Burger Model And Analytic Conditional Approach

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Abstract: In this paper the range of (x,t) for the corresponding u involved in Burger's equation is obtained. The used methodology is C-R equation along with conditions u_x=v_t, u_t=-v_x. When approached through this method easily we can solve v for the given u and u for the given v simultaneously without loss of generality.

Keywords: Absord, Burger, L'Hospital,, Modern C-R, Root

1. Introduction

Burgers' equation is also termed as Navier–Stoke's equation. This equation doesn't bear the stress term. Bateman first introduced this Burgers' equation [1]. Different types of discrepancies that are involved in physical flow like sound and shock wave theory, hydrodynamic turbulence, dispersion in porous media, vorticity transportation, wave processes in thermo elastic medium, mathematical modeling of turbulent fluid, and continuous stochastic processes can be easily explained by this model [2–5].

This research paper elaborates the one dimensional nonlinear Burgers' equation:

$$u_t + \alpha u u_x = \nu u_{xx} \qquad \text{………..(1)}$$

in detail.

Rigorous efforts have been made to compute the accuracy and efficiency of various numerical schemes for Burgers' equation with various values of kinematic viscosity. Several analytical and numerical schemes have been solved with the help of Burgers' equation, for instance, Hofe–Cole transformation [5&6], finite element method [7], finite difference method [8], implicit finite difference method [9], compact finite difference method [10–12], Fourier Pseudo spectral method [13], Several interested readers also speculated [11] and obtained the optimal error estimation and benefited by this rule.

2. Proposed methodology:

The aim of this present research paper is to find the solution of (1) for a given $u(x,t)$ by using analyticity and Cauchy's Riemann equations. In this attempt, we observe that we have two types of analytic functions in which one contains AC and the other with NAC. For a given $u(x,t)$, we construct the burger v and then we use Milne's Thompson method for the construction of an analytic function.

Let the burger's analytic function be $f(z) = u + iv$ ………. (2)

By assuming that the equation (2) satisfy the CR conditions to construct v (x,t) ………. (3)

If equations (1) and (3) equated then, $g(x,t) = 0$ ………. (4)

Solving the equation (4) gives spatial co-ordinate (x) and the temporal co-ordinate (t) of the speed of fluid u (x,t).

Nomenclature:

MT - Milne's Thomson

C-R – Cauchy's Riemann

AC – Absordable Condition

NAC – Non-absordable Condition

Example 1:

Construction of the viscosity of fluid for a given speed of fluid and find its solution

Let $u(x,t) = t^3 - 3x^2t + 3t^2 - 3x^2 + 1$ ……….(5)

Step 1: Finding v

The burger's equation is given by $v = \frac{u_t + u(u_x)}{u_{xx}}$ ………. (6)

$u_x = -6tx - 6x, \; u_t = 3t^2 - 3x^2 + 6t \; and \; u_{xx} = -6t - 6$ ………. (7)

Using (6) in (5) we get.

$$v = \frac{[6x^3 - x^2 - 2x - 2xt^4 - 8xt^3 - 6xt^2 + 12x^3t + 6x^3t^2 - 2xt + t^2 + 2t]}{(-2t-2)}$$ ……….(8)

Step 2: Construction of Burger's analytic function

By MT method

$f_2(z,0) = v_x(z,0) = -9z^2 + z + 1$ And $f_1(z,0) = v_t(z,0) = \left(-3z^3 - \frac{1}{2}z^2 - 1\right)$

$f'(z) = f_1 + if_2$

Where $f_1 = -3z^3 - \frac{1}{2}z^2 - 1, f_2 = -9z^2 + z + 1$

Integrating both sides with respect to z we get

$f(z) = \left(\frac{-3}{4}z^4 - \frac{1}{6}z^3 - z\right) + i\left(-3z^3 + \frac{1}{2}z^2 + z\right)$ Is called the burger's analytic function if it satisfy C-R equations

Assume that it satisfy C-R equations, that is $u_x = v_t$ and $u_t = -v_x$

Now $v_t = -6tx - 6x$

$v(x,t) = -3xt^2 - 6xt$ ………. (9)

From (8) and (9)

We have (8) = (9) = v(x, t)

In (8), if $x = 0$ then $t = 0 \; or \; -2$, and

If $t = 0$ then $x = \frac{2}{3} \; or \; \frac{-1}{2}$

Hence the solution space is (x,t)=$\left\{(0,0), (0,-2), \left(\frac{2}{3}, 0\right), \left(\frac{-1}{2}, 0\right)\right\}$

Example 2:

Construction of the viscosity of fluid for a given speed of fluid and find its solution

Let $u(x,t) = x^3t^2 + x^2t + x + t^3x^2 + t^2x + t$ ………. (10)

Step 1: Corresponding v using (10)

$$\left.\begin{aligned} u_x &= 3x^2t^2 + 2xt + 2t^3x + t^2 + 1, \\ u_t &= 2x^3t + 3x^2t^2 + 2xt + x^2 + 1 \\ u_{xx} &= 6xt^2 + 2t^3 + 2t \end{aligned}\right\}$$ ………. (11)

Using (11) in (6) we get

$$v = \frac{\left(5x^4t^5 + 8x^3t^4 + 9x^2t^3 + 3x^2t^5 + 2x^3t + 3x^2t^2 + 2xt + 6x^3t^2 + 3x^2t + 4xt^2 + 3xt^4 + x^2 + x + 1 + t^3 + t^2\right)}{\left(6xt^2 + 2t^3 + 2t\right)}$$ ………(12)

Step 2: Construction of $f_2(z,0)$

$v_1 = \frac{(2x^3+3x^2+2x+1)}{(6xt+2t^2+2)}$	$(v_1)_x(z,0) = 3z^2+3z+1$
$v_2 = \frac{(6x^3+3x^2+4x)}{\left(6x+2t+\frac{2}{t}\right)}$	$(v_2)_x(z,0)=\frac{\infty}{\infty}$ form, $(v_2)_{xt}=\frac{[18x^2+6x+4]}{\left(6x+2t+\frac{2}{t}\right)}$ (By L hospital 's rule) $(v_2)_{xt}(z,0) = 0$
$v_3 = \frac{(x^2+x+1)}{(6xt^2+2t^3+2t)}$	$(v_3)_x(z,0)=\frac{0}{0}$ form, $(v_3)_{xt}=\frac{[(2x+1)(12xt+6t^2+2)-12t(x^2+x+1)]}{[2(6xt^2+2t^3+2t)(12xt+6t^2+2)]}$ $(v_3)_{xt}(z,0) = \frac{\infty}{\infty}$ form $(v_3)_{xtt}(z,0)=\frac{3}{2}(z^2-1)$ (By L hospital 's rule)
$v_4 = \frac{(5x^4+9x^2+1)}{\left(\frac{2}{t^2}+\frac{6x}{t}+2\right)}$	$(v_4)_x = \frac{(20x^3+18x)}{\left(\frac{2}{t^2}+\frac{6x}{t}+2\right)}$ $(v_4)_x(z,0) = 0$
$v_5 = \frac{(3x^5+8x^3+3x)}{\left(\frac{1}{t^2}\left[6x+2t+\frac{2}{t}\right]\right)}$	$(v_5)_x(z,0)=$ $(v_5)_{xt}(z,0) = (v_5)_{xtt}(z,0)=$ etc….= AC
$v_6 = \frac{(5x^4+3x^2)}{\left(\frac{1}{t^3}\left[6x+2t+\frac{2}{t}\right]\right)}$	$(v_6)_x(z,0)=$ $(v_6)_{xt}(z,0) = (v_6)_{xtt}(z,0)=$ etc….= AC

Hence $f_2 =$ Sum of derivatives of all v's at the point (z,0)

$= 3z^2+3z+1+0+\frac{3}{2}(z^2-1)+0+AC$

$=\frac{9z^2+6z-1}{2}+AC$

Step 3: Construction of $f_1(z,0)$

$v_1 = \frac{(2x^3+3x^2+2x+1)}{(6xt+2t^2+2)}$	$(v_1)_t(z,0) = \frac{-3}{2}(2z^3+3z^2+2z+1)$
$v_2 = \frac{(6x^3+3x^2+4x)}{\left(6x+2t+\frac{2}{t}\right)}$	$(v_2)_t(z,0)=$ $((v_2)_t)_t(z,0) = (((v_2)_t)_t)_t(z,0)=$..etc= AC
$v_3 = \frac{(x^2+x+1)}{(6xt^2+2t^3+2t)}$	$(v_3)_x(z,0)$ is in - determined form
$v_4 = \frac{(5x^4+9x^2+1)}{\left(\frac{2}{t^2}+\frac{6x}{t}+2\right)}$	$(v_4)_t(z,0)=$ $(v_4)_{tt}(z,0) = (v_4)_{ttt}(z,0)=$..etc= AC
$v_5 = \frac{(3x^5+8x^3+3x)}{\left(\frac{1}{t^2}\left[6x+2t+\frac{2}{t}\right]\right)}$	$(v_5)_t(z,0)=$ $(v_5)_{tt}(z,0) = (v_5)_{ttt}(z,0)=$..etc= AC
$v_6 = \frac{(5x^4+3x^2)}{\left(\frac{1}{t^3}\left[6x+2t+\frac{2}{t}\right]\right)}$	$(v_6)_t(z,0)=$ $(v_6)_{tt}(z,0) = (v_6)_{ttt}(z,0)=$..etc= AC

Hence $f_1 =$ Sum of derivatives of all$v$'s with respect to 't' at the point (z,0)

$= \frac{-3}{2}(2z^3 + 3z^2 + 2z + 1) + AC$

Step 4: Construction of Burger's analytic function

By MT method

$f'(z) = f_1 + if_2$

Where $f_1 = \frac{-3}{2}(2z^3 + 3z^2 + 2z + 1) + AC,\quad f_2 = \frac{9z^2+6z-1}{2} + AC$

Integrating both sides with respect to z, we get,

$f(z) = \frac{-3}{2}(2z^3 + 3z^2 + 2z + 1) + i\left(\frac{9z^2+6z-1}{2}\right) + AC$ is called the burger's analytic function if it satisfy C-R equations

Assume that it satisfy C-R equations,

$v_t = 3x^2t^2 + 2xt + 2t^3x + t^2 + 1$

Integrating both sides with respect to t

$$v = x^2t^3 + xt^2 + \frac{xt^4}{2} + \frac{t^3}{3} + t \quad \text{……….}(13)$$

In (12) and (13), if $x = 0$ we get $t = -0.60947$ and $0.72957,$ and

If $t = 0$ we get $x^2 + x + 1 = 0$. $x = \frac{-1}{2} + i\frac{\sqrt{3}}{2}$ & $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$

The solution of space is (x,t) = $\left\{(0, -0.60947),\ (0,\ 0.72957), \left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}, 0\right), \left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}, 0\right)\right\}$

3. Future work:

In future we will be applied the same proposed algorithm for generalized Burger Equation to check analyticity for corresponding u(x,t) to determine v(x,t).

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Reference

1. H. Bateman, Some recent researches on the motion of fluids, Mon. Weather Rev. 43 (1915) 163–170.
2. J.M. Burgers, Mathematical example illustrating relations occurring in the theory of turbulent fluid motion, Trans. Roy. Neth. Acad. Sci. Amsterdam 17 (1939) 1–53.
3. J.M. Burgers, A mathematical model illustrating the theory of turbulence, Adv. Appl. Mech, vol. 1, Academic Press, New York, 1948, pp. 171–199.
4. S.E. Esipov, Coupled Burgers' equations: a model of poly- dispersive sedimentation, Phys. Rev. 52 (1995) 3711–3718.
5. J.D. Cole, On a quasilinear parabolic equations occurring in aerodynamics, Quart. Appl. Math. 9 (1951) 225–236.
6. C.A.J. Fletcher, Generating exact solutions of the two dimensional Burgers' equation, Int. J. Numer. Meth. Fluids 3 (1983) 213–216.
7. E.N. Aksan, Quadratic B-spline finite element method for numerical solution of the Burgers' equation, Appl. Math. Comput. 174 (2006) 884–896.
8. I.A. Hassanien, A.A. Salama, H.A. Hosham, Fourth-order finite difference method for solving Burgers' equation, Appl. Math. Comput. 170 (2005) 781–800.
9. M.K. Kadalbajoo, K.K. Sharma, A. Awasthi, A parameter- uniform implicit difference scheme for solving time dependent Burgers' equation, Appl. Math. Comput. 170 (2005) 1365–1393.

10. W. Liao, An implicit fourth-order compact finite difference scheme for one-dimensional Burgers' equation, Appl. Math. Comput. 206 (2008) 755–764.
11. H.P. Bhatt, A.Q.M. Khaliq, Fourth-order compact schemes for the numerical simulation of coupled Burgers' equation, Comput. Phys. Commun. 200 (2016) 117–138.
12. R.K. Mohanty, W. Dai, F. Han, Compact operator method of accuracy two in time and four in space for the numerical solution of coupled viscous Burgers' equations, Appl. Math. Comput. 256 (2015) 381–393.

Rashid, A.I.Md. Ismail, A fourier pseudospectral method for solving coupled viscous Burgers' equations, Comput. Methods Appl. Math. 9 (2009) 412–420.

13. A.H. Salas, Symbolic computation of solutions for a forced Burgers equation, Appl. Math. Comput. 216 (1) (2010) 18–26.
14. S. Lin, C. Wang, Z. Dai, New exact traveling and non traveling wave solutions for (2+1) dimensional Burgers equation, Appl. Math. Comput. 216 (10) (2010) 3105–3110
15. Xu Min, Ren-Hong Wang, Ji-Hong Zhang, Qin Fang, A novel numerical scheme for solving Burgers' equation, Appl. Math. Comput. 217 (2011) 4473–4482.
16. G.W. Wei, D.S. Zhang, D.J. Kouri, D.K. Hoffman, Distributed approximation functional approach to Burgers' equation in one and two space dimensions, Comput. Phys. Commun. 111 (1998) 93–109.