

Heat And Mass Transfer With Radiation In A Convective Flow Between Vertical Wavy Channels Due To Travelling Thermal Waves

R. Sakthikala⁽¹⁾, K. Sumathi⁽²⁾, T. Arunachalam⁽³⁾

(1) Assistant Professor of Mathematics, PSGR KRISHNAMMAL COLLEGE FOR WOMEN, Coimbatore-641 004, India

(2) Associate Professor in Mathematics, PSGR KRISHNAMMAL COLLEGE FOR WOMEN, Coimbatore-641 004, India

(3) Professor of Mathematics, KUMARAGURU COLLEGE OF TECHNOLOGY, Coimbatore-641 049, India

Article History: Received: 11 January 2021; Accepted: 27 February 2021; Published online: 5 April 2021

Abstract: The objective of this paper is to investigate analytically the convective flow of heat and mass transfer in vertical wavy channels due to travelling thermal waves. Effect of radiation, temperature dependent heat source/sink, concentration dependent mass source are taken into account. To tackle the highly complex non-linear problem, the perturbation technique is applied with long wave approximation.

Keyword: Heat transfer, convective flow, wavy channel, mass transfer, travelling thermal waves.

1. INTRODUCTION

The problem of heat and mass transfer offluid flows during a porous medium is of importance in geophysics, geothermal, metallurgy, aerodynamics, extrusion of plastic sheets and other engineering processes such as transpiration cooling of vehicles re-entry, missile launching, equilibrium ablation in chemically reacting flow fields and film atomization in combustion chambers.

In view of these applications, Lekoudis *et al* [1976] used a semi-analytical technique to work out the compressible viscous flows past wavy walls without restricting the mean flow which was linear within the disturbance sub-layer. Shankar and Sinha [1976] have made a thorough study of flow generated during a viscous fluid by impulsive motion of a wavy wall and received certain interesting conclusions, that in terms of low Reynolds numbers the waviness of the wall quickly terminate to be of much more importance because the liquid will be dragged along the wall, while at high Reynolds numbers the consequences of viscosity are confined to a thin layer close to the wall, and therefore the known potential solution emerges in time. Lessen and Gangwani [1976] have made a very interesting analysis of the results of small amplitude wall waviness upon the stability of the laminar boundary layers. Vajravelu and Sastri [1978] dedicated their work towards the effect of waviness on one of the walls on the flow and heat-transfer characteristics of an incompressible viscous fluid confined between two long vertical walls and set in motion by a temperature difference between the walls, with a focus on various parameters.

Later, Vajravelu and Sastri [1980] investigated the matter of natural convection heat transfer in vertical wavy channels. Vajravelu and Debnath [1986] made a stimulating and an in-depth study of nonlinear convection heat transfer and fluid flows, in an incompressible viscous fluid confined to a wavy channel in four geometrical configurations which gave a special attention to the heat transfer results which can have definite pertaining to the design of oil-or gas-fired boilers. Umavathi and Shekar [2011] paid attention towards the study of flow and heat transfer during a vertical wavy channel, containing porous layer saturated with a fluid and a transparent viscous fluid layer, where, the porous matrix is assumed to be sparse and therefore the Darcy-Brinkman model was adopted to explain the fluid flow in porous medium region. Raju and Muralidhar [2012] made an effort to analyse the unsteady convective heat and mass transfer flow of viscous, electrically conducting fluid confined during a vertical channel on whose walls an oscillatory temperature and concentration are prescribed with the effect of chemical reaction and heat source.

The objective of this study is to carry out research on the heat and mass transfer of a convective flow in vertical wavy channel with travelling thermal waves. Here we've extended the work of Vajravelu [1989] to understand the effect of radiation, temperature dependent heat source/sink, concentration dependent mass source. To tackle the highly complex non-linear problem, using radio emission approximation, the perturbation technique is applied to solve the governing equations. We've introduced Boussinesq approximation. Relative solutions for velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number are obtained.

2. Mathematical Formulation

We consider convective heat transfer fluid flow in an incompressible electrically conducting viscous fluid with radiation and mass transfer in a vertical wavy channel with the x-axis vertically upward and parallel to the direction of buoyancy and the y-axis normal to it. The wavy walls are represented in terms of $y = d + a \cos \lambda x$ and $y = -d + a \cos (\lambda x + \theta)$.

In the present work, the following assumptions are made:

- ❖ Two dimensional flow of a Newtonian fluid is considered, which is unsteady, viscous, laminar and oscillatory in nature.
- ❖ The viscous dissipation and work done by pressure are small when compared to the heat flow by conduction and the wall temperature.
- ❖ In the application of the Boussinesq approximation, all fluid properties are considered constant except density, which is assumed to differ with temperature.
- ❖ In the wavy walls, the wave length proportional to $\frac{1}{\lambda}$ is large and the electric field is zero.
- ❖ As compared to the applied magnetic field, the induced magnetic field is insignificant.
- ❖ A temperature dependent heat source and a variable mass source is assumed to present.

From the above mentioned assumptions, the governing equations like continuity, momentum and concentration are framed as follows,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma B_0^2 u - \rho g - \frac{u}{\kappa} \tag{2}$$

$$\rho \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{v}{\kappa} \tag{3}$$

$$\rho C_p \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q(T - \hat{T}_1) - \frac{\partial q_r}{\partial y} \tag{4}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + Q'(C - \hat{C}_1) \tag{5}$$

where u, v are the velocity components, T is the temperature, C is the concentration, p the pressure, B_0 is the transverse magnetic field, σ is the coefficient of electric conductivity, Q is the constant heat addition/absorption, ∇^2 is the two dimensional Laplacian, D is the molecular diffusive rate. The corresponding boundary conditions are

$$u = v = 0 ; T = \hat{T}_1 = T_1 [1 + \varepsilon \cos(\lambda x + wt)]; C = \hat{C}_1; \tag{6}$$

at $y = d + a \cos \lambda x$

$$u = v = 0 ; T = \hat{T}_2 = T_2 [1 + \varepsilon \cos(\lambda x + wt)]; C = \hat{C}_2; \tag{7}$$

at $y = -d + a \cos (\lambda x + \theta)$

The boundary conditions on the temperature field indicate physically that there are travelling thermal waves in the negative x direction. Introducing the non – dimensional variables

$$x^* = \frac{x}{d}; y^* = \frac{y}{d}; t^* = \frac{v}{d^2} t; u^* = \frac{d}{\nu} u; v^* = \frac{d}{\nu} v; \tag{8}$$

$$p^* = \frac{d^2}{\rho \nu^2} p; T^* = \frac{T - \hat{T}_1}{\hat{T}_2 - \hat{T}_1}; C^* = \frac{C - \hat{C}_1}{\hat{C}_2 - \hat{C}_1};$$

where $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity and the Boussinesq approximation

$$\rho = \rho_0 [1 - \beta(T - \hat{T}_1) - \beta'(C - \hat{C}_1)]$$

Radioactive heat flux is represented by the following form (Cogley *et al* [1968])

$$\frac{\partial q_r^*}{\partial y^*} = 4(T^* - T_\infty^*) I^*$$

where $I^* = \int K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T^*} d\lambda$

is the absorption coefficient at the plate and $e_{b\lambda}$ is plank constant.

Under this assumption and by introducing non-dimensional parameters, the basic equations with the boundary conditions (1) – (7) can be expressed in the non-dimensional form as

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{9}$$

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \left[\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right] - Mu^* +$$

$$GrT^* + GmC^* - \frac{u^*}{Da} \tag{10}$$

$$\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \left[\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right] - \frac{v^*}{Da} \tag{11}$$

$$\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Pr} \left[\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} \right] - \frac{1}{Pr} (PrF - Q)T^* \tag{12}$$

$$Sc \left[\frac{\partial C^*}{\partial t^*} + u^* \frac{\partial C^*}{\partial x^*} + v^* \frac{\partial C^*}{\partial y^*} \right] = \left[\frac{\partial^2 C^*}{\partial x^{*2}} + \frac{\partial^2 C^*}{\partial y^{*2}} \right] + QmC^* \tag{13}$$

with the boundary conditions

$$u^* = v^* = 0; T^* = \hat{T}_1 = T_1[1 + \varepsilon \cos(\lambda x + wt)]; C^* = \widehat{C}_1;$$

$$at y^* = d + a \cos \lambda x$$

$$u^* = v^* = 0; T^* = \hat{T}_2 = T_2[1 + \varepsilon \cos(\lambda x + wt)]; C^* = \widehat{C}_2;$$

$$at y^* = -d + a \cos (\lambda x + \theta) \tag{14}$$

Where $M, Gr, Gm, Pr, \alpha, Sc$ and ε are the Hartmann number, Grashoff number, modified Grashoff number, Prandtl number, the heat source/sink parameter, Schmidt number and the amplitude parameter, $\lambda = \lambda^* = \lambda d$, the non-dimensional wave number.

$$M = \frac{\sigma B_0^2 d^2}{\rho_0 \nu}; Gr = \frac{d^3 g \beta (\hat{T}_2 - \hat{T}_1)}{\nu^2}; Gm = \frac{d^3 g \beta' (\widehat{C}_2 - \widehat{C}_1)}{\nu^2};$$

$$Pr = \frac{\mu C_p}{k}; \alpha = \frac{Q d^2}{k(\hat{T}_2 - \hat{T}_1)}; Sc = \frac{\nu}{D}; \varepsilon = \frac{a}{d} \tag{15}$$

Removing the asterisks the governing equations (9) – (14) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{16}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - Mu + GrT + GmC - \frac{u}{Da} \tag{17}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{v}{Da} \tag{18}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{Pr} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{1}{Pr} (PrF - Q)T \tag{19}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{1}{Sc} + \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{Qm}{Sc} C \tag{20}$$

Corresponding boundary conditions are,

$$u = v = 0; T = \hat{T}_1 = T_1[1 + \varepsilon \cos(\lambda x + wt)]; C = \widehat{C}_1;$$

$$at y = d + a \cos \lambda x \tag{21}$$

$$u = v = 0; T = \hat{T}_2 = T_2[1 + \varepsilon \cos(\lambda x + wt)]; C = \widehat{C}_2;$$

$$at\ y = -d + \text{acos}(\lambda x + \theta) \tag{22}$$

Introducing the stream function ψ defined by

$$u = -\frac{\partial\psi}{\partial y} = -\psi_y; v = \frac{\partial\psi}{\partial x} = \psi_x; \tag{23}$$

Applying (23) in (17) – (22) and eliminating the non-dimensional pressure p , we get

$$-\psi_{yt} + \psi_y\psi_{xy} + \psi_x(-\psi_{yy}) = -\frac{\partial P}{\partial x} + (-\psi_{xxy} - \psi_{yyy}) - \left(M + \frac{1}{Da}\right)(-\psi_y) + \frac{GrT + GmC}{\tag{24}}$$

$$\psi_{xt} - \psi_y\psi_{xx} + \psi_x\psi_{xy} = -\frac{\partial P}{\partial y} + \psi_{xxx} + \psi_{xyy} - \frac{1}{Da}\psi_x \tag{25}$$

Differentiating (24) and (25) with respect to y and x , subtracting we get

$$\psi_{xxt} + \psi_{yxt} - \psi_y(\psi_{xxx} + \psi_{xyy}) + \psi_x(\psi_{xxy} + \psi_{yyy}) = (\psi_{xxxx} + 2\psi_{xxyy} + \psi_{yyy}) - \left(M + \frac{1}{Da}\right)\psi_{yy} - \frac{1}{Da}\psi_{xx} - GrT_y - GmC_y \tag{26}$$

$$Pr[T_t - \psi_y T_x + \psi_x T_y] = T_{xx} + T_{yy} + (PrF - Q)T \tag{27}$$

$$Sc[C_t - \psi_y C_x + \psi_x C_y] = C_{xx} + C_{yy} + Q_m C \tag{28}$$

where the subscripts denote partial differentiation. The boundary conditions are written in terms of ψ as

$$\psi_y = 0; \psi_x = 0; T = 0; C = 0; \text{ at } y = 1 + \varepsilon \cos \lambda x \tag{29}$$

$$0; \psi_y = 0; T = 1; C = 1; \text{ at } y = -1 + \varepsilon \cos(\lambda x + \theta) \tag{30}$$

$$\psi_y =$$

3. Method of Solution

Now assuming that the solution is of mean part and a perturbed part, applying method of perturbation to velocity, temperature and concentration equations, respectively

$$\psi(x, y, t) = \psi_0(y) + \varepsilon e^{i\lambda x + i\omega t} \psi_1(y) + \dots$$

$$T(x, y, t) = T_0(y) + \varepsilon e^{i\lambda x + i\omega t} T_1(y) + \dots$$

$$C(x, y, t) = C_0(y) + \varepsilon e^{i\lambda x + i\omega t} C_1(y) + \dots \tag{31}$$

Hence applying (23) in (26) – (30), we obtain

zeroth order or mean part equations as given below

$$\psi_{0\ yyy} - \left(M + \frac{1}{Da}\right)\psi_{0\ yy} - GrT_{0\ y} - GmC_{0\ y} = 0$$

$$T_{0\ yy} - (PrF - Q)T_0(y) = 0$$

$$C_{0\ yy} + Q_m C_0(y) = 0 \tag{32}$$

subject to the boundary conditions

$$\psi_0' = 0; \psi_0 = 0; T_0 = 0; C_0 = 0; \text{ at } y = 1; \psi_0' = 0; \psi_0 = 0; T_0 = 1; C_0 = 1; \text{ at } y = -1 \tag{33}$$

The first order equations or Perturbed Part equation are given as

$$\psi_{1\ yyy} - i\omega(\psi_{1\ yy} - \lambda^2\psi_1) + i\lambda\psi_{0\ y}(\psi_{1\ yy} - \lambda^2\psi_1) -$$

$$i\lambda\psi_1\psi_{0\ yyy} - 2\lambda^2\psi_{1\ yy} + \lambda^4\psi_1 - \left(M + \frac{1}{Da}\right)\psi_{1\ yy} -$$

$$-GrT_{1\ y} - GmC_{1\ y} = 0 \tag{34}$$

$$Pr[(i\omega)T_1(y) - \psi_{0\ y}i\lambda T_1(y) + i\lambda\psi_1 T_{0\ y}] =$$

$$-\lambda^2 T_1(y) + T_{1yy} - (PrF - Q)T_1 \quad (35)$$

$$Sc[(i\omega)C_1(y) - \psi_{0y}i\lambda C_1(y) + i\lambda\psi_1 C_{0y}] = -\lambda^2 C_1(y) + C_{1yy} + Q_m C_1 \quad (36)$$

subject to the boundary conditions

$$\psi_{1y} = -\psi_{0yy}e^{-i\omega t}; \psi_1 = 0; T_1 = -e^{-i\omega t}T_{0y}; C_1 = -e^{-i\omega t}C_{0y}$$

at $y = 1$

$$\psi_{1y} = -\psi_{0yy}e^{[i(v-\omega t)]}; \psi_1 = 0; T_1 = -e^{[i(v-\omega t)]}T_{0y}; C_1 = -e^{[i(v-\omega t)]}C_{0y}$$

at $y = -1$ (37)

The corresponding are the solutions in terms of zeroth order or mean solution

$$T_0 = A_1 e^{m_1 y} + A_2 e^{-m_1 y} \quad (38)$$

$$C_0 = B_1 e^{i\sqrt{Q_m} y} + B_2 e^{-i\sqrt{Q_m} y} \quad (39)$$

$$\psi_0 = A_3 + A_4 y + A_5 e^{m_2 y} + A_6 e^{-m_2 y} + E_6 e^{m_1 y} + E_7 e^{-m_1 y} + F_6 e^{i\sqrt{Q_m} y} + F_7 e^{-i\sqrt{Q_m} y} \quad (40)$$

For small values of λ , we can expand ψ_1, T_1, C_1 in terms of λ so that

$$\begin{aligned} \psi_1 &= \psi_{10} + \lambda\psi_{11} + \lambda^2\psi_{12} + \dots \\ T_1 &= T_{10} + \lambda T_{11} + \lambda^2 T_{12} + \dots \quad (41) \\ C_1 &= C_{10} + \lambda C_{11} + \lambda^2 C_{12} + \dots \end{aligned}$$

Substituting (38) between (34) – (37) we will obtain the following set of ordinary differential equations and boundary conditions in the order of λ

At $\lambda \ll 1$

$$\begin{aligned} T''_{10} - (Pri\omega + PrF - Q)T_{10} &= 0 \\ C''_{10} + (Q_m - Sci\omega)C_{10} &= 0 \\ \psi''_{10} - i\omega\psi'_{10} - \left(M + \frac{1}{Da}\right)\psi''_{10} - GrT'_{10} - GmC'_{10} &= 0 \end{aligned} \quad (42)$$

with the following boundary conditions

$$\begin{aligned} \psi'_{10} &= -\psi''_0 e^{-i\omega t}; \psi_{10} = 0; T_{10} = -e^{-i\omega t}T'_0; C_{10} = -e^{-i\omega t}C'_0 \quad \text{at } y = 1 \\ \psi'_{10} &= -\psi''_0 \exp(i(v - \omega t)); \psi_{10} = 0; T_{10} = -T'_0 \exp(i(v - \omega t)); \\ C_{10} &= -C'_0 \exp(i(v - \omega t)) \quad \text{at } y = -1 \end{aligned} \quad (43)$$

$$\begin{aligned} \psi'_{11} &= 0; \psi_{11} = 0; T_{11} = 0; C_{11} = 0 \quad \text{at } y = 1 \\ \psi'_{11} &= 0; \psi_{11} = 0; T_{11} = 0; C_{11} = 0 \quad \text{at } y = -1 \end{aligned} \quad (44)$$

Perturbed Part Solutions

$$T_{10} = A_7 e^{m_3 y} + A_8 e^{-m_3 y} \quad (45)$$

$$C_{10} = B_3 e^{im_4 y} + B_4 e^{-im_4 y} \quad (46)$$

$$\begin{aligned} \psi_{10} &= A_9 + A_{10}y + A_{11}e^{m_5 y} + A_{12}e^{-m_5 y} + E_{22}e^{m_3 y} + E_{23}e^{-m_3 y} \\ &+ F_{22}e^{im_4 y} + F_{23}e^{-im_4 y} \end{aligned} \quad (47)$$

T_{11}, ψ_{11} and the constants are not given for the sake of brevity

3A. Skin Friction

The shear stress at any point in the fluid is given by $\overline{\tau}_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$ and in non-dimensionless form $\tau = \left(\frac{d^2}{\rho \eta v^2} \right) \overline{\tau}_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$.

Shear stress at the wavy walls, $y = 1 + \epsilon \cos \lambda x$ and $y = -1 + \epsilon \cos(\lambda x - \nu)$ are calculated using the following expressions.

$$\begin{aligned} \tau_1 &= -\psi_0''(1) - Re(\epsilon \exp(i\lambda x)\psi_0'''(1) + \epsilon \exp(i\lambda x + \omega t)\psi_{10}''(1)) + \dots \\ \tau_2 &= -\psi_0''(-1) - \epsilon Re(\exp(i\lambda x)\psi_0'''(-1) + \exp(i\lambda x + \omega t)\psi_{10}''(-1)) + \dots \end{aligned}$$

3B. Nusselt Number

The rate of heat transfer measured as Nusselt number on the boundaries of the wavy channel $y = 1 + \epsilon \cos \lambda x$ and $y = -1 + \epsilon \cos(\lambda x - \nu)$ are given below

$$\begin{aligned} Nu_1 &= T_0'(1) + \epsilon Re[e^{i\lambda x}T_0''(1) + e^{i(\lambda x + \omega t)}T_{10}''(1)] + \dots \\ Nu_2 &= T_0'(-1) + \epsilon Re[e^{i\lambda x}T_0''(-1) + e^{i(\lambda x + \omega t)}T_{10}''(-1)] + \dots \end{aligned}$$

3C. Sherwood Number

The Sherwood number also called the mass transfer Nusselt number on the boundaries of the wavy channel $y = 1 + \epsilon \cos \lambda x$ and $y = -1 + \epsilon \cos(\lambda x - \nu)$ are given below

$$\begin{aligned} Sh_1 &= C_0'(1) + \epsilon Re[e^{i\lambda x}C_0''(1) + e^{i(\lambda x + \omega t)}C_{10}''(1)] + \dots \\ Sh_2 &= C_0'(-1) + \epsilon Re[e^{i\lambda x}C_0''(-1) + e^{i(\lambda x + \omega t)}C_{10}''(-1)] + \dots \end{aligned}$$

4. NUMERICAL RESULTS

The study of heat transfer in free and forced convection hydromagnetic flows in vertical wavy channels with travelling thermal waves has many technological applications, especially, in transpiration cooling of re-entry vehicles and rocket boosters. Combustion chambers involve such type of flows in film vaporization. Hence in the previous section we have considered heat transfer hydromagnetic porous flows bounded between wavy walls. This work extends the study of Vajravelu (1989) to investigate the effect of radiation and non-uniform heat source and a variable mass source. In the two wavy walls, one wall is assumed to have a phase advance or lag in comparison with the other wall. We have assumed that the wave length of the wavy walls which is proportional to $1/\lambda$ is large thereby reducing the complexity of the problem due to nonlinearity. We have used Boussinesq approximation. Under these assumptions, we have divided the problem into two parts, the mean part and the perturbed part. The asymptotic solution is found out for both the parts and the total solution is analysed using numerical computations. The effect of various parameters such as Hartmann number, Grashof number, Radiation parameter, Heat source parameter, mass source parameter, Schmidt number and Prandtl number on various important flow characteristics such as stream function, temperature profile, wall shear stress at both the walls, mass transfer and heat transfer at the walls are studied with the help of numerical results. For a clear visualization, these values are represented by graphs in Figures (1) to Figures (24). All the values used in the numerical computations involve values which will be applicable to practical situations. For example, Pr value is fixed as 2.45 to represent glycerine and 1.7 to 13.7 to represent various water specifications.

In Figures (1) to (4) we have shown the stream function as a dependent function of various parameters. From these graphs, we can observe that an increase in the Darcy number increases the stream function. Similarly increasing magnetic field is found to increase the magnitude of the stream function. Stream function is seen to be a decreasing function of the Grashof number and modified Grashof number. Presence of radiation is having a very significant effect on the stream function. The radiation parameter decreases the stream function when F is less than four and when F is between 4 and 6 the value of the stream function is found to be very high. Prandtl number is found to influence the stream function less significantly.

Figures (5) to (7) illustrate the temperature profile as a function of various non-dimensional parameters. Temperature profile is a decreasing function of Prandtl number, radiation parameter. The effect on radiation parameter on the temperature profile is qualitatively similar to that of the stream function.

Figures (8) – (10) depict the influence of various parameters on the Species concentration profile. Concentration profile is found to be an increasing function of the mass source parameter and it is decreasing with an increase in the radiation parameter and Schmidt number. Figures (11) – (16) show the skin friction at both the wavy walls as a function of various dimensionless parameters. From these figures it can be observed that an increase in the Darcy number, Prandtl number decreases the wall shear stress at the wavy walls while an increase in the Grashof number and modified Grashof number enhances the wall shear stress. Figures (17) – (24) show the rate of heat and mass transfer at the wavy walls. It can be inferred from these figures that an increase in porosity increases the heat transfer, radiation parameter decreases the rate of heat transfer and Prandtl number increase the heat transfer respectively; The rate of mass transfer characterized by Sherwood number is found to be decreasing with an increase in the heat source parameter, Schmidt number and modified Grashof number whereas the effect of these parameters on the wall $y=1+\epsilon\cos\lambda x$ is to enhance the rate of mass transfer.

5. Conclusions

In this chapter we have considered heat and mass transfer hydromagnetic flows bounded between wavy walls. The work extends the study of Vajravelu (1989) to investigate the effect of radiation, variable mass source and temperature dependent heat source. In the two wavy walls, one wall is assumed to have a phase advance or lag in comparison with the other wall. Due to the high nonlinearity of the problem, we have assumed that the wave length of the wavy walls which is proportional to $1/\lambda$ is large. We have used Boussinesq approximation. Under these assumptions, we have divided the problem into two parts, the mean part and the perturbed part. The asymptotic solution is found out for both the parts and the total solution is analysed using numerical computations.

Some of the significant results are summarized below.

- ❖ An increase in the Darcy number increases the stream function. Similarly increasing magnetic field is found to increase the magnitude of the stream function.
- ❖ Stream function is seen to be a decreasing function of the Grashof number and modified Grashof number
- ❖ Presence of radiation is having a very significant effect on the stream function. The radiation parameter decreases the stream function when F is less than four and when F is between 4 and 6 the value of the stream function is found to be very high.
- ❖ Prandtl number is found to influence the stream function less significantly.
- ❖ Temperature profile is a decreasing function of Prandtl number, radiation parameter
- ❖ Concentration profile is found to be an increasing function of the mass source parameter and it is decreasing with an increase in the radiation parameter and Schmidt number.
- ❖ An increase in the Darcy number, Prandtl number decreases the wall shear stress at the wavy walls while an increase in the Grashof number and modified Grashof number enhances the wall shear stress
- ❖ An increase in porosity increases the heat transfer, radiation parameter decreases the rate of heat transfer and Prandtl number increase the heat transfer respectively;
- ❖ The rate of mass transfer characterized by Sherwood number is found to be decreasing with an increase in the heat source parameter, Schmidt number and modified Grashof number whereas the effect of these parameters on the wall $y=1+\epsilon\cos\lambda x$ is to enhance the rate of mass transfer
- ❖ It is to be noted that the results are in good agreement with the results obtained by Vajravelu (1989) for vanishing radiation parameter and constant heat source parameter.

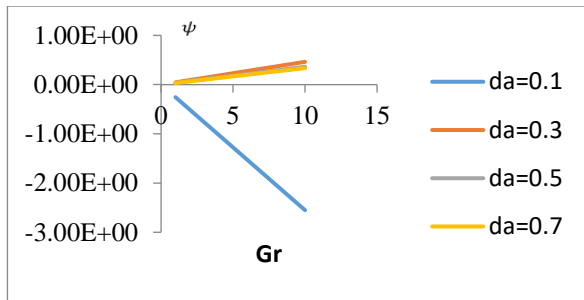


Figure: 1 Stream function as a function of Grashof number Gr for varying values of Darcy number Da, $M=6.0, Gr=10.0, Pr=2.45, F=10.0, Q = 4.0$

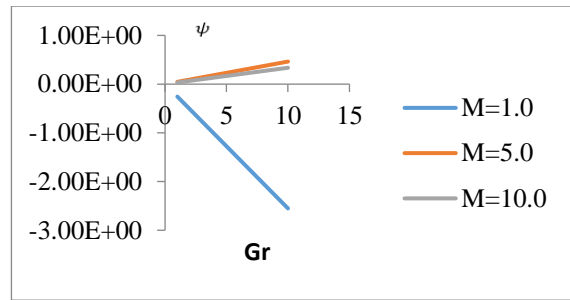


Figure: 2 Stream function as a function of Grashof number Gr for varying values of Hartmann number M, $Gr=10.0, Pr=2.45, F=10.0, Q = 4.0$

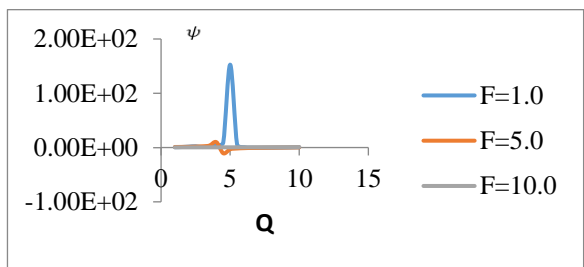


Figure: 3 Stream function as a function of heat source parameter Q for varying values of radiation parameter F, $M = 6.0, Gr=10.0, Pr=2.45, Q = 4.0$

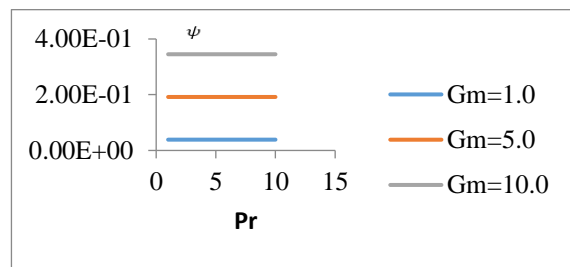


Figure: 4 Stream function as a function of Prandtl number Pr for varying values of modified Grashof number Gm, $M = 6.0, Gr=10.0, Pr=2.45, Q = 4.0$

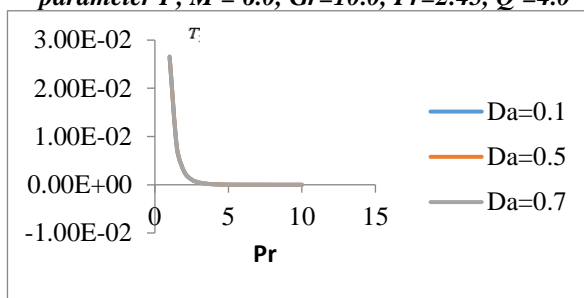


Figure:5 Temperature profile as a function of Prandtl number Pr for varying values of Darcy numbers D, $M = 6.0, Gr=10.0, F=10.0, Q = 4.0$

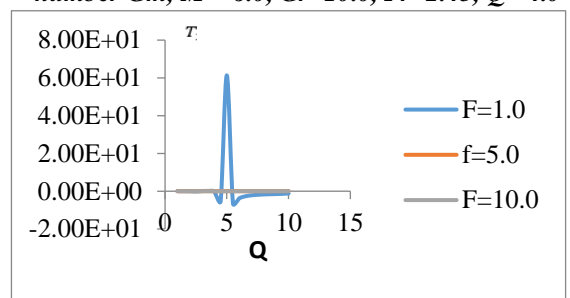


Figure: 6 Temperature profile as a function of heat source parameter Q for varying radiation parameter F, $M = 6.0, Gr=10.0, Pr=2.45, Q = 4.0$

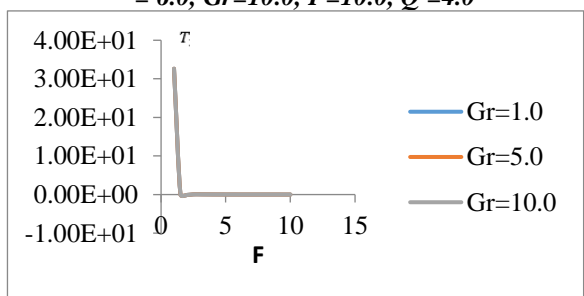


Figure:7 Temperature profile as a function of radiation parameter F for varying values of Grashof numbers Gr, $M = 6.0, F=10.0, Pr=2.45, Q = 4.0$

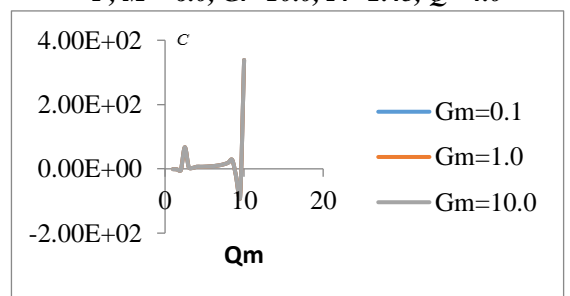


Figure:8 Concentration profile as a function of mass source parameter Q_m for varying values of modified Grashof numbers Gm, $M = 6.0, Gr=10.0, Pr=2.45, Q = 4.0$

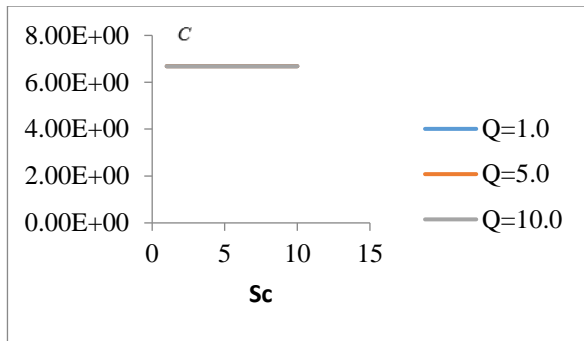


Figure:9 Concentration profile as a function of Schmidt number Sc for varying values of heat source parameter Q , $M = 6.0$, $Gr=10.0$, $Pr=2.45$, $F = 10.0$

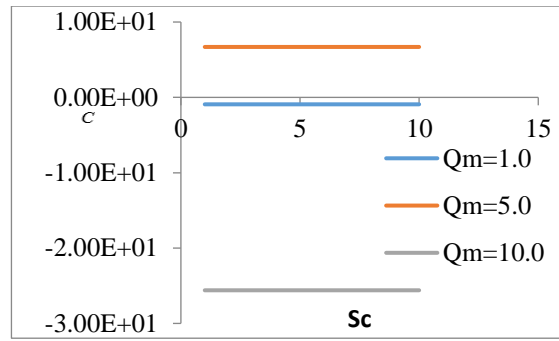


Figure: 10 Concentration profile as a function of Schmidt number Sc for varying values of mass source parameter Q_m , $Gr=10.0$, $Pr=2.45$, $F = 10.0$

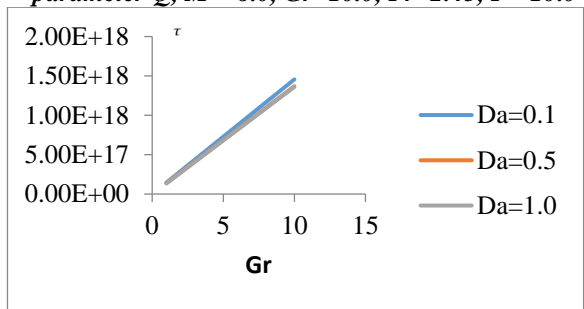


Figure: 11 Skin friction at the wavy wall $y=-1+\epsilon\cos(\lambda x-v)$ as a function of Grashof number for varying values of Darcy number Da , $M = 6.0$, $Gr=10.0$, $Pr=2.45$, $Q = 4.0$

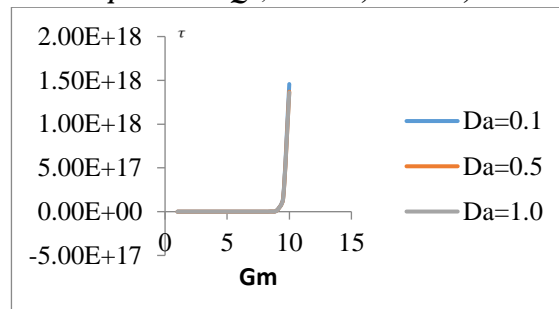


Figure: 12 Skin friction at the wavy wall $y=-1+\epsilon\cos(\lambda x-v)$ as a function of Modified Grashof number G_m for varying values of Darcy number Da , $M = 6.0$, $Gr=10.0$, $Pr=2.45$, $Q = 4.0$

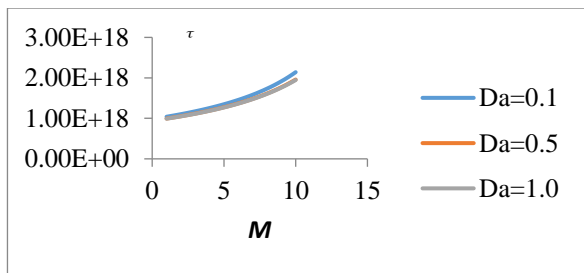


Figure: 13 Skin friction at the wavy wall $y=-1+\epsilon\cos(\lambda x-v)$ as a function of Hartmann number M for varying values of Darcy number Da , $M = 6.0$, $Gr=10.0$, $Pr=2.45$, $Q = 4.0$

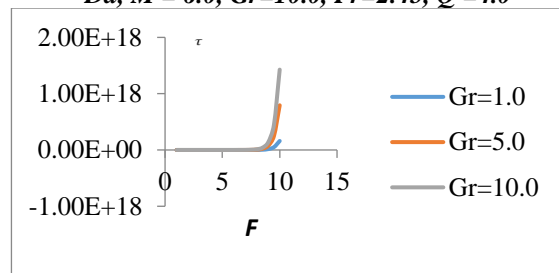


Figure: 14 Skin friction at the wavy wall $y=-1+\epsilon\cos(\lambda x-v)$ as a function of radiation parameter F for varying Grashof number Gr , $M = 6.0$, $F=10.0$, $Pr=2.45$, $Q = 4.0$

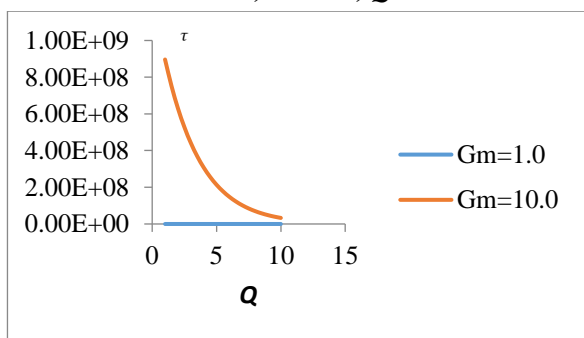


Figure: 15 Skin friction at the wavy wall $y=-1+\epsilon\cos(\lambda x-v)$ as a function of heat source parameter Q for varying values of modified Grashof number G_m , $M = 6.0$, $F=10.0$, $Pr=2.45$, $Q = 4.0$

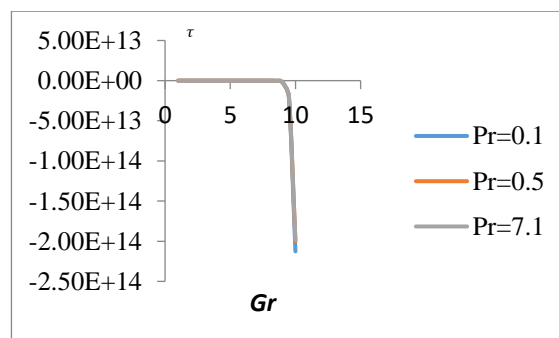


Figure: 16 Skin friction at the wavy wall $y=1+\epsilon\cos\lambda x$ as a function of Grashof number Gr for varying values of Prandtl number Pr , $M = 6.0$, $Gr=10.0$, $F=10.0$, $Q = 4.0$

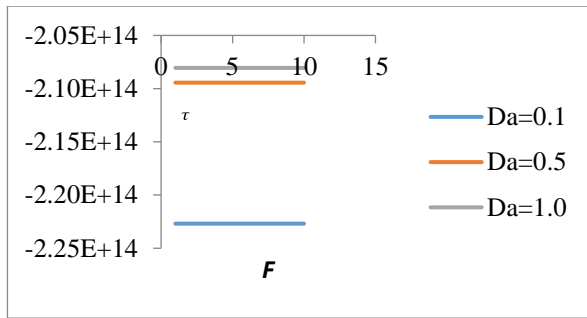


Figure: 17 Skin friction at the wavy wall $y=1+\epsilon\cos\lambda x$ as a function of radiation parameter F for varying values of Darcy number Da , $M = 6.0$, $Gr=10.0$, $F=10.0$, $Q = 4.0$

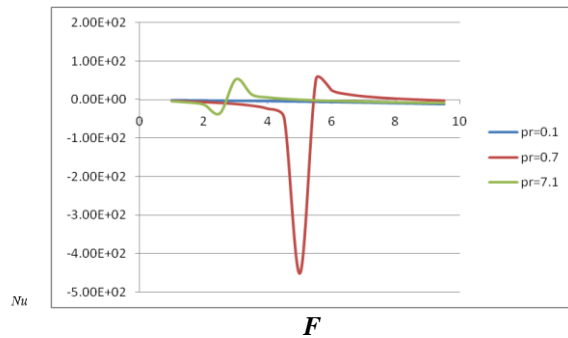


Figure: 18 Nusselt number at the wavy wall $y=-1+\epsilon\cos(\lambda x-\nu)$ as a function of radiation parameter F for varying values of Prandtl number Pr , $M = 6.0$, $Gr=10.0$, $F=10.0$, $Q = 4.0$

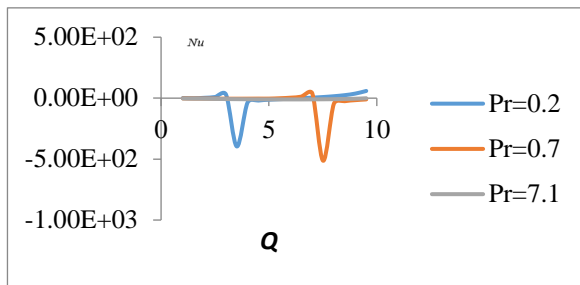


Figure:19 Nusselt number at the wavy wall $y=-1+\epsilon\cos(\lambda x-\nu)$ as a function of heat source parameter Q for varying values of Prandtl number Pr , $M = 6.0$, $Gr=10.0$, $F=10.0$, $Q = 4.0$

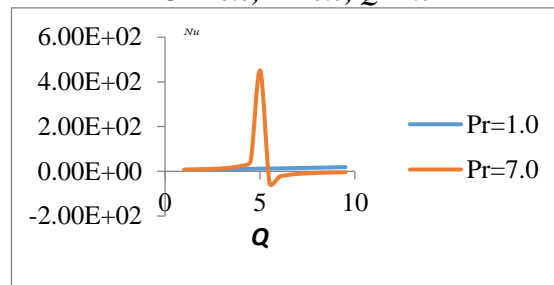


Figure: 20 Nusselt number at the wavy wall $y=1+\epsilon\cos\lambda x$ as a function of heat source parameter Q for varying values of Prandtl number Pr , $M = 6.0$, $Gr=10.0$, $F=10.0$, $Q = 4.0$

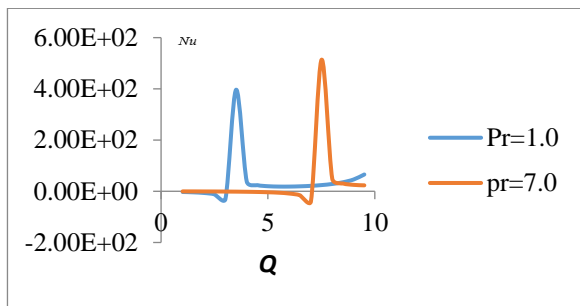


Figure: 21 Nusselt number at the wavy wall $y=1+\epsilon\cos\lambda x$ as a function of heat source parameter Q for varying Prandtl number Pr , $M = 6.0$, $Gr=10.0$, $F=10.0$, $Q = 4.0$

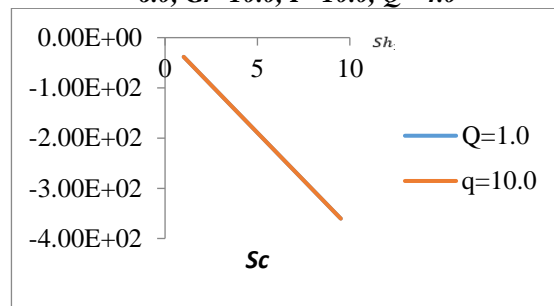


Figure: 22 Sherwood number at the wavy wall $y=-1+\epsilon\cos(\lambda x-\nu)$ as a function of Schmidt number Sc for varying values of heat source parameter Q , $M = 6.0$, $Gr=10.0$, $Pr=2.45$, $F=10.0$

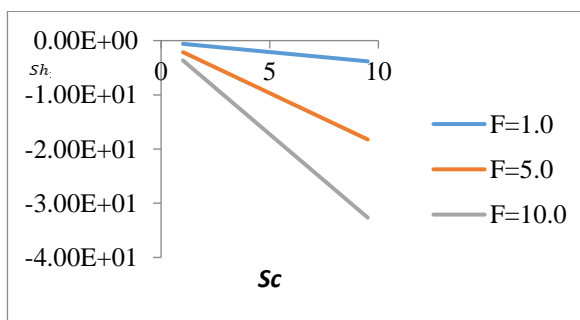


Figure: 23 Sherwood number at the wavy wall $y=-1+\epsilon\cos(\lambda x-\nu)$ as a function of Schmidt number Sc for varying values of Radiation parameter F , $M = 6.0$, $Gr=10.0$, $Pr=2.45$, $Q = 4.0$

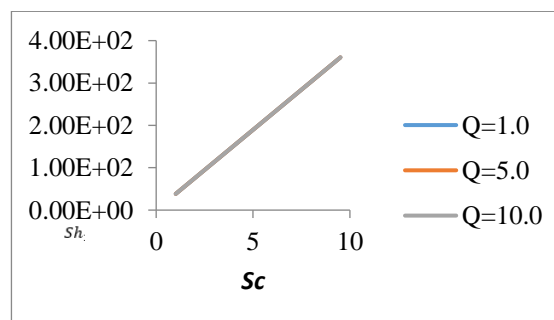


Figure: 24 Sherwood number at the wavy wall $y=1+\epsilon\cos\lambda x$ as a function of Schmidt number Sc for varying values of heat source parameter Q , $M = 6.0$, $Gr=10.0$, $Pr=2.45$, $F=10.0$

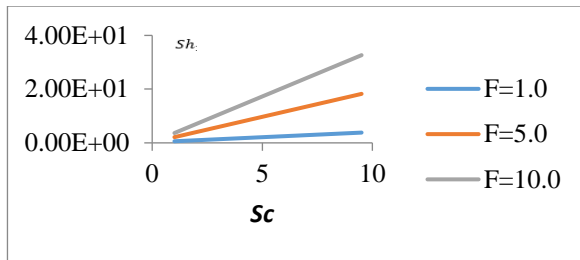


Figure: 25 Sherwood number at the wavy wall $y = -1 + \epsilon \cos(\lambda x - \nu t)$ as a function of Schmidt number Sc for varying values of Radiation parameter F , $M = 6.0$, $Gr = 10.0$, $Pr = 2.45$, $Q = 4.0$

References

1. **Cogley .A .C, Vincent .W .G and Giles .S .E**, “Differential approximation to radiative heat transfer in a non-grey gas near equilibrium”, *AIAA J*, [1968], Vol. 6, Issue 3, pp. 551-553.
2. **Lekoudis .S .G, Nayfeh .A .H and Saric .W .S**, “Compressible boundary layers over wavy walls”, *Phys. Fluids*, [1976], 19, 514—519
3. **Lessen .M and Gangwani .S .T** “Effect of small amplitude wall waviness upon the stability of the laminar boundary layer”, *Physics of Fluids*, [1976], Vol: 19(4), 510 – 513.
4. **Raju .L.T and Muralidhar .P**, “Unsteady Convective Heat and Mass Transfer Flow of a Viscous Fluid in a Vertical Wavy Channel with Variable Wall Temperature and Concentration”, *Advances in Applied Science Research*, [2012], Vol: 3 (5), 2947-2965.
5. **Shankar .P .N and Sinha .U .N**, “The Rayleigh problem for a wavy wall”, *Journal of Fluid Mechanics*, [1976], Vol: 77, 243–256.
6. **Umavathi .J .C and Shekar .M**, “Mixed convection flow and heat transfer in a vertical wavy channel containing porous and fluid layer with traveling thermal waves”, *International Journal of Engineering, Science and Technology*, [2011], Vol: 3(6), 196-219.
7. **Vajravelu .K and Sastri .K .S**, “Free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall”, *J. Fluid Mech*, [1978], Vol: 86(2), 365-383.
8. **Vajravelu .K and Sastri .K .S**, “Natural convective heat transfer in vertical wavy channels”, *Int. J. Heat Mass Transfer*, [1980], Vol: 23, 408-411.
9. **Vajravelu .K and Debnath .L**, “Non-linear study of convective heat transfer and fluid flows induced by travelling thermal waves”, *Acta Mechanica*, [1986], Vol: 59, 233–249.
10. **Vajravelu .K**, “Combined free and forced convection in hydromagnetic flows, in vertical wavy channels, with travelling thermal waves”, *Int. J. Engng Sci.* [1989], Vol: 27(3), 289-300.