On Strong Forms of Generalized Closed Sets in Micro Topological Spaces

R. Bhavani
PG and Research Department of Mathematics,MannarThirumalai Naicker College (AUTONOMOUS), Pasumalai, Madurai – 625004, Tamilnadu.
E-mail: bhavani120475@gmail.com.

Article History: Received: 11 January 2021; Revised: 12 February 2021; Accepted: 27 March 2021; Published online: 10 May 2021

Abstract. In this paper, we introduce different types of Micro generalized closed sets in Micro topological spaces and their properties. Also we introduce a new class of sets namely Micro-$\ddot{g}$– closed sets in Micro topological spaces and this is another generalization of closed sets and proved that the class of Micro- $\ddot{g}$– closed sets.

Key words and phrases. Micro-$\ddot{g}$ – closed sets, Micro -αg -closed sets, Micro -sg -closed sets, Micro-gs -closed sets.

1. INTRODUCTION

Levine **[7]** when he introduced generalized closed sets in general topology as a generalization of closed sets.This concept was comparing the closure ofa subset with its open supersets. Hariwan Z .Ibrahim **[1]** introduced the Micro -g-closed set and Micro -open set in Micro topological Spaces .O. Ravi and Ganesan **[8]** defined and studied $\ddot{g}$ -closed sets in general topology. S. Chandrasekar, G. Swathi **[6]** introduced the Micro -α – open sets, Micro -α -interior and Micro -α – closure in Micro topological Spaces. S.P. Arya and T. Nour **[2]** introduced the notion of generalized Semi- closed sets in topological Spaces. In 1987 Bhattacharyya and Lahiri**[3]** defined and studied the concept of Semi-generalized closed sets in topological Spaces. In1994, Maki, R. Devi and Balachandran**[4]** introduced the class of α – generalized closed sets in topological Spaces. Rough set theory was introduced by Pawlak **[9]** to organize and analyze various types of data in data mining. It was used the lower and upper approximations of decision classes. The concept of nano topology was introduced by M.L.Thivagar et al **[10,11]** which was defined in terms of lower and upper approximations. The concept of Micro -open set in Micro topological Space was introduced and investigated by S. Chandrasekar **[5]**.

2. PRELIMINARIES

Definition 2.1 [9]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.
That is, $L_R(X) = \bigcup_{x\in U} \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U} \{R(x): R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [10]

If (U, R) is an approximation space and $X, Y \subseteq U$; then
1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3 [10,11]

Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4 [10]

If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Throughout this thesis, $(U,\tau_R(X))$ (briefly, U) will denote the nano topological space. The elements of are called nano open sets.

Definition 2.2. [5]

The Micro topology $\mu_R(X)$ satisfies the following axioms

1. $U,\varphi \in \mu_R(X)$
2. The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$
3. The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$. Then $\mu_R(X) = \{N \cup (N' \cap \mu)\}: N, N' \in \tau_R(X)$ and $\mu \notin \tau_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U,\tau_R(X),\mu_R(X))$ is called Micro topological spaceand the elements of $\mu_R(X)$ are called Micro- open sets and the complement of a Micro -open set is called a Micro closed set.

Definition 2.3. [5]

The Micro closure of a set A is denoted by Micro-cl(A) and is defined as Mic-cl(A)=$\cap$\{B:B is Micro closed and $A\subseteq B$\}.

The Micro interior of a set A is denoted by Micro-int(A) and is defined as Mic-int(A)=$\cup$\{B:B is Micro open and $A\supseteq B$\} .

Definition 2.7. [5]

For any two Micro sets A and B in a Micro topological space$(U,\tau R(X),\mu R(X))$

1. A is a Micro closed set if and only if Mic-cl(A)=A.
2. A is a Micro open set if and only if Mic-int(A)=(A) .
3. $A \subseteq B$ implies Mic-int(A)$\subseteq$ Mic-int(B) and Mic-cl(A)$\subseteq$ Mic-cl(B)
4. Mic-cl(Mic-cl(A))=Mic-cl(A)and Mic-int(Mic-int(A))=Mic-int(A).
5. Mic-cl($A\cup B$)$\supseteq$Mic-cl(A)$\cup$ Mic-cl(B)
6. Mic-cl($A\cap B$)$\subseteq$Mic-cl(A)$\cap$Mic-cl(B)
7. Mic-int($A\cup B$)$\supseteq$ Mic-int(A) $\cup$Mic-int(B)
8. Mic-int($A\cap B$)$\subseteq$ Mic-int(A) $\cap$ Mic-int(B)
9. Mic-cl(A^C)=[Mic-int(A)]^C
10. Mic-int(A^C)=[Mic-cl(A)]^C

Definition 2.7

A subset A of a Micro topological space U is called

(i) Micro-semi-open set **[5]** if A$\subseteq$ Mic -cl(Mic-int(A));

(ii) Micro-α -open set **[6]** if A $\subseteq$Mic-int(Mic-cl(Mic-int(A)));

The complements of the above mentioned Micro-open sets are called their respective Micro-closed sets.

The Micro -semi-closure (resp. Micro-α -closure) of a subset A of U, denoted by Mic-scl(A) (resp. Mic-α -cl(A)) is defined to be the intersection of all Micro -semi-closed (resp. Mic-α -closed) sets of U containing A. It is known that Mic-scl(A) (resp. Mic-α cl(A)) is a Mic-semi-closed (resp. Mic-α -closed) set.

Definition 2.8

(i)Let $(U,\tau R(X),\mu R(X))$ be a Micro topological space. A subset A of a Microtopological space U is calledMic-g-closed set **[1]** if Mic-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is Mic-open in U. The complement of Micro -g-closed set is called Micro-g-open set;

(ii) sg-closed set **[3]** if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in U. The complement of sg-closed set is called sg-open set;

(iii) gs-closed set **[2]** ifscl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ-open in U. The complement of gs-closed set is called gs-open set;

(iv) α g-closed set **[4]** if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ-open in U. The complement of α g-closed set is called α g-open set;

The complements of the above mentioned closed sets are called their respective open sets.

3. Micro -$\ddot{g}$– Closed - sets

Definition 3.1

In this section, we define Micro-$\ddot{g}$– Closed sets adsome of their properties are discusedaboutclosed sets.

Definition 3.1

Let (U,τR(X),μR(X)) be a Micro topological space and A ⊂ U .Then A is said to be Micro-$\ddot{g}$– Closed sets if Mic-cl(A) ⊆ **U** whenever A ⊆U and U is Micro-sg-open in U.

Micro - $\ddot{g}_{\alpha}$- Closed - sets

Definition 3.2.

A subset A of a Micro topological space (U,τR(X),μR(X)) is called

Micro - $\ddot{g}_{\alpha}$ -closed set if Mic- α cl(A) ⊆ U whenever A ⊆ U and U is Micro - sg-open in U.

The complements of the above mentioned closed set is called Micro - $\ddot{g}_{\alpha}$ - open set

Micro - αg- Closed - sets

Definition 3.3

A subset A of a Micro topological space (U, τR(X), μR(X)) is called

Mic- α g-closed set if Mic-αcl(A) ⊆ U whenever A ⊆ U and U is Micro-open in U.

The complement of Mic- α g-closed set is called Mic- α g-open set;

Micro - sg- Closed – sets

Definition 3.4

A subset A of a Micro topological space (U, τR(X), μR(X)) is called Mic-sg-closed set if Mic-scl(A) ⊆ U whenever A ⊆ U and U is Mic-semi-open in U.

The complement of Mic-sg-closed set is called Mic- sg-open set.

Micro - gs- Closed - sets

Definition 3.5

A subset A of a Micro topological space (U, τR(X), μR(X)) is called Mic-gs-closed set if Mic-scl(A) ⊆ U whenever A ⊆ U and U is Micro-open in U.

The complement of Mic-gs-closed set is called Mic-gs-open set

Remark 3.6

(i) Every Micro-closed set is Micro-semi-closed but not conversely

(ii) Every Micro-closed set is Micro- α -closed but not conversely

(iii) Every Micro -semi-closed set is Micro- sg -closed but not conversely

(iv) Every Micro-sg-closed set is Micro-gs-closed but not conversely

(v) Every Micro-g-closed set is Micro- α g-closed but not conversely

(vi) Every Micro-g-closed set is Micro-gs-closed but not conversely

Proposition 3.7

Every Micro -α-closed set is Micro - $\ddot{g}_{\alpha}$ -closed set

Proof

If A is a Micro- α -closed subset of U and G is any Micro-sg-open set containing A, then G ⊇ A = Mic-αcl(A). Hence A is Micro- $\ddot{g}_{\alpha}$ -closed set in U.

The converse of the above Proposition 3.7 need not be true as seen from the following example.

Example 3.8

Let U ={a, b, c, d} with U/R={{a}, {c}, {b, d}}and X= {b, d}**)** ⊆ **U** . Then τ_R(X**)**{{ϕ, U, {b, d**}}.** Let μ = {b} Then Micro-O (X)=μR(X)= { φ,U {b}, {b, d}} Then Mic- α -closed ={φ,U,{a},{c},{d},{a,c},{a,d},{c,d},{a,c,d} and Mic- $\ddot{g}_{\alpha}$ -closed **=**

{ φ,U,{a},{b],{c},{d},{a,b},{a,c},{b,c}{a,d},{b,d},{c,d},{a,b,c},{a,c,d},{b,c,d}, {a,b,d}. Clearly,the set {a,b}is a Micro- $\ddot{g}_{\alpha}$ -closed set but not a Micro- α -closed set in U.

Proposition 3.9

Every Micro -closed set is Micro- $\ddot{g}$ -closed.

Proof

If A is a Micro- closed subset of U and G is any Micro-sg-open set containing A, then G ⊇ A =Mic-cl(A) . Hence A is Micro- $\ddot{g}$ -closed in U.

The converse of Proposition 3.9 need not be true as seen from the following example

Example 3.10

Let U = {1, 2, 3, 4} with U/R={{1}, {3}, {2, 4}}and X= {1, 2}**)** ⊆ **U** . Then τ_R(X)={ϕ, U, {1}, {2,4},{1,2,4**}}.** Let μ = {3} Then Micro-O (X)=μR(X)= { φ,U {1}, {3}, {2,4},{1,3},{1,2,4},{2,3,4}} Then Mic- closed (X) ={φ,U,{1},{3},{1,3},{2,4,},{1,2,4,},{2,3,4}} and Mic- $\ddot{g}$ **-closed(X)=**

{φ,U,{1},{2],{3},{4},{1,3},{2,3},{2,4}{1,2,4},{2,3,4}. Clearly,the set {2,3}is a Micro- $\ddot{g}$ -closed set but not a Micro -closed set in U

Proposition 3.11
Every Micro- $\ddot{g}$ -closed set is Micro-g-closed.
Proof
If A is a Micro- $\ddot{g}$ -closed subset of U and G is any Micro-open set containing A, since every Micro- open set is Mic-sg-open ,we have A⊆G= Micro-cl(A). Hence A is Micro -g-closed in U.
The converse of Proposition 3.11 need not be true as seen from the following example.
Example 3.12
Let U = {p, q, r, s} with U/R={{p}, {q,r,s}}and X= {p, q}**)** ⊆ **U** . Then τ_R(X**)** ={ϕ, U, {p}, {q},{p,q}**}.** Let μ = {s} Then Micro-O (X)=μR(X)= { φ,U {p}, {q}, {p,q},{p,s},{q,s},{p,q,s}} Then Mic-g -closed (X) ={φ,U,{r},{p,r},{q,r},{r,s,},{p,q,r},{p,r,s},{q,r,s}} and Mic- $\ddot{g}$ **-**closed **(X)=**
{φ,U,{r},{p,r},{q,r},{r,s}{p,r,s},{q,r,s}. Clearly,the set {p,q,r}is a Micro- g-closed set but not a Micro - $\ddot{g}$ **-** closed set in U
Proposition 3.13
Every Micro- $\ddot{g}_\alpha$ -closed set is Micro - αg-closed
Proof
If A is a Micro- $\ddot{g}_\alpha$ -closed subset of U and G is any Micro-open set containing A ,then G⊇A= Mic- α cl(A) ⊇ Mic-cl(A). Hence A is Micro - - αg-closed in U
The converse of Proposition 3.13 need not be true as seen from the following example.
Example 3.14
In Example3.10. Clearly , the set {3,4} is a Micro -αg-closed but not in Micro- $\ddot{g}_\alpha$ -closed set.
Proposition 3.15
Every Micro-semi closed set is Micro- sg -closed.
Proof
If A is a Micro-semi closed subset of U and G is any Micro-semi open set containing A, we have Mic-scl(A) = A ⊆ G. Hence A is Micro-sg-closed in U.
The converse of Proposition 3.15 need not be true as seen from the following example.
Example 3.16
In Example 3.10. Clearly, the set {1, 2, 3} is a Micro-sg -closed but not inMicro- semi closed set in U.
Remark 3.17
The concept of Micro- $\ddot{g}_\alpha$ -closed sets and Micro- $\ddot{g}$ -closed sets are independent.
Example 3.18
(i) In Example 3.8. Then {a,d} is Micro- $\ddot{g}_\alpha$ -closed set but not in Micro- $\ddot{g}$ -closed set.
(ii) In Example 3.10. Then {4} is Micro- $\ddot{g}$ -closed set but not in Micro- $\ddot{g}_\alpha$ -closed set.
Proposition 3.19
Every Micro -sg- closed set is Micro-gs-closed.
Proof
If A is a Micro-sg- closed subset of U and G is any Micro-open set containing A, since every Micro-open set is Micro-semi-open set, we have G ⊇ Mic-scl(A). Hence A is Micro -sg-closed in U.
The converse of Proposition 3.19 need not be true as seen from the following example.
Example 3.20
In Example 3.12. Clearly, the set {p, q, r} is a Micro-gs-closed set but not inMicro-sg- closed set in U.
Proposition 3.21
Every Micro- g -closed set is Micro- αg -closed.
Proof
If A is a Micro-g- closed subset of U and G is any Micro-open set containing A, we have Micro- α cl(A) ⊆U. Hence A is Micro - αg -closed in U.
The converse of Proposition 3.21 need not be true as seen from the following example.
Example 3.22
In Example 3.12, Clearly, the set{s} is aMicro- α g-closed set but not in Micro -g -closed.
Proposition 3.23
Every Micro- $\ddot{g}$ -closed set is Micro-sg -closed.
Proof

If A is a Micro-$\ddot{g}$ -closed subset of U and G is any Micro- semi-open set containing A, since every Micro-semi-open set is Micro-sg-open, we have G $\supseteq$ Mic-cl(A) $\supseteq$ Mic-scl(A). Hence A is Micro-sg-closed in U.
The converse of Proposition 3.23 need not be true as seen from the following example.

Example 3.24

In Example 3.10. Clearly, the set {3,4} is a Micro-sg-closed but not in Micro-$\ddot{g}$ -closed set in U.

Proposition 3.25

Every Micro- g -closed set is Micro-gs-closed.

Proof

If A is a Micro-g- closed subset of U and G is any Micro-open set containing A, we have G $\supseteq$Mic-cl(A) $\supseteq$Mic-scl(A). Hence A is Micro-gs-closed set in U.
The converse of Proposition 3.25 need not be true as seen from the following example.

Example 3.26

In Example 3.8. Clearly, the set {d} is a Micro-gs-closed but not inMicro-g- closed set in U.

Proposition 3.27

Every Micro -closed set is Micro-semi -closed.

Proof

If A is a Micro- closed subset of U and G is any Micro-semi -open set, we have Mic-(cl (Mic int (A)))$\subseteq$ G. Hence A is Micro-semi- closed set in U.
The converse of Proposition 3.27 need not be true as seen from the following example.

Example 3.28

In Example 3.12, clearly the set{p, s} is a Micro-semi -closed set but not in Micro -closed set in U.

Proposition 3.29

Every Micro -closed set is Micro-α closed.

Proof

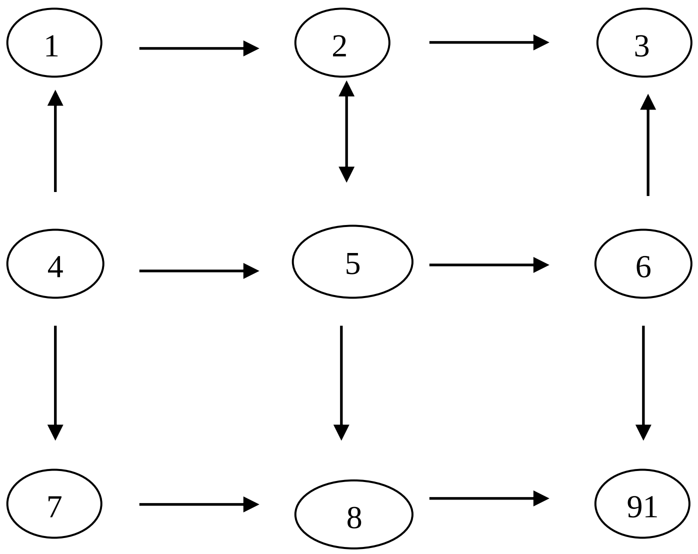
If A is a Micro-closed subset of U and G is any Micro-α- open set containing A, we have Mic(cl(Mic(int Mic cl(A))))= A $\subseteq$ G. Hence A is Micro-α -closed set in U.
The converse of Proposition 3.29 need not be true as seen from the following example.

Example 3.30

In Example 3.8. Then { c, d} is Micro-α -closed set but not in Micro -closed

Remark 3.34

From the above Propositions, Examples and Remark, we obtain the following diagram, where A $\rightarrow$ B (resp. A B) represents A implies B but not conversely (resp. A and B are independent of each other).



where

(1)	Micro-α -closed	(6)	Micro- g-closed
(2)	Micro-$\ddot{g}_{\alpha}$ -closed	(7)	Micro-semi-closed
(3)	Micro-α g-closed	(8)	Micro-sg- closed
(4)	Micro -closed	(9)	Micro-gs-closed

(5) Micro- $\ddot{g}$ -closed

4. Conclusion

In this paper we presented some strong forms of generalized closed sets, We discussed about properties and various new type of Micro-generalized closed sets in Micro topological spaces.Also we introduced Micro-$\ddot{g}$ –closed sets, Micro -αg -closed sets, Micro -sg -closed sets and Micro-gs -closed sets in Micro topological spaces. Later on Research be reached out with certain applications.

Reference

1. Hariwan Z . Ibrahim, On Micro $T_{1/2}$ Space , International Journal of Applied Mathematics, Volume 33 , No. 3, 2020,369-384.
2. S.P. Arya and T. Nour, Characterizations of S -normal Space ,Indian Journal of Pure and Applied Mathematics, 21(1990),717-719.
3. Bhattacharyya and Lahiri, Semi generalized closed sets in topology, Ind.J.Math., 29(1987), 375-382.
4. Maki, Devi and Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets,Mem.Fac. Sci. Kochi Uni.Ser.A.Math., 15(1994),51-63.
5. S. Chandrasekar, On Micro Topological Spaces, Journal of New Theory, 26(2019), pages 23-31.
6. S. Chandrasekar, G. Swathi, Micro- α-open Sets in Micro Topological Spaces, International Journal of Research in Advent Technology, Vol.6, No.10. October 2018 ,2633-2637.
7. Levine,N.: Generalized Closed Sets in topology , Rend. Circ. Math. Palermo, 19(2)(1970), 89-96.
8. Ravi. O. and Ganesan, S.: $\ddot{g}$ -closed sets in topology , International Journal of Computer Science and Emerging Technologies, 2(2011),330-337.
9. Z. Pawlak, Rough Sets, International Journal of Information and Computer Sciences , 11(1982), 341-356.
10. M.L.Thivagar and C.Richard, Nano forms of Weakly Open Sets, International journal of Mathematics and Statistics Invention,Vol.1, Issue 1,Aug(2013). Pp 31-37.
11. M.L. Thivagar, C. Richarad and N.R.Paul, Mathematical Innovations of a Modern Topology in Medical Events, international Journal of Information Science, Vol.2, No.4(2012),33-36