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Research Article

Approximation methods to solve a single machine scheduling problem with fuzzy due date to minimize multi-objective functions

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Abstract*:* In this paper present three methodant colony optimization (ACO), particle swarm optimization (PSO) and bees algorithm optimization (BAO) to the solution multi-objective function of single machine problem with the fuzzy due date. The objective function to minimize total completion time and maximum lateness with a fuzzy due date. By a computer simulation used to compare the performance of each algorithm with another one from where accuracy and time.

1. Introduction

The concept of fuzzy decision making introduced by bellman and zadeh[1]in 1970, different requisitions of the fluffy principle will choice making issues have been introduced. In 1974 Tanaka et al. [2] and in 1979 Zimmermann[3] were detailed fluffy mathematical programming issues. In 1989 W.Szwarc and J.J.Liu[4] were found approximation solution to flow shop of m machine and n jobs where $m \leq 8$ and $n \leq 8$. In 1990 Inuiguchi, M., Ichihashi, H. and Tanaka [5] were propose many approaches in the field of fuzzy mathematical programming. In any case, many promising and intriguing areas stay to be explored in the field of fluffy combinatorial improvement. In 1992 Ishii et, al. [6] for scheduling problem were introduced the concept of fuzzy due date. In 1994 HisaoIshibuchi, Naohisa Yamamoto.[7] were solve NP-hard problem by approximation mathods and compere between descent, simulated annealing and taboo search algorithms are applied to the problem. In 1999Andreas Bauer, Bernd Bullnheimer [8] were solve NP-hard problem by used ant colony optimization methods and developed it. In 2003[9] G. Celano, A. Costa And S. Fichera were developed genetic algorithm to solved fuzzy flow shop scheduling problem. In 2005 [10] Hong Wang was applied branch and bound method to got exact solution and approach to artificial intelligence search techniques and compare between them. In 2006[11] Hamid Allaoui and Samir Lamouri were using Johnson's algorithms to found approximation solution for some flow shop scheduling problem formatted by makespan for two machine. In 2008 [12] BabakJavadi and al. were proposed model to solved minimize the weighted mean completion time and the weighted mean earliness to no wait flow shop scheduling problem . in 2010[13] K Sheibani was The proposed technique comprises of two stages: masterminding the positions in need request and afterward building a grouping for flow shop scheduling problem to makespan criterion. In 2012[14] H. F. Abdullah was found approximation solution for two machine flow shop scheduling problem to minimized total earliness by proposes a new algorithms. CengizKahraman a, OrhanEngin and Mustafa KerimYilmaz[15] were solved multi objective function formatted by minimized the average tardiness and the number of tardy jobs to fuzzy flow shop scheduling problem by found new artificial immune system algorithms. In 2014 [16] J.Behnamiana , S.M.T. FatemiGhomi were solved bi- objective hybrid scheduling problem formulated by minimized maximum completion time and sum of trainees and earliness for flow shop scheduling problem by using some algorithms of local search as genetic algorithms and particle swarm optimization to found approximation solution. DonyaRahmani, Reza Ramezanianand Mohammad Saidi-Mehrabad[17] were studied fuzzy flow shop scheduling problem formulated by minimized total flow shop and total tardiness to considered provide release time, process time and a more realistic model by using genetic algorithm. B. Naderi, M. Aminnayeri, M. Piri and M.H. Ha'iriYazdi [18] were studied multi-objective no-wait flow shop scheduling problem to makespan and total tardiness formatted by F/nwt/TT,Cmax by using three type of local search greedy, moderate and curtailed fashions. In 2017[19] they studied development in flow shop scheduling problem under uncertainties depicts the distinctive arrangement draws near introduced in the writing and present status of exploration. At last, a few headings for future examination . In 2018 [20] ChiwenQu ,Yanming Fu,Zhongjun Yi,and Jun Tanwere solved no-wait flow shop scheduling problem to minimize the maximum accomplished time

2. Preliminaries

In this section we review the concept of fuzzy set theory

2-1Definition(fuzzy set**)**[9]

The subset S of X is a fuzzy set if$\tilde{S} = \{(x, \mu(x)): x\epsilon X\}$where $\mu(x)$is membership function define by $\mu(x): X \to [0,1]$

2-2 Definition(support)[9]

A fuzzy set $\tilde{S}$ is said to be support if $\tilde{S}$ is a set of all a point $x \in X$ such that

Supp$(\tilde{S}) = \{x \in X: \mu(x) > 0\}$

2-3 Definition (core)

Let $\tilde{S}$is a fuzzy set a core of $\tilde{S}$ is a set of all $x \in X$ such that $\mu(x) = 1$

core $(\tilde{S}) = \{x \in X: \mu(x) = 1\}$

2-4 Definition (normal)

Let $\tilde{S}$is a fuzzy set is said to be normal if $\exists x \in X$ such that $\mu(x) = 1$

2-5 Definition (α cut)

Let $\tilde{S}$is a fuzzy set α −cut define by the following

$S_\alpha = \{x \in X: \mu(x) \geq \alpha\}$ where $\alpha \in [0,1]$

2-6 Definition (convex fuzzy set)

Let $\tilde{S}$ is fuzzy set is said to be convex fuzzy set if every $x_1, x_2 \in S_\alpha$ and $\alpha \in [0,1]$ and satisfy the following condition

then $f(\gamma\chi_1 + (1-\gamma)\chi_2) \geq f(\chi_1) \wedge f(\chi_2)$

2-7 Definition (fuzzy number)[10]

Let $\tilde{S} \in R$ is a fuzzy subset is said to be fuzzy number if satisfy the following condition :

i. If a fuzzy set is normal
ii. If the member ship $\mu(x)$ is quasi concave this mean

$$\mu(sx + (1-s)y \geq \min\{\mu(x), \mu(y)$$

iii. The member ship function $\mu(x)$is semi continuous this mean

$\{x \in R: \mu(x) \geq \alpha$ this set is closed in R for $\alpha\epsilon[0,1]$

2-8Definition (triangular fuzzy number)

Let $\tilde{S}$ be a fuzzy set define by $\tilde{S} = (s_1, s_2, s_3)$ with a membership function define by

$$\mu_{\tilde{S}}(\chi) = \begin{cases} 0 & if\ \chi < s_j{}^I \\ \dfrac{\chi - s_j{}^I}{s_j{}^c - s_j{}^I} & if\ s_j{}^I \leq \chi < s_j{}^c \\ \dfrac{s_j{}^u - \chi}{s_j{}^u - s_j{}^c} & if\ s_j{}^c \leq \chi < s_j{}^u \\ 0 & if\ s_j{}^u \leq \chi \end{cases}$$

Is called triangular fuzzy number.

3. Problem formulation

Suppose there are n-jobs scheduling on single machine each job has a processing timep_j and triangular fuzzy due date $\widetilde{d_j}$. On a machine all a jobs are available to be processed and starts without interrupted. Let a sequence σ be a sequence of jobs processed on single machine to minimized total completion time and maximum lateness with a fuzzy due date.

Now, let the triangular fuzzy number (s_1, s_2, s_3), we using distance measure

Let$\tilde{A} = [\underline{a}_\alpha, \overline{a}_\alpha]$ and $\tilde{B} = [\underline{b}_\alpha, \overline{b}_\alpha]$, than

$$\tilde{d}(\tilde{A}, \tilde{B}) = \frac{1}{2}\int_0^1 \{(\underline{a}_\alpha - \underline{b}_\alpha)^+ + (\overline{a}_\alpha - \overline{b}_\alpha)^+\} d\alpha + \frac{1}{2}\int_0^1 \{(\underline{a}_\alpha - \underline{b}_\alpha)^- + (\overline{a}_\alpha - \overline{b}_\alpha)^-\} d\alpha$$

Where

$$(\chi)^+ = \begin{cases} \chi & if\ \chi \geq 0, \\ 0 & if\ \chi < 0 \end{cases}$$

And

$$(\chi)^- = \begin{cases} 0 & if\ \chi \geq 0, \\ \chi & if\ \chi < 0 \end{cases}$$

By changing $\tilde{A}$ with C_j where C_j is a completion time and $\tilde{B}$ with $\tilde{D}_j$ where $\tilde{D}_j$ is fuzzy due date we can evaluated the following lateness function:

$$\tilde{L}(C_j, \tilde{D}_j) = \frac{1}{2}\int_0^1 \{(C_j - \underline{d_j}_\alpha)^+ + \left(C_j - \overline{d_j}_\alpha\right)^+\} d\alpha + \frac{1}{2}\int_0^1 \{(C_j - \underline{d_j}_\alpha)^- + \left(C_j - \overline{d_j}_\alpha\right)^-\} d\alpha$$

Where $[\underline{d_j}_\alpha, \overline{d_j}_\alpha]$ according to α-cut

To derive the fuzzy lateness cost function we have four cases:

Case(1) :

if $C_j < {d_j}^I$

For $C_j - ({d_j}^I + ({d_j}^c - {d_j}^I)\alpha$

If $\alpha = 0$ then $C_j - {d_j}^I < 0$

If $\alpha = 1$ then $C_j - {d_j}^c < 0$

For $C_j - ({d_j}^u + ({d_j}^c - {d_j}^u)\alpha$

If $\alpha = 0$ then $C_j - {d_j}^u < 0$

If $\alpha = 1$ then $C_j - {d_j}^c < 0$

Then by equation (1) we get

$$\tilde{L}(C_j, \tilde{D}_j) = \frac{1}{2}\int_0^1 \{(C_j - \underline{d_j}_\alpha)^- + \left(C_j - \overline{d_j}_\alpha\right)^-\} d\alpha$$

$$= \frac{1}{2}\int_0^1 \{\left(C_j - \left({d_j}^I + \left({d_j}^c - {d_j}^I\right)\alpha\right) + C_j - \left({d_j}^u + \left({d_j}^c - {d_j}^u\right)\alpha\right)\right\} d\alpha$$

$$= \frac{1}{2}[C_j\alpha - {d_j}^I\alpha - \frac{1}{2}{d_j}^c\alpha^2 + \frac{1}{2}{d_j}^I\alpha^2 + C_j\alpha - {d_j}^u\alpha - \frac{1}{2}{d_j}^c\alpha^2 + \frac{1}{2}{d_j}^I\alpha^2]_0^1$$

$$= \frac{1}{2}[2C_j - \frac{1}{2}{d_j}^I - {d_j}^c - \frac{1}{2}{d_j}^u]$$

$$= C_j - \frac{1}{4}\left[{d_j}^I + 2{d_j}^c + {d_j}^u\right]$$

Case 2:

If ${d_j}^I \leq C_j < {d_j}^c$ then:

For $C_j - ({d_j}^I + ({d_j}^c - {d_j}^I)\alpha$

If $\alpha = 0$ then $C_j - {d_j}^I \geq 0$

If $\alpha = 1$ then $C_j - {d_j}^c < 0$

For $C_j - ({d_j}^u + ({d_j}^c - {d_j}^u)\alpha$

If $\alpha = 0$ then $C_j - {d_j}^u < 0$

If $\alpha = 1$ then $C_j - {d_j}^c < 0$

Than $C_j - \left({d_j}^c - {d_j}^I\right)\alpha - {d_j}^I) \geq 0$

$C_j - {d_j}^I \geq \left({d_j}^c - {d_j}^I\right)\alpha$ Than $\alpha \leq \frac{C_j - {d_j}^I}{{d_j}^c - {d_j}^I}$

Than $[0, \frac{C_j - {d_j}^I}{{d_j}^c - {d_j}^I}] \geq 0$, $[\frac{C_j - {d_j}^I}{{d_j}^c - {d_j}^I}, 1]$

by using equation (1)

$$\tilde{L}(C_j, \tilde{D}_j) = \frac{1}{2}\int_0^1 \{(C_j - \underline{d_j}_\alpha)^+ + \left(C_j - \overline{d_j}_\alpha\right)^+\} d\alpha + \frac{1}{2}\int_0^1 \{(C_j - \underline{d_j}_\alpha)^- + \left(C_j - \overline{d_j}_\alpha\right)^-\} d\alpha$$

$$= \frac{1}{2}\int_{0}^{\frac{C_j - d_j^{\,I}}{d_j^{\,c}-d_j^{\,I}}} (C_j - (d_j^{\,I} + (d_j^{\,c} - d_j^{\,I})\alpha))d\alpha$$

$$+\frac{1}{2}\int_{\frac{C_j - d_j^{\,I}}{d_j^{\,c}-d_j^{\,I}}}^{1} (C_j - (d_j^{\,I} + (d_j^{\,c} - d_j^{\,I})\alpha))d\alpha + \frac{1}{2}\int_0^1 (C_j - (d_j^{\,u} + (d_j^{\,c} - d_j^{\,u})\alpha))d\alpha$$

$$= \frac{1}{2}[C_j\alpha - d_j^{\,I}\alpha - \frac{1}{2}d_j^{\,c}\alpha^2 + \frac{1}{2}d_j^{\,I}\alpha^2]_0^{\frac{C_j - d_j^{\,I}}{d_j^{\,c}-d_j^{\,I}}} + = \frac{1}{2}[C_j\alpha - d_j^{\,I}\alpha - \frac{1}{2}d_j^{\,c}\alpha^2 + \frac{1}{2}d_j^{\,I}\alpha^2]_{\frac{C_j - d_j^{\,I}}{d_j^{\,c}-d_j^{\,I}}}^{1}$$

$$+\frac{1}{2}[C_j\alpha - d_j^{\,u}\alpha - \frac{1}{2}d_j^{\,c}\alpha^2]_0^1$$

$$= \frac{1}{2}\left[2C_j - \frac{1}{2}d_j^{\,I} - \frac{1}{2}d_j^{\,c}\right] + \frac{1}{2}\left[2C_j - \frac{1}{2}d_j^{\,u} - \frac{1}{2}d_j^{\,c}\right]$$

$$= C_j - \frac{1}{4}[d_j^{\,I} + 2d_j^{\,c} + d_j^{\,u}]$$

Case (3)

If $d_j^{\,c} \le C_j < d_j^{\,u}$ then:

For $C_j - (d_j^{\,I} + (d_j^{\,c} - d_j^{\,I})\alpha$

If $\alpha = 0$ then $C_j - d_j^{\,I} > 0$

If $\alpha = 1$ then $C_j - d_j^{\,c} > 0$

For $C_j - (d_j^{\,u} + (d_j^{\,c} - d_j^{\,u})\alpha$

If $\alpha = 0$ then $C_j - d_j^{\,u} < 0$

If $\alpha = 1$ then $C_j - d_j^{\,c} > 0$

Than $C_j - (d_j^{\,u} + (d_j^{\,c} - d_j^{\,u})\alpha \ge 0$

$$\Rightarrow (d_j^{\,u} - d_j^{\,c})\alpha \ge d_j^{\,u} - C_j$$

$$\Rightarrow \alpha \ge \frac{d_j^{\,u} - C_j}{d_j^{\,u} - d_j^{\,c}}$$

Than $\left[0, \frac{d_j^{\,u} - C_j}{d_j^{\,u} - d_j^{\,c}}\right] \le 0, \quad [\frac{d_j^{\,u} - C_j}{d_j^{\,u} - d_j^{\,c}}, 1] \ge 0$

Then by using equation (1)

$$\tilde{L}(C_j, \tilde{D}_j) = \frac{1}{2}\int_0^1 \{(C_j - \underline{d_j}_\alpha)^+ + (C_j - \overline{d_j}_\alpha)^+\} d\alpha + \frac{1}{2}\int_0^1 \{(C_j - \underline{d_j}_\alpha)^- + (C_j - \overline{d_j}_\alpha)^-\} d\alpha$$

$$= \frac{1}{2}\int_0^1 [C_j - (d_j^{\,I} + (d_j^{\,c} - d_j^{\,I})\alpha]d\alpha + \frac{1}{2}\int_0^{\frac{d_j^{\,u} - C_j}{d_j^{\,u} - d_j^{\,c}}} [C_j - (d_j^{\,u} + (d_j^{\,c} - d_j^{\,u})\alpha]\, d\alpha$$

$$+\frac{1}{2}\int_{\frac{d_j^{\,u} - C_j}{d_j^{\,u} - d_j^{\,c}}}^{1} [C_j - (d_j^{\,u} + (d_j^{\,c} - d_j^{\,u})\alpha]\, d\alpha$$

$$= \frac{1}{2}[C_j\alpha - d_j^{\,I}\alpha - \frac{1}{2}(d_j^{\,c} - d_j^{\,I})\alpha^2]_0^1 + \frac{1}{2}[C_j\alpha - d_j^{\,u}\alpha - \frac{1}{2}(d_j^{\,c} - d_j^{\,u})\alpha^2]_0^{\frac{d_j^{\,u} - C_j}{d_j^{\,u} - d_j^{\,c}}}$$

$$\frac{1}{2}[C_j\alpha - d_j^{\,u}\alpha - \frac{1}{2}(d_j^{\,c} - d_j^{\,u})\alpha^2]_{\frac{d_j^{\,u} - C_j}{d_j^{\,u} - d_j^{\,c}}}^{1}$$

Than we get

$$C_j - \frac{1}{4}[d_j^{\,I} + 2d_j^{\,c} + d_j^{\,u}]$$

Case (4)

: if ${d_j}^u < C_j$
For $C_j - ({d_j}^I + ({d_j}^c - {d_j}^I)\alpha$
If $\alpha = 0$ then $C_j - {d_j}^I > 0$
If $\alpha = 1$ then $C_j - {d_j}^c > 0$
For $C_j - ({d_j}^u + ({d_j}^c - {d_j}^u)\alpha$
If $\alpha = 0$ then $C_j - {d_j}^u > 0$
If $\alpha = 1$ then $C_j - {d_j}^c > 0$
Then by equation (1) we get

$$\tilde{L}(C_j, \tilde{D}_j) = \frac{1}{2}\int_0^1 \{(C_j - \underline{d_j}_\alpha)^+ + \left(C_j - \overline{d_j}_\alpha\right)^+\} d\alpha$$

$$= \frac{1}{2}\int_0^1 \{\left(C_j - \left({d_j}^I + \left({d_j}^c - {d_j}^I\right)\alpha\right) + C_j - \left({d_j}^u + \left({d_j}^c - {d_j}^u\right)\alpha\right)\right\} d\alpha$$

$$= \frac{1}{2}[C_j\alpha - {d_j}^I\alpha - \frac{1}{2}{d_j}^c\alpha^2 + \frac{1}{2}{d_j}^I\alpha^2 + C_j\alpha - {d_j}^u\alpha - \frac{1}{2}{d_j}^c\alpha^2 + \frac{1}{2}{d_j}^I\alpha^2]_0^1$$

$$= \frac{1}{2}[2C_j - \frac{1}{2}{d_j}^I - {d_j}^c - \frac{1}{2}{d_j}^u]$$

$$= C_j - \frac{1}{4}\left[{d_j}^I + 2{d_j}^c + {d_j}^u\right]$$

Than from case (1, 2, 3, 4) we get

$$\tilde{L}(C_j, \tilde{D}_j) = C_j - \frac{1}{4}\left[{d_j}^I + 2{d_j}^c + {d_j}^u\right]$$

Where C_j is completion time of jobs j under a sequence δ
Using the traditional notion, we denote by
$1\backslash\tilde{D}_j = TFN\backslash\sum_{j=1}^{n} C_j + \tilde{L}_{max}$
the problem formulated by

Min F =Min$\sum_{j=1}^{n} C_j + \tilde{L}_{max}$
subject to :

$$\left.\begin{array}{lll} C_j \geq P_j; & j=1,2,\ldots,n & \\ C_j = C_{j-1} + P_j; & j=2,3,\ldots,n & \\ & & L_j = C_j - d_j \end{array}\right\} \ldots\ldots\ldots\ldots Q$$

4. Local search algorithms

In this section we will used three different methods of local search to solved multi-objective function on single machine formulated by $1\backslash\tilde{D}_j = TFN\backslash\sum_{j=1}^{n} C_j + \tilde{L}_{max}$

Suppose T is a finite set and let a function $f: T \to R$ has a solution $t' \in T$ with $f(t') \in f(t)$ for all $t \in T$[21]. Local search is an iterative strategy which moves starting with one arrangement in S then onto the next as long as essential. To methodically look through S. The potential moves from an answer s to next arrangement ought to be limited somehow or another. A local search(neighborhood) define by moves from initial solution by some sequence neighborhood change until found local optimum with improve each time value of the objective function. A local search starting with any feasible solution there exists a sequence of move to reach optimal solution. Neighborhood structures assume a vital part in local search as the time intricacy of a hunt depends on the size of the Neighborhood and the computational cost of the moves.This choice leads to the well-known iterative improvement method which may be formulated as follows:

4-1Ant Colony Optimization Algorithms

Step 0: Initialization. Define the user specified parameters; the number of decision variables (n) (this number is sum of the number of green times as stage numbers at each intersection, the number of offset times as intersection numbers and common cycle time), the constraints for each decision variable, the size of ant colony (m), search space value (β) for each decision variable.

Step 1: Set t=1
Step 2: Generate the random initial signal timings, ψ(c,θ,φ)
within the constraints of decision variables.
Step 3: Distribute to the initial green timings to the stages according to distribution rule as mentioned above. At this step, randomly generated green timings at Step 2 are distributed to the stages according to generated cycle time at the same step, minimum green and intergreen time.
Step 4: Get the network data and fixed set of link flows for TRANSYT-7F traffic model.
Step 5: Run TRANSYT-7F.
Step 6: Get the network PI. At this step, the PI is determined using TRANSYT-7F traffic model.
Step 7: If t=tmax
then terminate the algorithm; otherwise, t=t+1
and go to Step 2

4-2The Bees Colony Optimization Algorithm

INPUT: n, ss, e, nep, nsp, Maximum of iterations.
Step1. Initialize population with random solutions.
Step2. Evaluate fitness of the population.
Step3. REPEAT
Step4. Select sites for neighborhood search.
Step5. Recruit bees for selected sites (more bees for best e sites) andevaluate fitness's.
Step6. Select the fittest bee from each patch.
Step7. Assign remaining bees to search randomly and evaluate theirfitness's.
Step8. UNTIL stopping criterion is met.
4-3 Particle Swarm Optimization (PSO) Algorithm
step1. Initialize a population of particles with random positions and velocities on d-dimensions in the problem space.
step2. PSO operation includes:
a. For each particle, evaluate the desired optimization fitness function in d variables.
b. Compare particle's fitness evaluation with its pbest. If current value is better than pbest, then set pbest equal to the current value, and pai equals to the current location xi.
c. Identify the particle in the neighborhood with the best success so far, and assign it index to the variable g.
d. Change the velocity and position of the particle according to equations (2.1a) and (2.1b).
step3. Loop to step (2) until a criterion is met.

5. Computational results

In this section we using local search methods by using coding of matlab virgin R2017a were tasted and runs on computer Pentium IV at 2.400GH$_z$, 4.00GB. in the below table given the results of optimal values by ant colony optimization algorithms(ACO), the bees colony optimizationalgorithm(BA) and particle swarm optimization algorithm(PSO) were n=10,50,100,150,200,300,400,500,600,700,800,900,1000,1500 as the following table

n: no. of jobs,
Ex: no. of examples,
PSO: particle swarm optimization algorithm
ACO: ant colony optimization algorithms
BA: the bees colony optimization algorithm
Time: The execution time of the problem (by seconds).

n	**X**	**ACO**	**TIME**	**BA**	**TIME**	**PSO**	**TIME**
10		359.25	1.515	332.75	79.033	335.75	2.713

		547.25	1.541	535.5	80.415	538.5	2.772
		475.75	1.432	447.75	78.841	475.75	2.997
		482	1.504	481	80.359	482	2.712
		327.25	1.520	321.25	79.450	321.25	2.566
50		19047.75	5.371	12219	315.685	12898.5	9.250
		11367	4.032	10794	314.183	11375.25	10.376
		11394.75	4.106	10943	305.246	11351.75	9.460
		10158.5	4.184	9264.25	328.664	9957	9.580
		9486.75	4.145	8655.25	317.797	8834.75	9.558
100		40466.25	7.947	40045.5	640.410	40356	18.707
		40746	7.692	39718.75	667.534	40353.25	18.925
		47280.5	16.254	47090.25	720.777	47819.75	18.811
		41339.75	16.811	39605.25	888.121	40185.5	26.512
		42929.75	0.230	41354	895.917	41590	19.122
150		106426.75	12.704	101898	1035.324	102993.25	29.996
		107677.5	12.643	104638.75	976.389	106106.75	29.279
		98332.5	11.621	94492.5	1094.254	97691	36.839
		105958.5	14.306	102151.25	1016.971	104651	29.078
		105167.5	11.713	103512.75	1014.326	104676	29.548
200		194745.75	13.483	189046	1214.389	192709.5	36.380
		197056.75	13.807	192809.75	1217.649	194437.5	35.132
		185655.75	13.405	180032	1202.807	181268.25	34.713
		177984.75	13.260	170486	1221.464	173903.25	34.989
		195065.75	12.449	185542.75	1236.416	189810	35.042
300		458314	15.151	449284	1319.829	454636.5	37.921
		439736.5	15.196	434399.25	1315.083	439248.25	38.041
		442944.5	13.564	437213.5	1313.724	440301.25	38.925
		456717	14.818	450442.5	1031.251	454151	37.634
		464816.5	15.309	460248.25	1316.386	461392	39.110

400		744315	15.879	735855	1479.297	745847	44.631
		783312.25	15.808	781248.75	1468.212	782342.5	44.284
		781578	15.501	767425.25	1450.819	774938.25	41.870
		785648	16.215	771059.5	1356.176	775431.5	40.409
		785648	17.130	771059.5	1381.142	775431.5	41.457
500		1203537.25	17.349	1180203.25	1488.563	1197999.25	45.540
		1188662	15.987	1177537.25	1489.234	1182720.75	44.830
		1242205.25	16.018	1238348	1486.792	1251027.5	42.651
		1216479.25	16.662	1218784.8	1488.936	1222257	42.877
		1153541.25	16.499	1145806.75	1492.290	1157400.25	42.892
600		1728530.75	17.593	1710070.75	1631.703	1724363.75	46.377
		1728553	17.661	1714604.5	1563.504	1725398.5	47.658
		1791190	19.074	1782975.5	1562.558	1789210.25	46.155
		1742749.75	18.370	1716733	1559.189	1737821.5	46.089
		1727570.25	18.896	1712956.5	1558.625	1733046	46.586
700		2256608.75	18.942	2234514.75	1749.084	2249483.75	51.035
		2489438.5	19.565	2451308.75	1741.403	2474523.25	49.727
		2484713	19.717	2473856.25	1748.992	2494122.5	52.060
		2413287.5	19.003	2385101.25	1745.376	2405379.5	51.769
		2457635.25	19.265	2445536.5	1767.265	2452116.75	50.072
800		3104373.5	21.660	3077404.75	1913.805	3093468.5	54.059
		3262420.75	20.521	3229612.5	1893.188	3262025.5	52.988
		3116125.25	20.541	3086304.5	1839.503	3127753.25	54.074
		3211890	20.490	3182970	1814.507	3199328.25	52.170
		3058229	20.328	3042580.25	1848.626	3060590.5	53.444
900		3889877.5	23.545	3847421.75	2070.062	3865368.5	59.861
		4120634	23.630	4095036.75	2081.621	4116886	58.882
		3953056.5	33.680	3954258.25	2233.430	3962795	82.996
		3829251	24.487	3796587.5	2906.174	3799322.75	62.339

		3945676.5	24.784	3905834.75	2137.487	3932860.5	62.094
1 000		4709484	25.889	4695182.75	2295.268	4729960.25	66.164
		4823125	25.543	4770742.25	2255.058	4817326.5	65.346
		4780853.5	25.303	4759938.75	2269.823	4798746.75	66.744
		5043940.25	25.419	5000244.75	2276.391	5037509.75	65.716
		4868213.25	25.643	4800091.75	2268.094	4868999.5	65.406
1 500		10956394.25	36.948	10856001.75	3266.125	10908603	88.450
		10950683.75	37.552	10854738.5	3209.933	10935584.25	98.699
		10998562.25	36.251	10913708.5	3217.297	10983254	98.128
		11752364	35.127	11720364	3277.125	11746520	97.555
		9125361	34.123	9012651	3310.128	9062154	96.332

6. Conclusion

In this paper we solved problem Q which NP-heard on single machine with fuzzy due date by using local search methods to get approximation solution and a compare between ant colony optimization algorithms(ACO), the bees colony optimizationalgorithm(BA) and particle swarm optimization algorithm(PSO) form whereAccuracy value of objective function and time of processing in coding of matlab.

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