Mean Waiting time Analysis on Bukl Arrival Fuzzy Queueing System Using Maximum Likelihood Estimation

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Abstract: Consider the Erlang k-phase queueing system on fuzzy environment and to obtain the optimal value of mean waiting time for the concern system. In this regard, here we proposed new algorithm, using alpha - cut method to construct the different value in the range of uncertain and analyze the system performance level by the MINLP method and discussed a numerical example.

Keywords: Alpha cut, Bulk Queue, State Diagram, Fuzzy membership function.

1. Introduction

The present world is very highly competitive. Due to the situation decision maker decisions are uncertain. In particular, consider industrial people they would expect to deliver the product as on time. But some time not possible to do, it's based on the delay parameters (Service, whether, Cost etc.). Consider a general queueing system, we can provide the service on time the level of queue cannot be increased otherwise not possible, because service time taken in more than the expected service time the entire system will collapse. So, we rectify the above problem by fuzzy environment and the decision parameters are considered TFN's. In this case, we estimate the queue parameter and using a statistical procedure can help to overcome the above behavior or obtained the error of the system performance based on the existing data. In recent years, some authors did the work in statistical manner and also stated the results but did not occur the optimum level (system Performance). Generally, the basic queueing system has used for Markov processes. In the year 1957, Clarke has published in some sequence of papers on MLE for general queueing system in addition to the covariance matrix – variance in the statistics. Benes(1957) followed by the Clarke work and stated the results in special distribution on a queueing model in fuzzy environment.Li and Lee [5] have to analyze the principle of fuzzy queue infinite capacity queueing system. . Negi and Lee [6] simulated the two variable fuzzy queueing systems using an alpha cut method for $FM/FE^k/1/\infty$. Kao et al [4] constructed the membership functions and analyzed the system performance of the special methods as follows (i) $FM/M/1$ arrival parameters are uncertain (ii) $M/FM/1$ Uncertain service parameter (iii) $FM/FM/1$ both queue parameters are uncertain. Especially Buckley.J.J and Li.R.J [8,3,5] analyzed the queueing parameter via possibility theory and obtained the optimization levels in fuzzy numbers. Chen.S.P [1,2] analyzed the system performance level Fuzzy environment. Nakamura [7]] has developed nonlinear programs to calculate the range of the system and measured the possibility level of the system performance. In this paper, we proposed an algorithm and obtained system concert level with the help of using statistical approach.

2. Generalized Erlang K-phase service distribution

The entire system consists as on k – phase Erlang k phase distribution. We know that the phase of the service time is $k\mu$. Here $P_{n.i}(t)$ is probability of n^{th} customer in the i^{th} phase service channel system. The numbers of phases are considering the process in backward substitution method. Here, we provided the first service in the 'k^{th}' node. So the phase "1" is leaves last channel. The parameters are considered as $A(t) = \exp(-\lambda t)\lambda,\ t \in [0,\infty]$ & $B(t) = \frac{\exp(-k\mu t)\, t^{k-1} (\mu t)^k}{1.2.3.4.......(k-1)}$ .The mean and variance value of the distribution is $\frac{k\lambda}{\mu}$ & $\frac{k\lambda}{\mu^2}$ respectively.

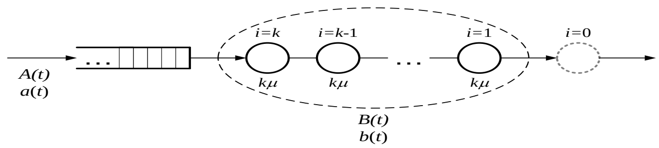


Figure 2.1 Flow Diagram

The probability of "n" service customer being served at the 'ith' phase stage is $P(n)=\sum_i P(n,i) \quad i=1,2,3..k$

Initially the kth service channel customer has been removed in the initial stage and the first service channel customer leaves the system at the end of the stage.

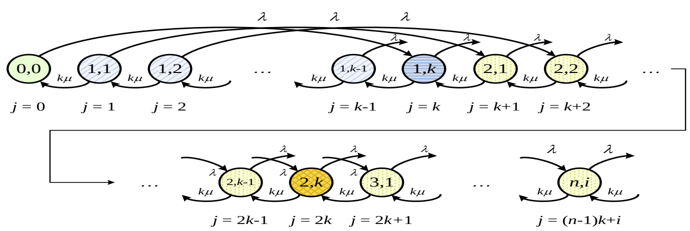


Figure 2.2 Service pattern of the system

$$P(n,i)(t+\Delta t)=P(n,i)(1-\lambda\Delta t-\mu k\Delta t)+P(n,i+1)(t)(\mu k\Delta t)+P(n-1,i)(t)(\lambda\Delta t), where\ i\in[1,k]$$
$$P(n,k)(t+\Delta t)=P(n,k)(1-\lambda\Delta t-\mu k\Delta t)+P(n,k+1)(t)(\mu k\Delta t)+P(n-1,k)(t)(\lambda\Delta t), where\ i\in[1,k]$$

If $n>0\ \ for\ i\in[1,k]$ **then we get** $k\mu P(1,1)=\lambda P(0,0)$

If $n=1\ \ for\ i\in[1,k-1]$

then we get $k\mu P(1,i+1)=(\lambda+k\mu)\ P(1,i)$ **and** $k\mu P(2,1)+\lambda P(0)=(\lambda+k\mu)\ P(1,k)$

If $n=2\ \ for\ i\in[1,k-1]$

$$\lambda P(n-1,i)+\mu k P(n,i+1)=(k\mu+\lambda)P(n,i)\ \ \&\lambda P(n-1,k)+\mu k P(n+1,1)=(k\mu+\lambda)P(n,k)$$

The relation about the nth customer service channel is

$G(z)=\sum_n\sum_i P(n,i)z^{k(n-1)+i}+P(0), 1<n<\infty, 1<i<k$ and obtain the cumulative value of the total phase is

$\dfrac{(1-z)P(0)}{Z^{k+1}r-(r+1)z+1}$. The queue waiting time this system is $E(Nq)=\sum_n\sum_i P(n,i)\left[\dfrac{(n-1)k+i}{\mu k}\right]$

The probability on nth customer in the ith phase of the system is $P(n)=\sum_{j=(n-1)k+1}^{nk} P_j^{(p)}$.

The balance equation of $P_j^{(p)}=\begin{Bmatrix} k\mu P(1)=\lambda P(0) \\ \lambda P(n-k)+\mu k P(n+1)=(\mu k+\lambda)P(n) \end{Bmatrix}$,

therefore the expected queue waiting time is $E(wq)=\dfrac{\rho(k+1)}{2k(1-\rho)\mu}$

3. Fuzzy Queues with k-phase infinite capacity

Here we consider two cases, First one is time axis it may be decaying in two sequences because the system is busy n (t)>0 or its free n(t)=0. Consider the interval time $t_2>t_1$ and system parameter is independent. Now, we discussed only in busy periods. So, we fixed the some pre-assigned value, the initial queue size is $n(0) = \tau$, 'm' denotes the total number of servers, T be period of server, n be the arrival of T. The given system is balanced ($\rho < 1$) and τ taken to the Erlang distribution with the ratio value of $x_i, y_i \; \& \; z_i \;\; i = 0,1,2....$. If $y_i = \sup[y_{i-1}, x_{i-1}] + [z_i - z_{i-1}], \;\; i = 0$, here $x_i \; \& \; z_i$ is the transition time of queue parameter, Using likelihood function L as follows;

(i) The index of the maximum value consider as $z_m \le \tau$, here 'm' is the function of z and τ is the independent of 'm'. The frequency function considers Erlang distribution as follows: $B(t) = \dfrac{\exp(-k\mu t)\, t^{k-1} \, (\mu t)^k}{1.2.3.4.......(k-1)}$

(ii) τ has a Erlang distribution with the frequency function $(1-\rho)\,\rho^k \;\; k = 0,1,2....$

(iii) z_i is the conditional distribution of given 'm'. The random subdivision of the fixed interval length τ into m+1 parts and it's fixed.

(iv) Depends on time the number of arrival is $(n-m+\tau)$ and the frequency domain of Erlang distribution is $\dfrac{\left[k\lambda(T - x_{m-\tau})\right]^{n-m+\tau}}{(n-m+\tau)!} e^{-k\lambda(\tau - z_{m-\tau})}, \; n-m+\tau = 0,1,2...$.

(v) If $m > \tau$, then the likelihood function value from (ii) is $L = (1-\rho) e^{-(k\mu r + \lambda r)} (k\mu)^{m-r} \lambda^{n+r} \theta$, where θ not depends for the basic queue parameters.

(vi) Obtained the MLE of $\bar{\lambda}$ and $\bar{\mu}$ is

$\lambda^1 = \begin{cases} (n+\tau-\lambda T)(k\,\mu-\lambda) & \lambda > \mu \\ (n+\tau-k\mu T)(k\,\mu-\lambda) & \lambda < \mu \end{cases}$ and eliminating the service parameter we obtain the MLE of ρ in the quadratic equation.

$f(\rho') = (m-\tau-1)T\,\rho'^2 - \left[(m-\tau)T + (n+\tau+1)r\right]\rho' + (n+\tau)\,r = 0$.

(vii) We estimate the unique solution in the interval $\rho' = (0,1)$. Since the following condition is exists $f(0) = (n+\tau)r > 0, \; f(1) = -\tau - T < 0$.

(viii) Obtained two roots, One root is unit and the other root is $\rho_1 = \dfrac{(n+\tau)r}{(m-\tau)T}$ and $0 < \rho_1 - \rho' < \dfrac{2\rho_1}{(1-\rho_1)(m-\tau)}$

(ix) we get the bounded value of ρ_1 because it provides a better approximation value and 'm' get larger than the.

(x) Using back substation method we obtained the MLE value of queue parameters. But $\lambda' = \dfrac{(n+m)\rho'}{\rho' T + r}$ and $\mu' = \dfrac{(n+m)}{\rho' T + r}$.

(xi) The result of $\lambda' \approx \dfrac{n+\tau}{T}$ and $\mu' \approx \dfrac{m-\tau}{T}$

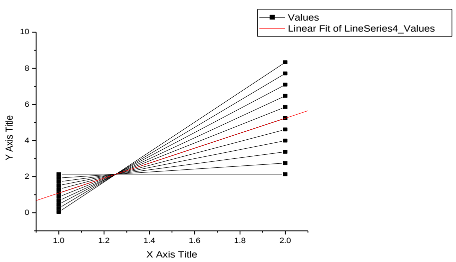
4. Numerical Example

A three engine airline maintenance base schedules the service providing one by one. Hence, to return the airplanes into use at the earliest, the policy is to stumble the overhaul. Under this policy, arrival and service parameter consider in Poisson and Erlang distribution. The parameter $\bar{\lambda}$ =[1,4,5] and $\bar{\mu}$ =[2,5,7] per hour, respectively. Find all the waiting time character. $\overline{L_q} = \left[L^L_{q_\alpha}, L^U_{q_\alpha}\right]$= [2.089 α +0.045, 8.338-6.204 α] & $\overline{W_q} = \left[W^L_{q_\alpha}, W^U_{q_\alpha}\right]$ = [0.4866 α +0.0474, 1.668-1.134 α],

Table: 1 Range of Uncertainty Value:

α	L_q^L	L_q^U	L_q	W_q^L	W_q^U	W_q
0	0.045	8.338	4.1915	0.0474	1.668	0.8577
0.1	0.2539	7.7176	3.9857	0.0961	1.5546	0.8253
0.2	0.4629	7.0972	3.7800	0.1447	1.4412	0.7929
0.3	0.6717	6.4768	3.5742	0.1934	1.3278	0.7606
0.4	0.8806	5.8564	3.3685	0.2420	1.2144	0.7282
0.5	1.0895	5.236	3.1627	0.2907	1.101	0.6958
0.6	1.2984	4.6156	2.957	0.3394	0.9876	0.6635
0.7	1.5073	3.9952	2.7512	0.3880	0.8742	0.6311
0.8	1.7162	3.3748	2.5455	0.4367	0.7608	0.5987
0.9	1.9251	2.7544	2.3397	0.4853	0.6474	0.5663
1	2.134	2.134	2.13	0.534	0.534	0.534

Using Origin software from table (i), we occurred the error value by statistical approach and also obtained the curve linear value of Ls and Wq in Fig(i) , Fig(ii) and Table(ii)



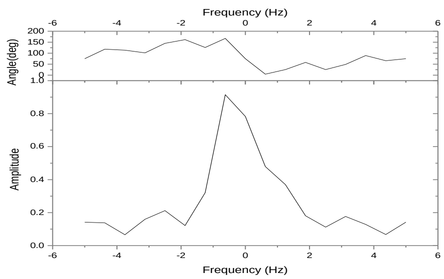


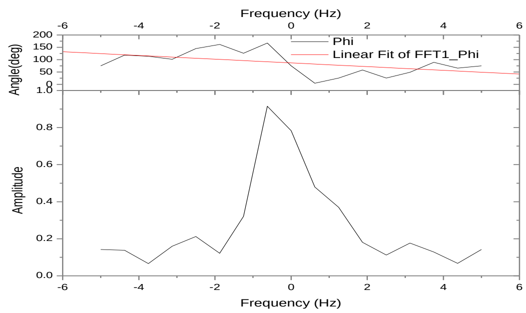
Table (ii) – Statistical Data of $\overline{L_q}$:

(x)	E(y)	V(y)	Error V(y)	L(y)	U(y)	Range(y)	N
1	0.8577	1.1459 4	0.8103	0.0474	1.668	1.6206	2
2	0.8253	1.0313	0.7295	0.0961	1.5546	1.4585	2
3	0.7929	0.9167	0.6482	0.1447	1.4412	1.2965	2
4	0.7606	0.8021	0.5672	0.1934	1.3278	1.1344	2
5	0.7282	0.6875	0.4862	0.242	1.2144	0.9724	2
6	0.6958	0.5927	0.4051	0.2907	1.101	0.8103	2
7	0.6635	0.4583	0.3241	0.3394	0.9876	0.6482	2
8	0.6311	0.3428	0.2431	0.388	0.8742	0.4862	2
9	0.5987	0.2291	0.1620	0.4367	0.7608	0.3241	2
10	0.5663	0.1146	0.0810	0.4853	0.6474	0.1621	2
11	0.534	0	0	0.534	0.534	0	2

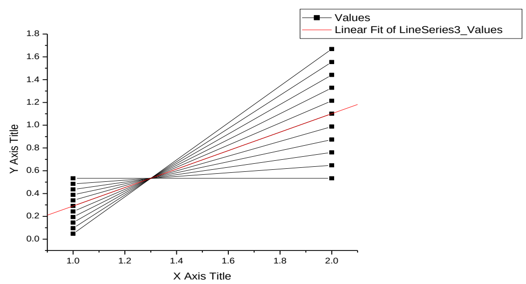
From table (1) represented as the $\overline{L_q}$ error value

P	Value	Er($\overline{L_q}$)	R($\overline{L_q}$)	Squ (V(y))
A	-0.5012	0.1226	0.6245	0.254
B	0.9032	0.1043		

Using Origin software from table (i), we occurred the error value by statistical approach and also obtained the curve linear value Wq in Fig(i) , Fig(iii) and Table(ivi)



Fig(iii) Frequency level (Wq)



Fig(iv) Cureve Regression value

(x)	E(y)	V(y)	Error V(y)	L(y)	U(y)	Range(y)	N
1	4.1915	5.8640	4.1465	0.045	8.338	8.293	2
2	3.9857	5.2776	3.7318	0.2539	7.7176	7.4637	2
3	3.7800	4.6911	3.3171	0.4629	7.0972	6.6343	2
4	3.5742	4.1048	2.9025	0.6717	6.4768	5.8051	2
5	3.3685	3.5182	2.4879	0.8806	5.8564	4.9758	2
6	3.1625	2.9320	2.07325	1.0895	5.236	4.1465	2
7	2.957	2.3456	1.6586	1.2984	4.6156	3.3172	2
8	2.7512	1.7592	1.2439	1.5073	3.9952	2.4879	2
9	2.54551	1.1728	0.8293	1.7162	3.3748	1.6586	2
10	2.3397	0.5864	0.4146	1.9251	2.7544	0.8293	2
11	2.134	0	0	2.134	2.134	0	2

From table (i) from table (1) represented as the $\overline{L_q}$ error value

Parameter	Value	Error	Range	SD
A	-3.0569	1.0350	0.8169	1.5352
B	4.1464	0.6546		

5. Conclusion

The system considered as bulk arrival queueing system on fuzzy environment and system performance level can be obtained in different level. Finally, we concluded that the optimum queue length are exist: (i) If α =0 then $\overline{L_q}$ =[0.045,6.238] and α =1 then $\overline{L_q}$ 2.134 and (ii) If α =0 then $\overline{W_q}$ =[0.0474, 1.458] and α =1 the range of α is 0.354. The curve linear value $\overline{L_q}$ is A= 0.1342 & B= 0.1043 and $\overline{W_q}$ is A= 1.0265 & B = 0.5646. The error range of $\overline{L_q}$ & $\overline{W_q}$ is 0.6238 & 0.7196.In our proposed method easily obtained the error value and it's helpful to simulate the system performance level.

References

1. Chen.S.P. "Parametric Nonlinear Programming Approach to Fuzzy Queues with bulk service".European Journal of Operational Research, Volume 163 (2005): pp: 434–444.
2. Chen, S. P. "A Bulk Arrival Queuing Model with Fuzzy Parameters and Varying Batch Sizes",Applied Mathematical Modeling. Volume 30 (2006): pp: 920–929.
3. De Cooman, G. "A behavioral model for vague probability assessments", Fuzzy Sets and Systems, Volume 154 (2005): pp: 305–358.
4. Kao.C, Li.C. C., and Chen, S. P., 1999, "Parametric programming to the analysis of fuzzy queues," Fuzzy Sets and Systems, Volume 107(1999): pp: 93-100.
5. Li.R.J and Lee.E.S. "Analysis of fuzzy queues", Computers and Mathematics with Applications, Volume 17 (1989): pp: 1143–1147.
6. Negi,D.S and Lee.E.S 1992, Analysis and simulation of fuzzy queues, Fuzzy Sets and Systems, (46), 321–330.
7. Nakamura K. "Preference relations on a set of fuzzy Utilities as a basis for decision making", Fuzzy Sets and Systems, Volume 20 (1986): pp: 147–162.
8. Stanford.R.E 1982, The set of limiting distributions for a Markov chain with fuzzy transition probabilities, Fuzzy Sets and Systems,(7),71–78