alpha(t_n + h)W(t_n + h)\} + hw(h) \tag{3.11}$$

Equation (3.10) has the following form,

$$\|e_{n+1}\| \leq a_n\|e_n\| + b_n, \quad n = 0, 1, 2, ... \tag{3.12}$$

By using the successive substitution of equation (13),

$$\|e_{n+1}\| \leq (\prod_{i=0}^{n} a_i)\|e_0\| + \sum_{i=0}^{n}\left(\prod_{j=i+1}^{n} a_j\right)b_i, \quad n = 0, 1, 2, ... \tag{3.13}$$

Equation (3.11) can be rewrite as,

$$\prod_{i=0}^{n} a_i \leq exp\left[(n+1)\frac{h}{6L}\left\{V(t_i) + \alpha(t_i)W(t_i) + \left(\frac{2h}{3L} - \frac{h^2}{6L^2}\right)\left(V(t_i + \frac{h}{2}\right) + \alpha\left(t_i + \frac{h}{2}\right)W\left(t_i + \frac{h}{2}\right)\right\} + \frac{h}{6L}\{V(t_i + h) + \alpha(t_i + h)W(t_i + h)\}\right] \tag{3.14}$$

By using Geometric Progression, equation (3.14) can be written as

$$\sum_{i=0}^{n}\left(\prod_{j=i+1}^{n} a_j\right) \leq \frac{\exp(n+1)\left[\frac{h}{6}\{V(t_n)+\alpha(t_n)W(t_n)\} + \left(\frac{2h}{3L}-\frac{h^2}{6L^2}\right)\left\{V\left(t_i+\frac{h}{2}\right)+\alpha\left(t_i+\frac{h}{2}\right)W\left(t_i+\frac{h}{2}\right)\right\}+ \frac{h}{6L}\{V(t_i+h)+\alpha(t_i+h)W(t_i+h)\}\right]-1}{\frac{h}{6}\{V(t_n)+\alpha(t_n)W(t_n)\} + \left(\frac{2h}{3L}-\frac{h^2}{6L^2}\right)\left\{V\left(t_i+\frac{h}{2}\right)+\alpha\left(t_i+\frac{h}{2}\right)W\left(t_i+\frac{h}{2}\right)\right\}+ \frac{h}{6L}\{V(t_i+h)+\alpha(t_i+h)W(t_i+h)\}} \tag{3.15}$$

Substitute the equation (3.11), (3.14), (3.15) in equation (3.13), hence

$$\|e_{n+1}\| \leq \frac{\exp(n+1)\left[\frac{h}{6}\{V(t_n)+\alpha(t_n)W(t_n)\} + \left(\frac{2h}{3L}-\frac{h^2}{6L^2}\right)\left\{V\left(t_i+\frac{h}{2}\right)+\alpha\left(t_i+\frac{h}{2}\right)W\left(t_i+\frac{h}{2}\right)\right\}+ \frac{h}{6L}\{V(t_i+h)+\alpha(t_i+h)W(t_i+h)\}\right]-1}{\frac{h}{6}\{V(t_n)+\alpha(t_n)W(t_n)\} + \left(\frac{2h}{3L}-\frac{h^2}{6L^2}\right)\left\{V\left(t_i+\frac{h}{2}\right)+\alpha\left(t_i+\frac{h}{2}\right)W\left(t_i+\frac{h}{2}\right)\right\}+ \frac{h}{6L}\{V(t_i+h)+\alpha(t_i+h)W(t_i+h)\}} \times \frac{Mh^2}{6L}\left(V(t_n) + \alpha(t_n)W(t_n)\right) + \left(\frac{Mh^2}{L} - \frac{Mh^3}{4L^2}\right)\left\{V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right\} + \frac{Mh^2}{3L}\{V(t_n + h) + \alpha(t_n + h)W(t_n + h)\} + hw(h) \tag{3.16}$$

By theorem 2.3, the inequality of equation (3.16), the sequence $\{e_n\}$ is mean square convergent to zero as w (h) $\to$0, h$\to$0.

4. Numerical Example

Consider the RL Circuit,

$$\begin{cases} L\frac{dI(t)}{dt} + RI(t) = V(t) + \beta(t)B(t) \\ I(0) = I_0 \end{cases} \tag{4.1}$$

Here, I_0 is an exponential random variable which is independent of W (t) with parameter $\lambda = \frac{1}{2}$, I(t) is the current at time t, for each t$\in$[0,2] ,V(t) and β(t) indicates the non-randomized functions and intensity of noise at time t, $B(t) = \frac{d\zeta(t)}{dt}$ and ζ(t) are the 1-dimensional white noise and Brownian motion.

By solving the equation (4.1) we have,

$$e^{\frac{Rt}{L}}dI(t) + \frac{R}{L}e^{Rt/L}I(t)dt = \frac{V(t)}{L}e^{\frac{Rt}{L}}dt + \frac{\alpha(t)}{L}e^{\frac{Rt}{L}}dB(t) \tag{4.2}$$

Assume g (t, x) and using Theorem-2.1, we get,

$$d\left(e^{\frac{Rt}{L}}I(t)\right) = \frac{R}{L}e^{\frac{Rt}{L}}I(t)dt + e^{\frac{Rt}{L}}dI(t) \tag{4.3}$$

By using equation (2) & equation (3),

$$I(t) = e^{-\frac{Rt}{L}}\left[I_0 + \frac{1}{L}\int_0^t e^{\frac{Rs}{L}}V(s)ds + \frac{1}{L}\int_0^t \beta(s)e^{\frac{Rs}{L}}dW(s)\right] \tag{4.4}$$

To find Mean & Variance:

$$E[I(t)] = e^{-\frac{Rt}{L}}\left[2 + \frac{1}{L}\int_0^t e^{\frac{Rs}{L}} V(s)ds\right] \tag{4.5}$$

$$E[I^2(t)] = E[e^{\frac{-2Rt}{L}}(4 + \frac{1}{L^2}\int_0^t e^{\frac{2Rs}{L}} V(s)(ds)^2 + \frac{1}{L^2}\int_0^t \beta^2(s)e^{\frac{2Rs}{L}} (dW(s))^2$$

$$E[I^2(t)] = e^{-\frac{2Rt}{L}}\left[4 + \frac{1}{L^2}\int_0^t e^{\frac{2Rs}{L}} \beta^2(s)ds\right]$$

$$Var[I(t)] = E[I^2(t)] - E[I(t)]^2$$

$$[VarI(t)] = e^{\frac{-2Rt}{L}}[4 + \frac{1}{L^2}\int_0^t \beta^2(s)e^{\frac{2Rs}{L}} ds \tag{4.6}$$

Table 1. Expectation of Mean & Variance of I (t):
When R=1, L=1, and $V(t) = e^t$, $\alpha(t) = \frac{cost}{25}$

t	E[I(t)]	Var I(t)
0	2.0000	4.0000
0.2	1.8387	2.6814
0.4	2.1391	1.7975
0.6	2.5872	1.2051
0.8	3.2556	0.8079
1.0	4.2528	0.5417
1.2	5.7404	0.3632
1.4	7.9597	0.2436
1.6	11.2704	0.1634
1.8	16.2095	0.1098
2.0	23.5778	0.0737

Using random Runge Kutta fourth order method,

$$I_{n+1} = I_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = \frac{h}{L}[-RI(t) + V(t_n) + \beta(t_n)W(t_n)]$$

$$k_2 = \frac{h}{L}\left[-R\left(1 - \frac{h}{2L}\right)I(t) - \frac{h}{2L}\left(V(t_n) + \beta(t_n)W(t_n)\right) + V\left(t_n + \frac{h}{2}\right) + \beta\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right]$$

$$k_3 = \frac{h}{L}\left[-R\left(1 - \frac{h}{2L} + \frac{h^2}{4L^2}\right)I(t) + \frac{h^2}{4L^2}[V(t_n) + \beta(t_n)W(t_n)] + \left(1 - \frac{h}{2L}\right)\left[V\left(t_n + \frac{h}{2}\right) + \beta\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right]\right]$$

$$k_4 = \frac{h}{L}\left[-R\left(1 - \frac{h}{L} + \frac{h^2}{2L^2} - \frac{h^3}{4L^3}\right)I(t) - \frac{h^3}{4L^3}\left(V(t_n) + \alpha(t_n)W(t_n)\right) - \frac{h}{L}\left(1 - \frac{h}{2L}\right)\left[V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right] + V(t_n + h) + \alpha(t_n + h)W(t_n + h)\right]$$

By setting

$$a = 1 - \frac{hR}{L} + \frac{h^2R}{2L^2} - \frac{h^3R}{6L^3} + \frac{h^4R}{24L^4}$$

$$b_n = \frac{h}{6L}\left(1 - \frac{h}{L} + \frac{h^2}{2L^2} - \frac{h^3}{4L^3}\right)(V(t_n) + \alpha(t_n)W(t_n)) + \frac{h}{3L}\left(1 + \left(1 - \frac{h}{2L}\right)^2\right)\left(V\left(t_n + \frac{h}{2}\right) + \alpha\left(t_n + \frac{h}{2}\right)W\left(t_n + \frac{h}{2}\right)\right) + \frac{h}{6L}\left(V(t_n + h) + \alpha(t_n + h)W(t_n + h)\right)$$

$I_{n+1} = aI_n + b_n, \; n = 0, 1, 2 \ldots.$

where

$$I_n = a^n I_0 + \sum_{i=0}^{n-1} a^{n-i-1} b_i, \quad n = 1, 2, 3, \ldots.. \tag{4.7}$$

The expectation of equation (4.7) is,

$$E[I_n] = 2a^n + \sum_{i=0}^{n-1} a^{n-i-1} \left(\frac{h}{6L} \left(1 - \frac{h}{L} + \frac{h^2}{2L^2} - \frac{h^3}{4L^3} \right) V(t_i) + \frac{h}{3L} \left(1 + \left(1 - \frac{h}{2L} \right)^2 \right) V\left(t_i + \frac{h}{2} \right) + \frac{h}{6L} V(t_i + h) \right)$$

$$Var[I_n] = 4a^{2n} + \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} a^{2n-i-j-2} Cov[b_i, b_j] \tag{4.8}$$

where,

$$Cov[b_i, b_j] = A_{i,j}\gamma(t_i - t_j) + B_{i,j}\gamma\left(t_i - t_j - \frac{h}{2}\right) + B_{j,i}\gamma\left(t_i - t_j + \frac{h}{2}\right) + C_{i,j}\gamma(t_i - t_j - h) + C_{j,i}\gamma(t_i - t_j + h)$$

$$A_{i,j} = \frac{h^2}{36L^2}\left(1 - \frac{h}{L} + \frac{h^2}{2L^2} - \frac{h^3}{4L^3}\right)^2 \alpha(t_i)\alpha(t_j) + \frac{h^2}{9L^2}\left[1 + \left(1 - \frac{h}{2L}\right)^2\right]^2 \alpha\left(t_i + \frac{h}{2}\right) \alpha\left(t_j + \frac{h}{2}\right) + \frac{h^2}{36L^2} \alpha(t_i + h)\alpha(t_j + h)$$

$$B_{i,j} = \frac{h^2}{18L^2}\left(1 - \frac{h}{L} + \frac{h^2}{2L^2} - \frac{h^3}{4L^3}\right)\left[1 + \left(1 - \frac{h}{2L}\right)^2\right]\alpha(t_i)\alpha\left(t_j + \frac{h}{2}\right) + \frac{h^2}{18L^2}\left[1 + \left(1 - \frac{h}{2L}\right)^2\right]\alpha\left(t_i + \frac{h}{2}\right)\alpha(t_j + h)$$

$$C_{i,j} = \frac{h^2}{36L^2}\left(1 - \frac{h}{L} + \frac{h^2}{2L^2} - \frac{h^3}{4L^3}\right)\alpha(t_i)\alpha(t_j + h) \qquad i, j = 0,1,2,3 \ldots \ldots . n - 1$$

Here, L=1, R=1, $V(t) = e^t$ & $\alpha(t) = \frac{cost}{25}$

Table 2. Expectation of Mean & Variance of I_n (t):

t	$h = \frac{1}{10}$		$h = \frac{1}{20}$		$h = \frac{1}{30}$		$h = \frac{1}{40}$		$h = \frac{1}{50}$	
	Mean	**Variance**	**Mean**	**Variance**	**Mean**	**Variance**	**Mean**	**Variance**	**Mean**	**Variance**
0	1.8504	3.2748	1.9538	3.6192	1.9677	3.7420	1.9756	3.8048	1.9804	3.8432
0.2	1.6999	2.6809	1.8707	3.2744	1.9117	3.5008	1.9330	3.6192	1.9461	3.6924
0.4	1.5580	2.1948	1.7959	2.9628	1.8594	3.2744	1.8928	3.4428	1.9134	3.5476
0.6	1.4339	1.7968	1.7284	2.6808	1.8110	3.0632	1.8552	3.2748	1.8827	3.4084
0.8	1.3250	1.4708	1.6687	2.4256	1.7671	2.8656	1.8205	3.1152	1.8543	3.2748
1.0	1.2368	1.2040	1.6173	2.1944	1.7279	2.6808	1.7892	2.9432	1.8282	3.1464
1.2	1.1633	0.9856	1.5752	1.9856	1.6943	2.5080	1.7618	2.6816	1.8051	3.0230
1.4	1.1064	0.8068	1.5431	1.7964	1.6669	2.3460	1.7387	2.5508	1.7854	2.9047
1.6	1.0670	0.6604	1.5227	1.6252	1.6465	2.1948	1.7208	2.4264	1.7696	2.7909
1.8	1.0460	0.5408	1.5154	1.4704	1.6345	2.0532	1.7086	2.3080	1.7586	2.6814
2.0	1.0446	0.4428	1.5231	1.3306	1.6322	1.9205	1.7038	2.3073	1.7528	2.5762

5. Conclusion

In this paper, the random stochastic initial value problem for a RL circuit is considered whose mean square convergence is proved. Numerical examples show that, even though the sufficient convergence conditions are not satisfied, the random Runge Kutta fourth order of RL circuit gives good results.

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